Basic Physics 1 Lecture Module

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Basic Physics 1
Lecture Module

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Measurements, Units, Dimension

Measurements, Units, Dimension

- Measurements
- Quantities
- SI units
- Unit conversion
- Dimension

Measurements

- To be quantitative in Physics requires measurements
- How tall is Ming Yao? How about his weight?
  - Height: 2.29 m (7 ft 6 in)
  - Weight: 141 kg (310 lb)

- Number + Unit
  - "thickness is 10." has no physical meaning
  - Both numbers and units necessary for any meaningful physical quantities

Type Quantities

- Many things can be measured: distance, speed, energy, time, force ……
- These are related to one another: speed = distance / time
- Choose three basic quantities (DIMENSIONS):
  - LENGTH
  - MASS
  - TIME
- Define other units in terms of these.

SI Unit for 3 Basic Quantities

- Units in physics equations must always be consistent. Converting units is a matter of multiplying the given quantity by a fraction, with one unit in the numerator and its equivalent in the other units in the denominator, arranged so the unwanted units in the given quantity are cancelled out in favor of the desired units.

- In 1960, standards bodies control and define Système Internationale (SI) unit for 3 main basic quantities as
  - LENGTH: Meter
  - MASS: Kilogram
  - TIME: Second

Fundamental Quantities and SI Units

Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Luminous Intensity	candela	cd
Amount of Substance	mole	mol

SI Length Unit: Meter

- French Revolution Definition, 1792
- 1 Meter = XY/10,000,000
- 1 Meter = about 3.28 ft
- 1 km = 1000 m, 1 cm = 1/100 m, 1 mm = 1/1000 m
- Current Definition of 1 Meter: the distance traveled by light in vacuum during a time of 1/299,792,458 second.

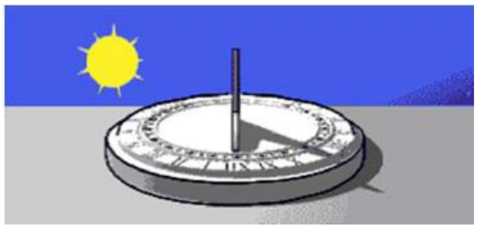
distance XY = 1 quadrant of the Earth

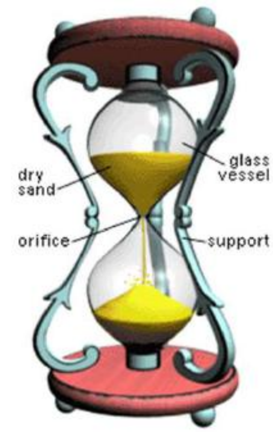
Values of Length

Approximate Values of Some Measured Lengths

	Length (m)
Distance from Earth to most remote known quasar	1×10^{26}
Distance from Earth to most remote known normal galaxies	4×10^{25}
Distance from Earth to nearest large galaxy (M31, the Andromeda galaxy)	2×10^{22}
Distance from Earth to nearest star (Proxima Centauri)	4×10^{16}
One light year	9×10^{15}
Mean orbit radius of Earth about Sun	2×10^{11}
Mean distance from Earth to Moon	4×10^{8}
Mean radius of Earth	6×10^{6}
Typical altitude of satellite orbiting Earth	2×10^{5}
Length of football field	9×10^{1}
Length of housefly	5×10^{-3}
Size of smallest dust particles	1×10^{-4}
Size of cells in most living organisms	1×10^{-5}
Diameter of hydrogen atom	1×10^{-10}
Diameter of atomic nucleus	1×10^{-14}
Diameter of proton	1×10^{-15}

SI Time Unit: Second





- 1 Second is defined in terms of an “atomic clock”– time taken for 9,192,631,770 oscillations of the light emitted by a ^{133}Cs atom.
- Defining units precisely is a science (important, for example, for GPS):
  - This clock will neither gain nor lose a second in 20 million years.

Values of Time Intervals

Approximate Values of Some Time Intervals

	Time Interval (s)
Age of Universe	5×10^{17}
Age of Earth	1×10^{17}
Average age of college student	6×10^{8}
One year	3×10^{7}
One day	9×10^{4}
Time between normal heartbeats	8×10^{-1}
Period[a] of audible sound waves	1×10^{-3}
Period[a] of typical radio waves	1×10^{-6}
Period[a] of vibration of atom in solid	1×10^{-13}
Period[a] of visible light waves	2×10^{-15}
Duration of nuclear collision	1×10^{-22}
Time required for light to travel across a proton	3×10^{-24}

[a]A *period* is defined as the time required for one complete vibration.

SI Mass Unit: Kilogram

- 1 Kilogram – the mass of a specific platinum-iridium alloy kept at International Bureau of Weights and Measures near Paris.

- Copies are kept in many other countries.

- Yao Ming is 141 kg, equivalent to weight of 141 pieces of the alloy cylinder.

Values of Masses

Approximate Values of Some Masses

	Mass (kg)
Observable Universe	1×10^{52}
Milky Way galaxy	7×10^{41}
Sun	2×10^{30}
Earth	6×10^{24}
Moon	7×10^{22}
Shark	1×10^{2}
Human	7×10^{1}
Frog	1×10^{-1}
Mosquito	1×10^{-5}
Bacterium	1×10^{-15}
Hydrogen atom	2×10^{-27}
Electron	9×10^{-31}

Prefixes for SI Units

- 3,000 m = 3 $\times$ 1,000 m
  = 3 $\times$ 10^3 m = 3 km
- 1,000,000,000 = 10^9 = 1G
- 1,000,000 = 10^6 = 1M
- 1,000 = 10^3 = 1k
- 141 kg = 141,000 g
- 1 GB = 1,000 MB
- 4 MB = 4,000 KB

If you are rusty with scientific notation, see appendix B.1 of the text

10^x	Prefix	Symbol
x=18	exa	E
15	peta	P
12	tera	T
9	giga	G
6	mega	M
3	kilo	k
2	hecto	h
1	deca	da

Prefixes for SI Units

10^x	Prefix	Symbol
x=-1	deci	d
-2	centi	c
-3	milli	m
-6	micro	µ
-9	nano	n
-12	pico	p
-15	femto	f
-18	atto	a

- 0.003 s = 3 × 0.001 s = 3 × 10^{-3} s = 3 ms
- 0.01 = 10^{-2} = centi
- 0.001 = 10^{-3} = milli
- 0.000 001 = 10^{-6} = micro
- 0.000 000 001 = 10^{-9} = nano
- 0.000 000 000 001 = 10^{-12} = pico = p
- 1 nm = ? m = ? cm
- 3 cm = ? m = ? mm

Derived Quantities and Units

- Multiply and divide units just like numbers
- Derived quantities: area, speed, volume, density ……
  - Area = Length × Length SI unit for area = m^2
  - Volume = Length × Length × Length SI unit for volume = m^3
  - Speed = Length / time SI unit for speed = m/s
  - Density = Mass / Volume SI unit for density = kg/m^3

- In 2008 Olympic Game, Usain Bolt sets world record at 9.69 s in Men's 100 m Final. What is his average speed ?

$$\text{speed} = \frac{100\ \text{m}}{9.69\ \text{s}} = \frac{100}{9.69} \cdot \frac{\text{m}}{\text{s}} = 10.32\ \text{m/s}$$

Other Unit System

- U.S. customary system: foot, slug, second
- Cgs system: cm, gram, second
- We will use SI units in this course, but it is useful to know conversions between systems.
  – 1 mile = 1609 m = 1.609 km   1 ft = 0.3048 m = 30.48 cm
  – 1 m = 39.37 in. = 3.281 ft   1 in. = 0.0254 m = 2.54 cm
  – 1 lb = 0.465 kg   1 oz = 28.35 g   1 slug = 14.59 kg
  – 1 day = 24 hours = 24 * 60 minutes = 24 * 60 * 60 seconds

  – More can be found in Appendices A & D in your textbook.

Unit Conversion

Example: On the garden state parkway of Jakarta, a car is traveling at a speed of 38.0 m/s. Is the driver exceeding the speed limit (if the limit is 80 mph)?

Since the speed limit is in miles/hour (mph), we need to convert the units of m/s to mph. Take it in two steps.

- Step 1: Convert m to miles. Since 1 mile = 1609 m, we have two possible conversion factors, 1 mile/1609 m = $6.215\text{x}10^{-4}$ mile/m, or 1609 m/1 mile = 1609 m/mile. What are the units of these conversion factors? Since we want to convert m to mile, we want the m units to cancel => multiply by first factor:
- Step 2: Convert second (s) to hour (hr). Since 1 hr = 3600 s, again we could have 1 hr/3600 s = $2.778\text{x}10^{-4}$ hr/s, or 3600 s/hr. Since we want to convert s to hr, we want the s units to cancel =>

$$38.0\ \text{m/s} = 2.36\times10^{-2}\ \frac{\text{mile}}{\text{s}}\cdot\frac{3600\ \text{s}}{\text{hr}} = 85.0\ \text{mile/hr} = 85.0\ \text{mph}$$

Dimension

- "Dimension" is characteristic of the object, condition, or event and is described quantitatively in terms of defined "units".

- A physical quantity is equal to the product of two elements:
  - A quality or dimension
  - A quantity expressed in terms of "units"

- Physical things are measurable in terms of three primitive qualities (Maxwell 1871)
    - Mass (M)
    - Length (L)
    - Time (T)

- Quantities have dimensions:
  - Length – L, Mass – M, and Time - T

- Quantities have units: Length – m, Mass – kg, Time – s

- To refer to the dimension of a quantity, use square brackets, e.g. $[F]$ means dimensions of force.

- *Temperature, electrical charge, chemical quantity, and luminosity were added as "primitives" some years later (after Maxwell 1871).*

Quantity	Area	Volume	Speed	Acceleration
Dimension	$[A]$ = L^2	$[V]$ = L^3	$[v]$ = L/T	$[a]$ = L/T^2
SI Units	m^2	m^3	m/s	m/s^2

Dimensional Analysis

- Necessary either to derive a math expression, or equation or to check its correctness.
- Quantities can be added/subtracted only if they have the same dimensions.
- The terms of both sides of an equation must have the same dimensions.
- The three fundamental physical dimensions of mechanics are length, mass and time, which in the SI system have the units meter (m), kilogram (kg), and second (s), respectively
- The method of dimensional analysis is very powerful in solving physics problems.
- Please analyze these !
  - a, b, and c have units of meters, $s = a$, what is $[s]$?
  - a, b, and c have units of meters, $s = a + b$, what is $[s]$?
  - a, b, and c have units of meters, $s = (2a + b)b$, what is $[s]$?
  - a, b, and c have units of meters, $s = (a + b)^3/c$, what is $[s]$?
  - a, b, and c have units of meters, $s = (3a + 4b)^{1/2}/9c^2$, what is $[s]$?

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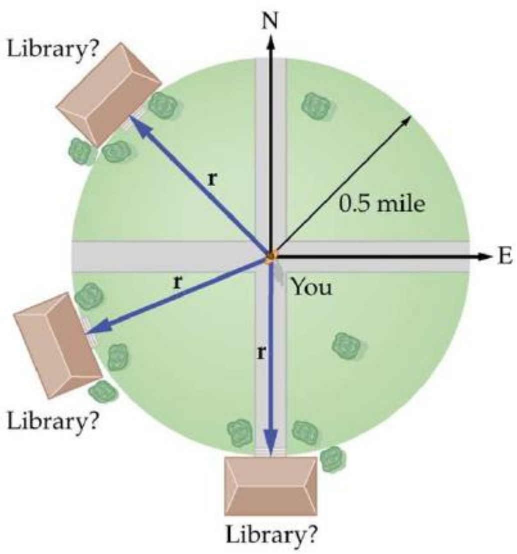
Vectors

Vectors

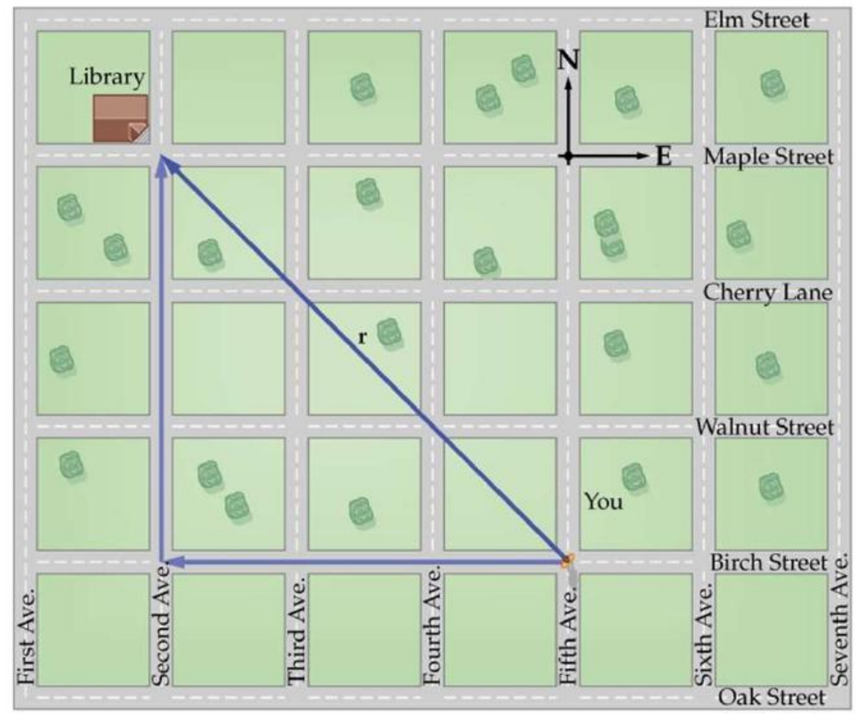
- Vectors and scalars
- Describe vectors geometrically
- Components of vectors
- Unit vectors
- Vectors addition and subtraction
- Scalar product of two vectors
- Cross product of two vectors
- The Properties of Vector Products

Vector vs. Scalar Review

A library is located 0.5 mi from you. Can you point where exactly it is?



You also need to know the direction in which you should walk to the library!



- All physical quantities encountered in this text will be either a scalar or a vector
- A **vector** quantity has both magnitude (value + unit) and direction
- A **scalar** is completely specified by only a magnitude (value + unit)

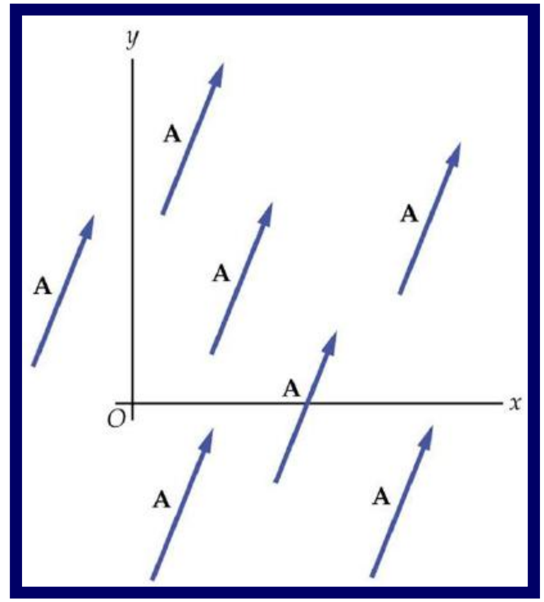
Vector and Scalar Quantities

- **Vectors**
  - Displacement
  - Velocity (magnitude and direction!)
  - Acceleration
  - Force
  - Momentum
- **Scalars:**
  - Distance
  - Speed (magnitude of velocity)
  - Temperature
  - Mass
  - Energy
  - Time

To describe a vector we need more information than to describe a scalar! Therefore vectors are more complex!

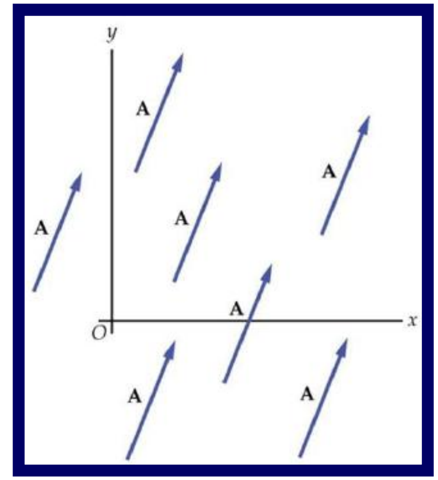
Important Notation

- To **describe vectors** we will use:
  - The bold font: Vector A is **A**
  - Or an **arrow** above the vector**:** $\vec{A}$
  - In the pictures, we will always show vectors as arrows
  - Arrows point the direction
  - To describe the magnitude of a vector we will use absolute value sign: $|\vec{A}|$ or just A,
  - Magnitude is always positive, the magnitude of a vector is equal to the length of a vector.



Properties of Vectors

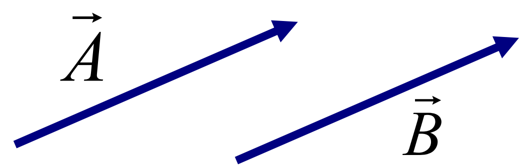
- Equality of Two Vectors
  - Two vectors are **equal** if they have the same magnitude and the same direction
- Movement of vectors in a diagram
  - Any vector can be moved parallel to itself without being affected



Negative Vectors

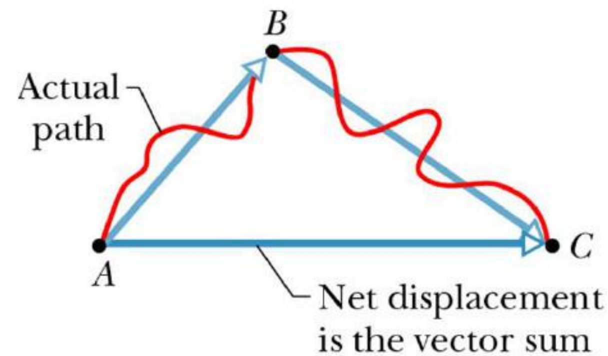
- Two vectors are **negative** if they have the same magnitude but are 180° apart (opposite directions)

$$\vec{\mathbf{A}} = -\vec{\mathbf{B}};\ \vec{\mathbf{A}} + \left(-\vec{\mathbf{A}}\right) = 0$$

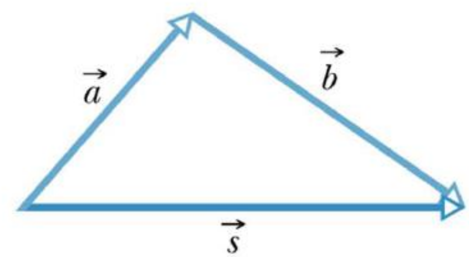


Adding Vectors

- When adding vectors, their directions must be taken into account
- Units must be the same
- Geometric Methods
  - Use scale drawings
- Algebraic Methods
  - More convenient



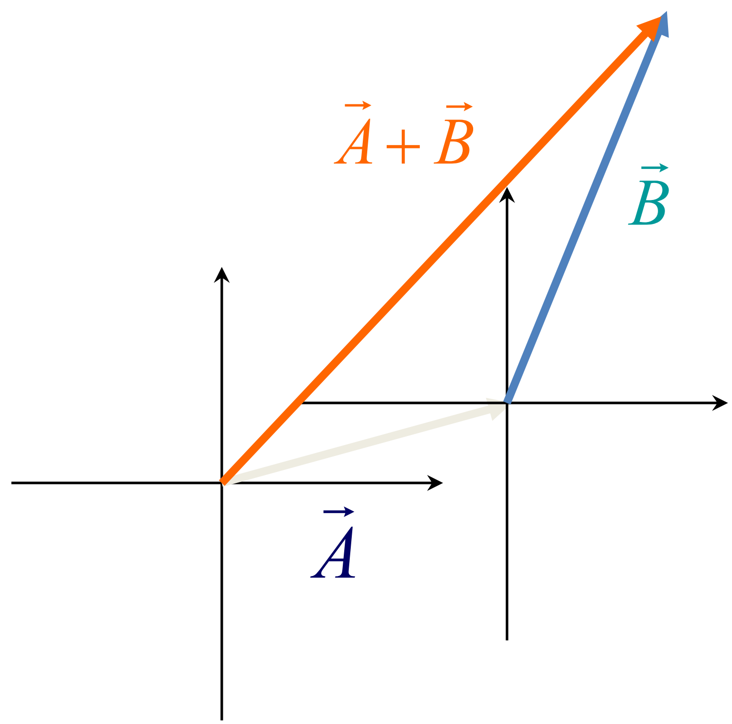
(a)



(b)

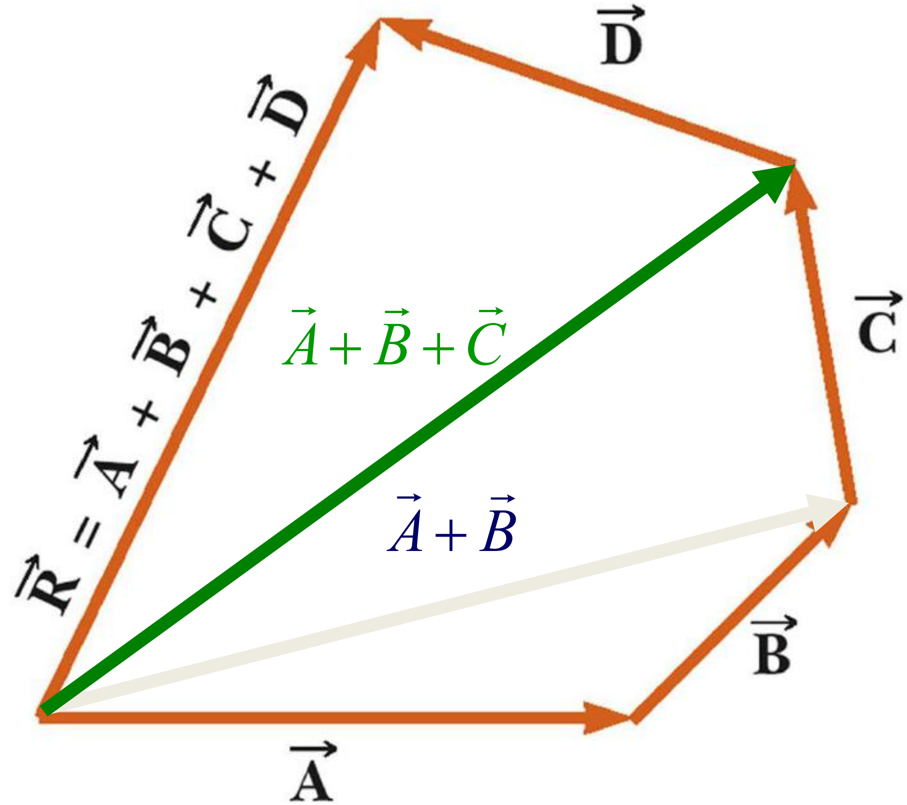
Adding Vectors Geometrically (Triangle Method)

- Draw the first vector $\vec{A}$ with the appropriate length and in the direction specified, with respect to a coordinate system
- Draw the next vector $\vec{B}$ with the appropriate length and in the direction specified, with respect to a coordinate system whose origin is the end of vector $\vec{A}$ and parallel to the coordinate system used for $\vec{A}$: "tip-to-tail".
- The resultant is drawn from the origin of $\vec{A}$ to the end of the last vector $\vec{B}$



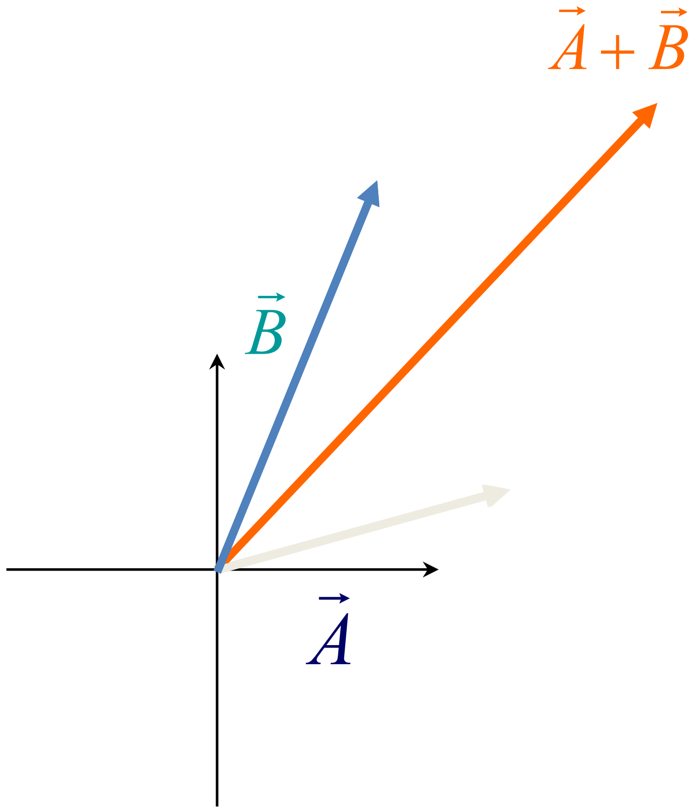
Adding Vectors Graphically

- When you have many vectors, just keep repeating the process until all are included
- The resultant is still drawn from the origin of the first vector to the end of the last vector



Adding Vectors Geometrically (Polygon Method)

- Draw the first vector $\vec{A}$ with the appropriate length and in the direction specified, with respect to a coordinate system
- Draw the next vector $\vec{B}$ with the appropriate length and in the direction specified, with respect to the same coordinate system
- Draw a parallelogram
- The resultant is drawn as a diagonal from the origin



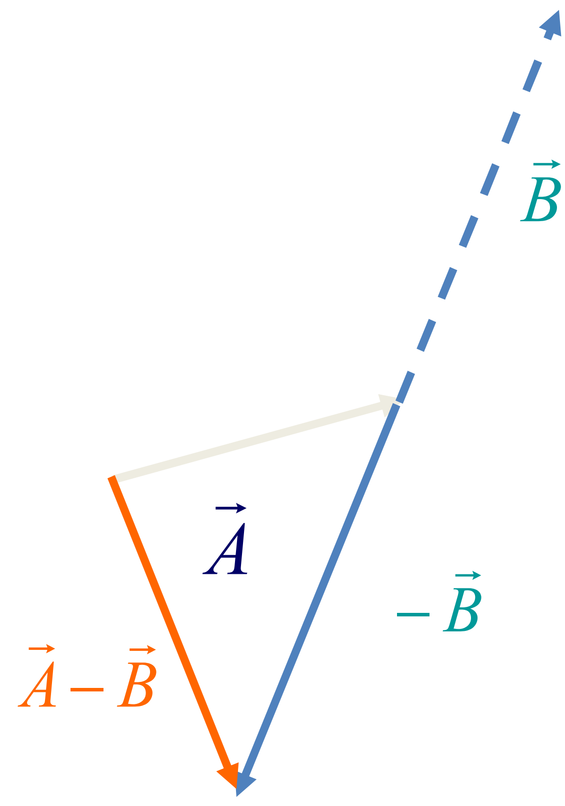
$$\vec{A}+\vec{B}=\vec{B}+\vec{A}$$

Vector Subtraction

- Special case of vector addition
  - Add the negative of the subtracted vector

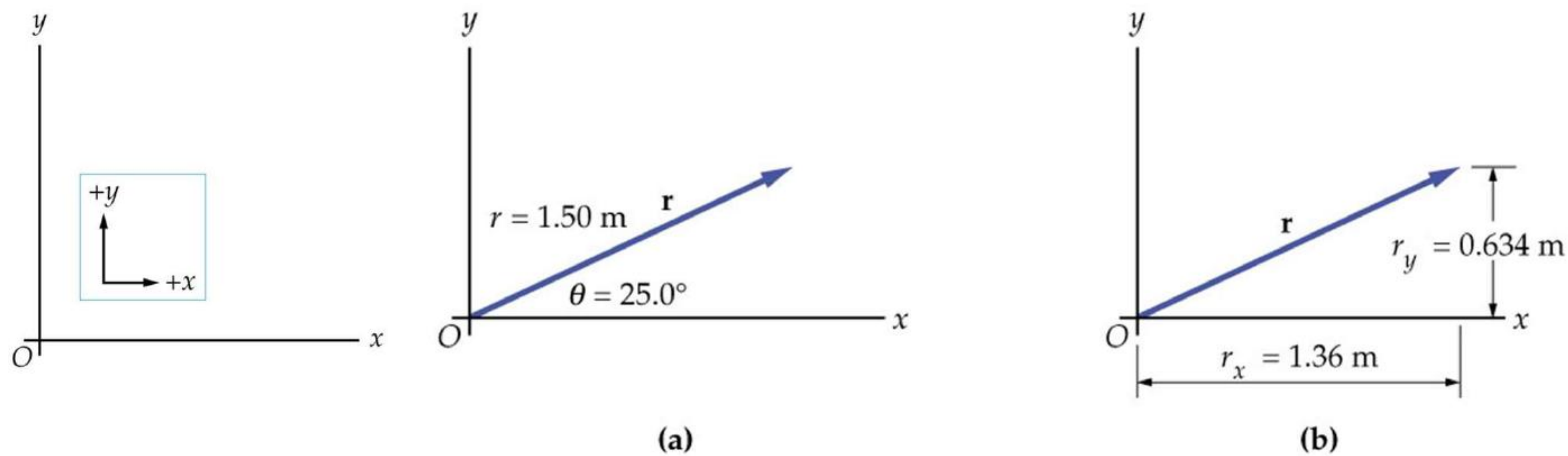
$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \vec{\mathbf{A}} + \left(-\vec{\mathbf{B}}\right)$$

- Continue with standard vector addition procedure



Describing Vectors Algebraically

Vectors: Described by the number, units and direction!

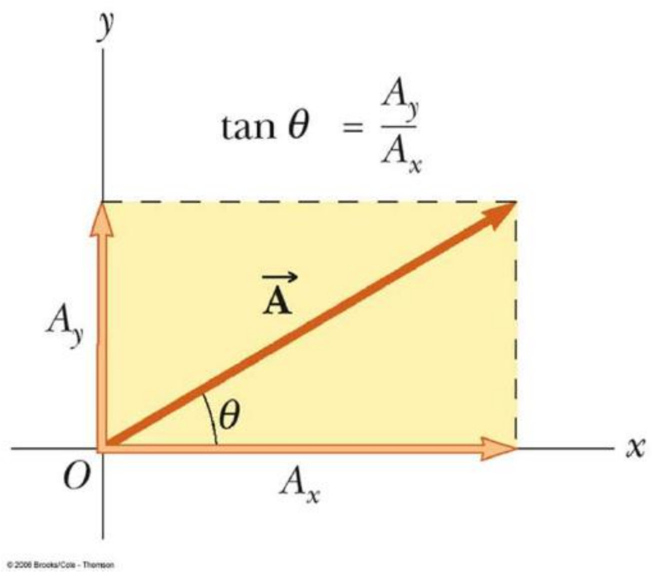


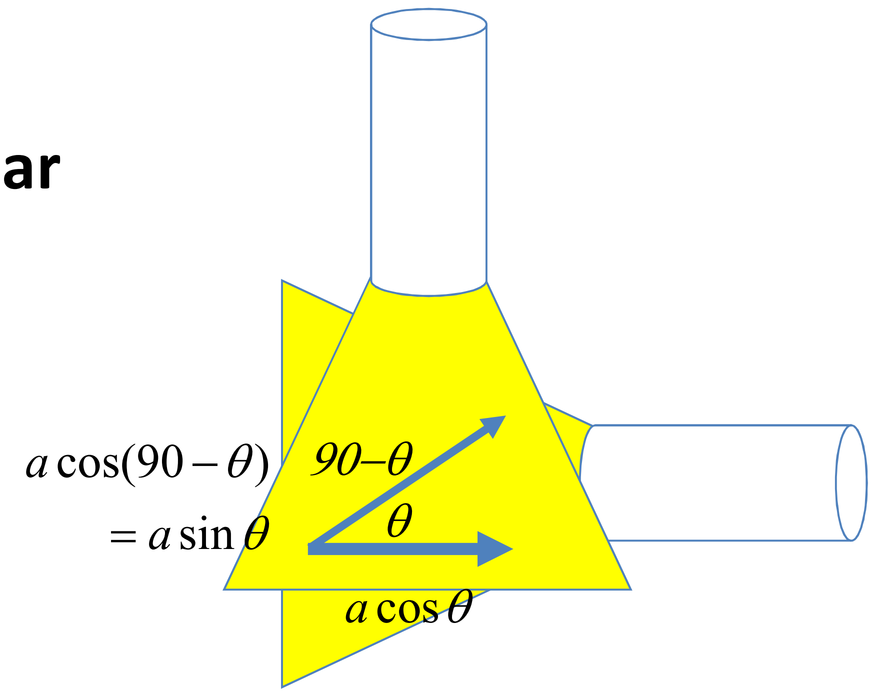
Vectors: Can be described by their magnitude and direction. For example: Your displacement is 1.5 m at an angle of 25^0.

Can be described by components? For example: your displacement is *1.36 m* in the positive x direction and *0.634 m* in the positive y direction.

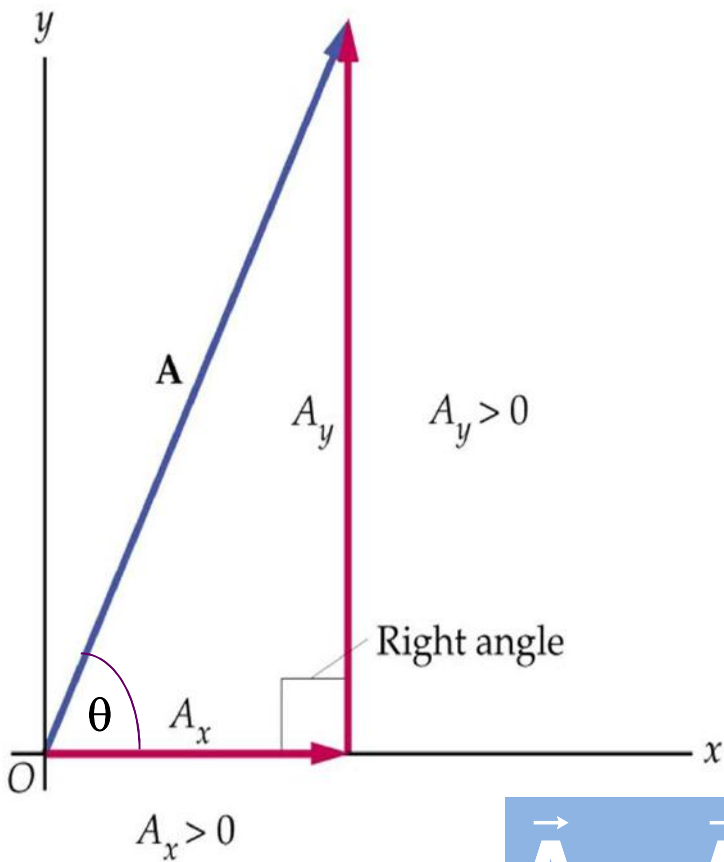
Components of a Vector

- A **component** is a part
- It is useful to use **rectangular components** These are the projections of the vector along the x- and y-axes





Components of a Vector



- The x-component of a vector is the projection along the x-axis

$$\cos\theta = \frac{A_x}{A} \qquad A_x = A\cos\theta$$

- The y-component of a vector is the projection along the y-axis

$$\sin\theta = \frac{A_y}{A} \qquad A_y = A\sin\theta$$

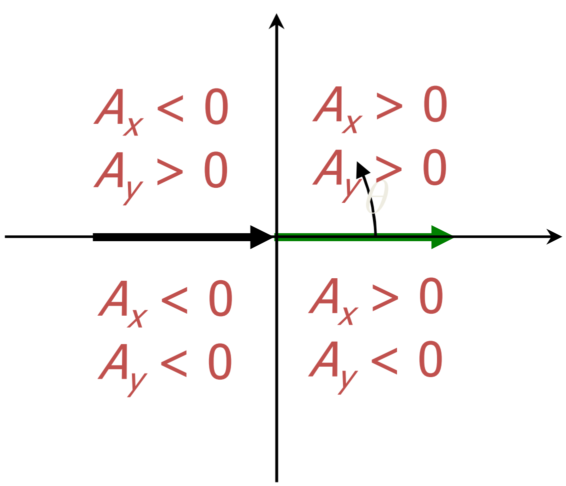
- Then,

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

$$\boxed{\vec{A} = \vec{A}_x + \vec{A}_y}$$

Components of a Vector

- The previous equations are valid ***only if ϑ is measured with respect to the x-axis***
- The components can be positive or negative and will have the same units as the original vector



$\theta=0,\ A_x=A>0,\ A_y=0$

$\theta=45^\circ,\ A_x=A\cos 45^\circ>0,\ A_y=A\sin 45^\circ>0$

$\theta=90^\circ,\ A_x=0,\ A_y=A>0$

$\theta=135^\circ,\ A_x=A\cos 135^\circ<0,\ A_y=A\sin 135^\circ>0$

$\theta=180^\circ,\ A_x=-A<0,\ A_y=0$

$\theta=225^\circ,\ A_x=A\cos 225^\circ<0,\ A_y=A\sin 225^\circ<0$

$\theta=270^\circ,\ A_x=0,\ A_y=-A<0$

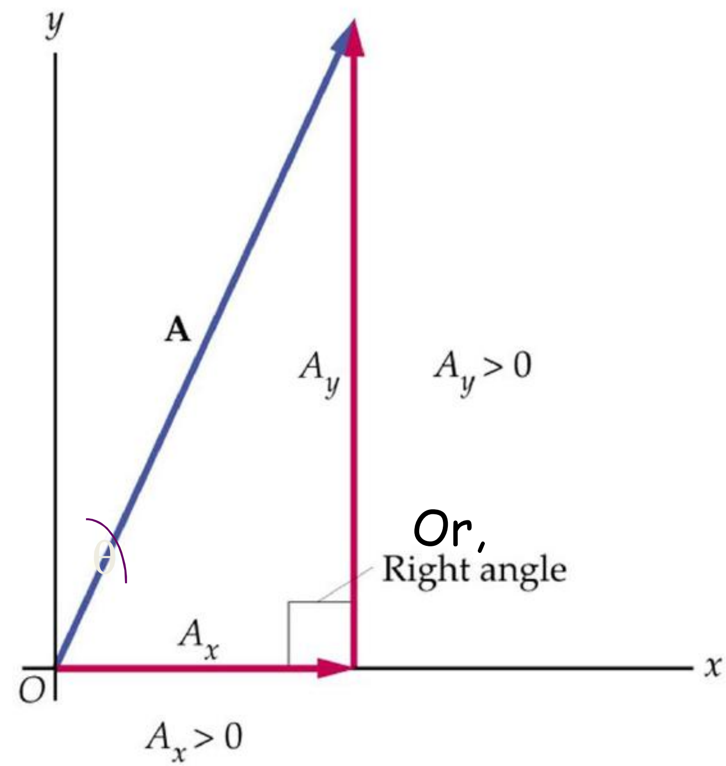
$\theta=315^\circ,\ A_x=A\cos 315^\circ<0,\ A_y=A\sin 315^\circ<0$

More About Components

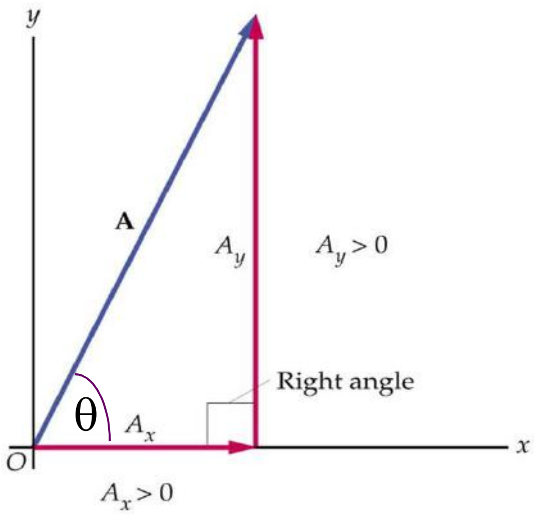
- The components are the legs of the right triangle whose hypotenuse is *A*

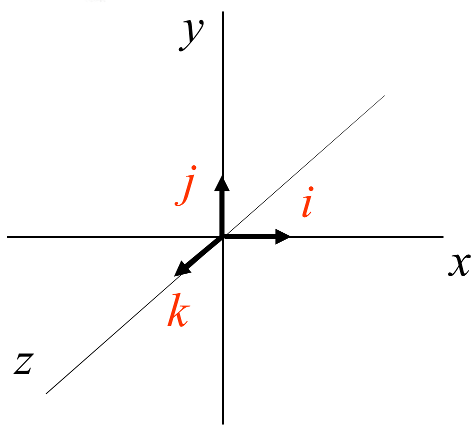
$$\begin{cases} A_x = A\cos(\theta) \\ A_y = A\sin(\theta) \end{cases}$$

$$\begin{cases} \left|\vec{A}\right| = \sqrt{(A_x)^2 + (A_y)^2} \\ \tan(\theta) = \dfrac{A_y}{A_x} \quad \text{or} \quad \theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right) \end{cases}$$



Unit Vectors





- Components of a vector are vectors

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

- Unit vectors i-hat, j-hat, k-hat

$$\hat{i} \rightarrow x \quad \hat{j} \rightarrow y \quad \hat{k} \rightarrow z$$

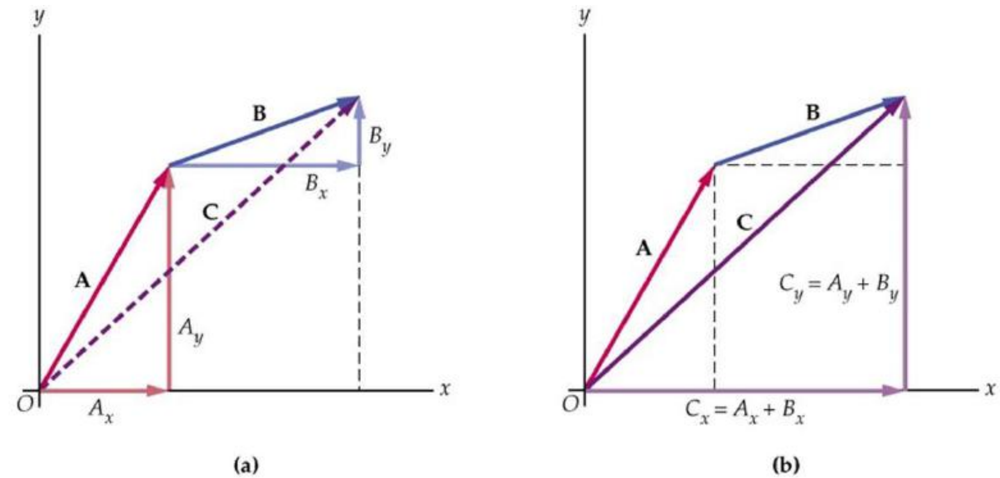
- Unit vectors used to specify direction
- Unit vectors have a magnitude of 1
- Then

$$\vec{A} = A_x \hat{i} + A_y \hat{j}$$

Magnitude + Sign

Unit vector

Adding Vectors Algebraically



- Consider two vectors

$$\vec{A} = A_x\hat{i} + A_y\hat{j}$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j}$$

- Then

$$\vec{A} + \vec{B} = (A_x\hat{i} + A_y\hat{j}) + (B_x\hat{i} + B_y\hat{j})$$

$$= (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

- If $\vec{C} = \vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$
- so $C_x = A_x + B_x \qquad C_y = A_y + B_y$

Example : Operations with Vectors

- Vector **A** is described algebraically as **(-3, 5),** while vector **B** is **(4, -2).** Find the value of **magnitude** and **direction** of the sum (**C**) of the vectors **A** and **B**.

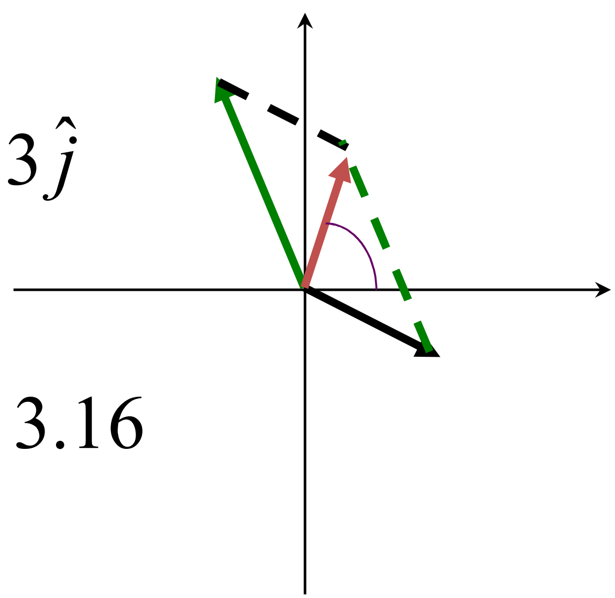
$$\vec{A} = -3\hat{i} + 5\hat{j} \qquad \vec{B} = 4\hat{i} - 2\hat{j}$$

$$\vec{C} = \vec{A} + \vec{B} = (-3+4)\hat{i} + (5-2)\hat{j} = 1\hat{i} + 3\hat{j}$$

$$C_x = 1 \qquad C_y = 3$$

$$C = (C_x{}^2 + C_y{}^2)^{1/2} = (1^2 + 3^2)^{1/2} = 3.16$$

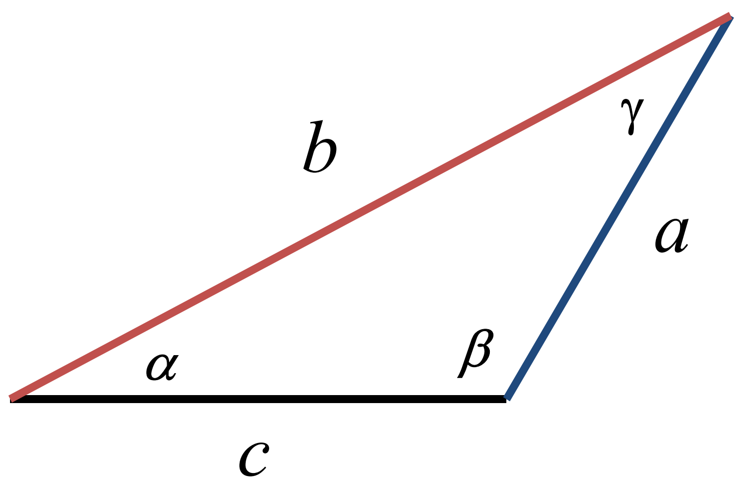
$$\theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1} 3 = 71.56^\circ$$



Law of Cosines

This will be used often in balancing forces

$$\alpha + \beta + \gamma = 180^\circ$$



$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

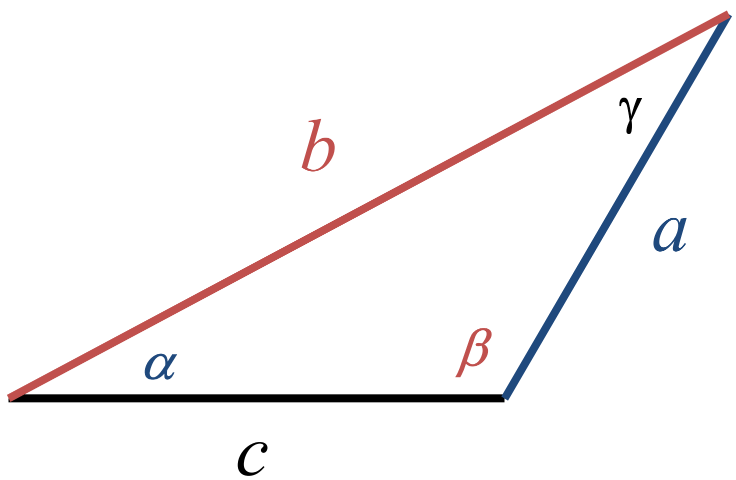
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Law of Sines

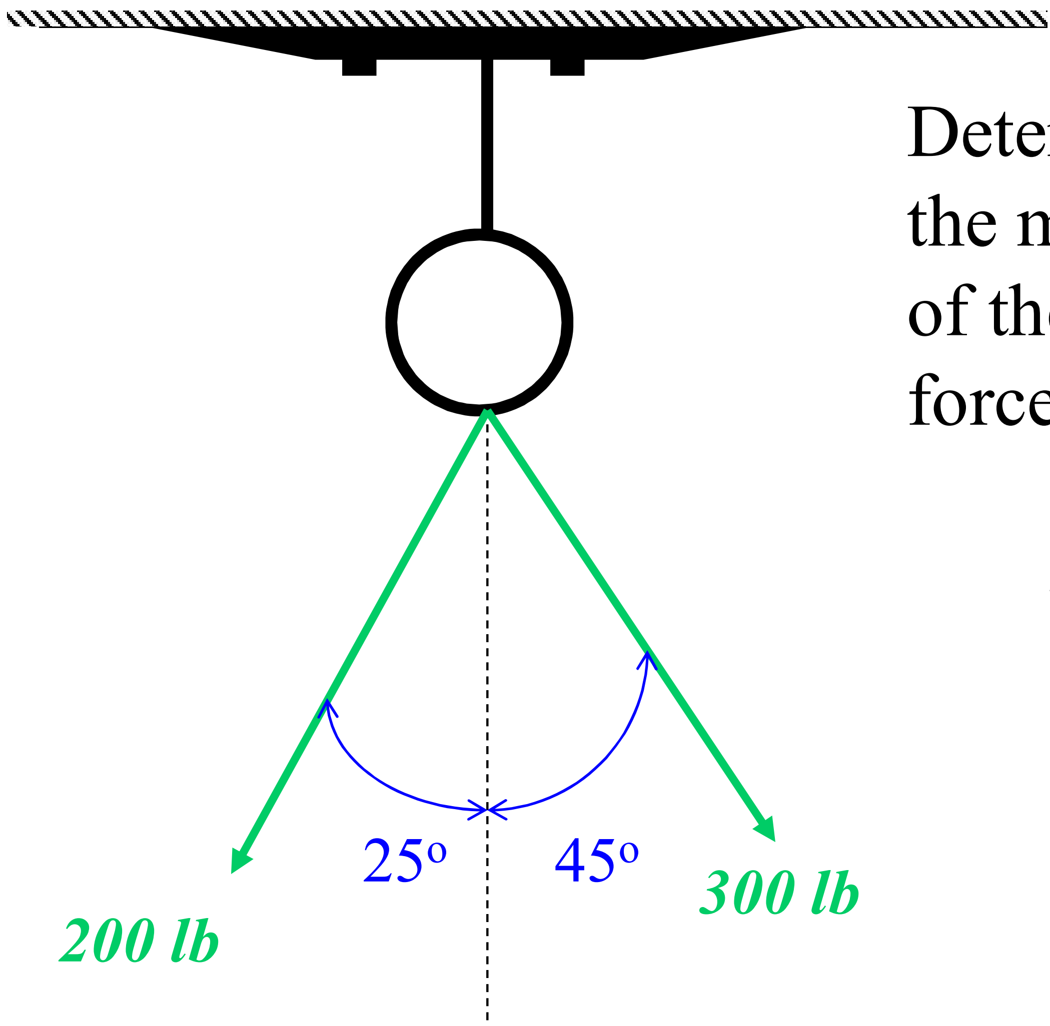
Again, used throughout this and other classes

Start with the same triangle:



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

Example

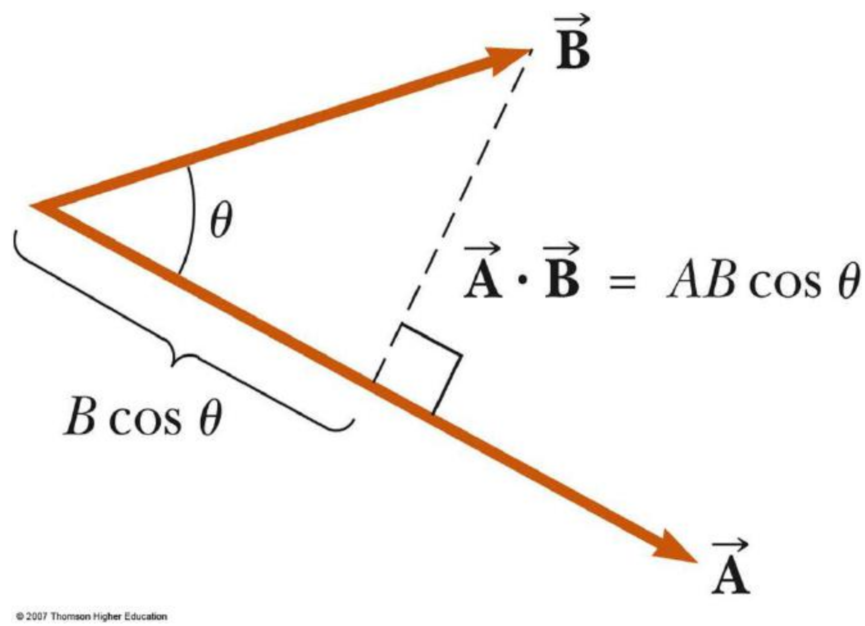


Determine by trigonometry the magnitude and direction of the resultant of the two forces shown

Note: resultant of two forces is the sum of the two vectors

Scalar (Dot) Product of Two Vectors

- The scalar product of two vectors is written as $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$
  - It is also called the dot product
- $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv A\,B\,cos\,\theta$
  - θ is the angle *between A* and *B*
- Applied to work, this means



$$W = F\Delta r\cos\theta = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$$

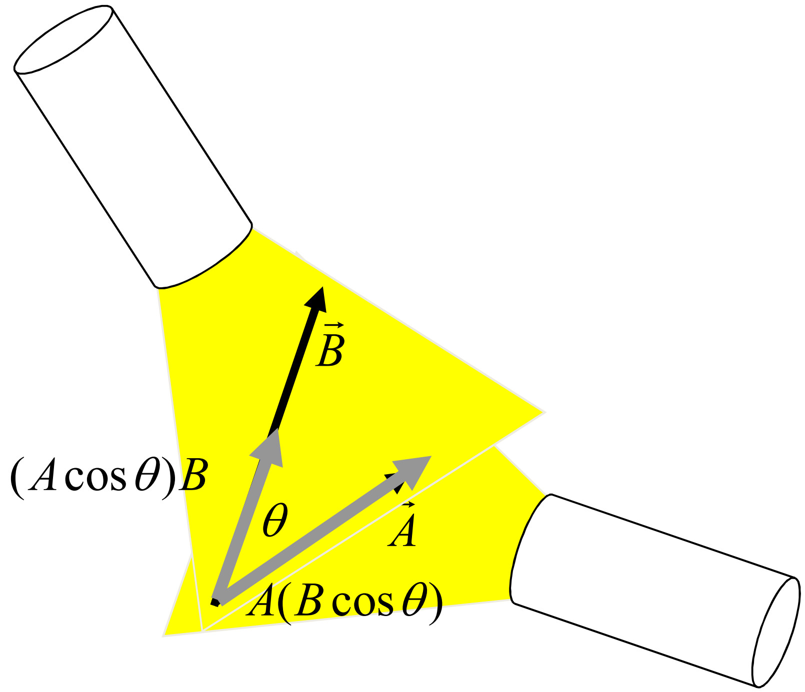
Dot Product

- The dot product says something about how parallel two vectors are.
- The dot product (scalar product) of two vectors can be thought of as the projection of one onto the direction of the other.

$$\vec{A}\cdot\vec{B} = AB\cos\theta$$
$$\vec{A}\cdot\hat{i} = A\cos\theta = A_x$$

- Components

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$



Projection of a Vector: Dot Product

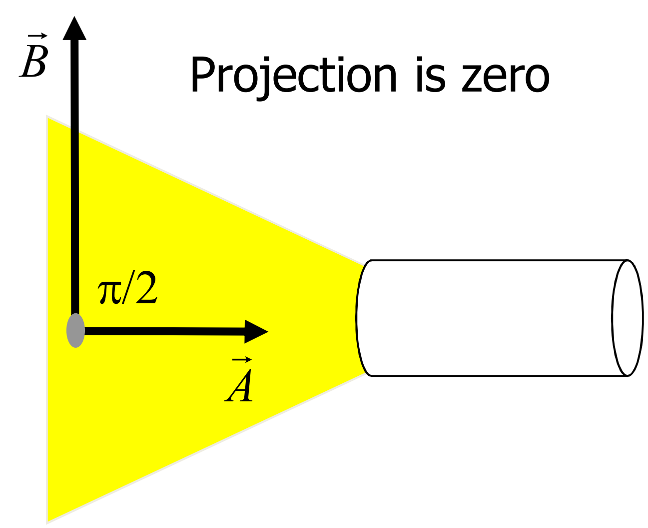
- The dot product says something about how parallel two vectors are.
- The dot product (scalar product) of two vectors can be thought of as the projection of one onto the direction of the other.

$$\hat{i}\cdot\hat{j}=0;\ \ \hat{i}\cdot\hat{k}=0;\ \ \hat{j}\cdot\hat{k}=0$$
$$\hat{i}\cdot\hat{i}=1;\ \ \hat{j}\cdot\hat{j}=1;\ \ \hat{k}\cdot\hat{k}=1$$

- Components

$$\vec{A}\cdot\vec{B}=AB\cos\theta$$
$$\vec{A}\cdot\hat{i}=A\cos\theta=A_x$$

$$\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$$



Derivation

- How do we show that $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$
- Start with
$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$
$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$
- Then
$$\begin{aligned}\vec{A}\cdot\vec{B} &= (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) \\ &= A_x\hat{i}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_y\hat{j}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k}) + A_z\hat{k}\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})\end{aligned}$$
- But
$$\hat{i}\cdot\hat{j} = 0;\ \ \hat{i}\cdot\hat{k} = 0;\ \ \hat{j}\cdot\hat{k} = 0$$
$$\hat{i}\cdot\hat{i} = 1;\ \ \hat{j}\cdot\hat{j} = 1;\ \ \hat{k}\cdot\hat{k} = 1$$
- So
$$\begin{aligned}\vec{A}\cdot\vec{B} &= A_x\hat{i}\cdot B_x\hat{i} + A_y\hat{j}\cdot B_y\hat{j} + A_z\hat{k}\cdot B_z\hat{k} \\ &= A_xB_x + A_yB_y + A_zB_z\end{aligned}$$

Scalar Product

- The vectors $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = -\hat{i} + 2\hat{j}$
- Determine the scalar product $\vec{A} \cdot \vec{B} = ?$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y = 2 \cdot (-1) + 3 \cdot 2 = -2 + 6 = 4$$

- Find the angle θ between these two vectors

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{2^2 + 3^2} = \sqrt{13} \qquad B = \sqrt{B_x^2 + B_y^2} = \sqrt{(-1)^2 + 2^2} = \sqrt{5}$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{4}{\sqrt{13}\sqrt{5}} = \frac{4}{\sqrt{65}}$$

$$\theta = \cos^{-1}\frac{4}{\sqrt{65}} = 60.3^\circ$$

The Cross Product

If $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$

The cross product of a and b is

$$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$$

Notice that the cross product a × b of two vectors a and b, unlike the dot product, is a vector. For this reason it is also called the vector product. Note that a × b is defined only when a and b are *three-dimensional* vectors.

Determinant of Order 2

- In order to make Definition 4 easier to remember, we use the notation of determinants.

- A **determinant of order 2** is defined by

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

- For example, $\begin{vmatrix} 2 & 1 \\ -6 & 4 \end{vmatrix} = 2(4) - 1(-6) = 14$

Determinant of Order 3

- A **determinant of order 3** can be defined in terms of second-order determinants as follows:

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

- Observe that each term on the right side of Equation 5 involves a number a_i in the first row of the determinant, and a_i is multiplied by the second-order determinant obtained from the left side by deleting the row and column in which a_i appears.

Example Order 3

Notice also the minus sign in the second term. For example,

$$\begin{vmatrix} 1 & 2 & -1 \\ 3 & 0 & 1 \\ -5 & 4 & 2 \end{vmatrix} = 1\begin{vmatrix} 0 & 1 \\ 4 & 2 \end{vmatrix} - 2\begin{vmatrix} 3 & 1 \\ -5 & 2 \end{vmatrix} + (-1)\begin{vmatrix} 3 & 0 \\ -5 & 4 \end{vmatrix}$$

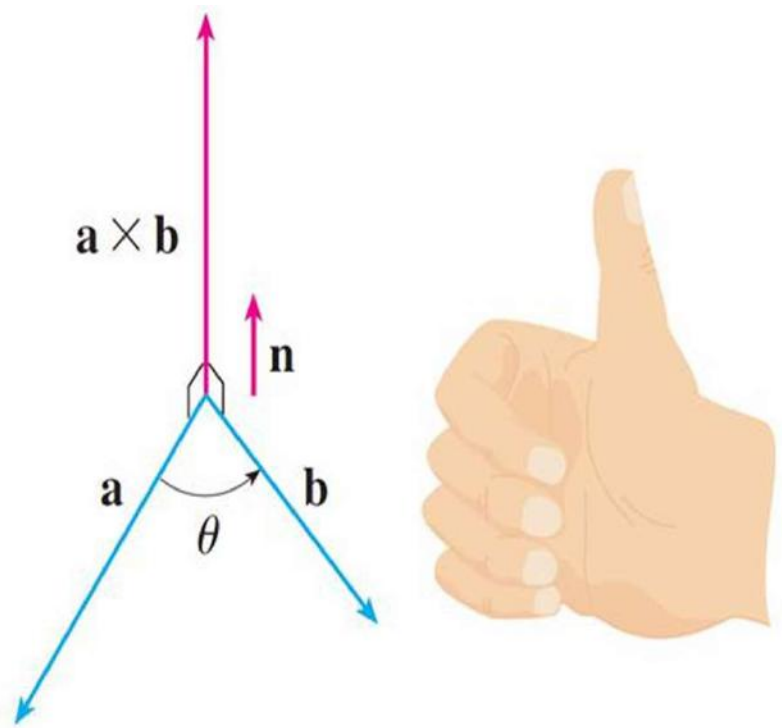
$$= 1(0 - 4) - 2(6 + 5) + (-1)(12 - 0)$$

$$= -38$$

The Cross Product: Geometric Description

- We constructed the cross product **a** × **b** so that it would be perpendicular to both **a** and **b**. This is one of the most important properties of a cross product.

- If **a** and **b** are represented by directed line segments with the same initial point (as in Figure), then the cross product **a** × **b** points in a direction perpendicular to the plane through **a** and **b**.



- It turns out that the direction of $\mathbf{a} \times \mathbf{b}$ is given by the *right-hand rule*: If the fingers of your right hand curl in the direction of a rotation (through an angle less than 180°) from to **a** to **b**, then your thumb points in the direction of $\mathbf{a} \times \mathbf{b}$.

- Now that we know the direction of the vector $\mathbf{a} \times \mathbf{b}$, the remaining thing we need to complete its geometric description is its length $|\ \mathbf{a} \times \mathbf{b}\ |$. This is given by the following theorem.

$$|\ \mathbf{a} \times \mathbf{b}\ | = |a|\ |b| \sin \theta$$

- Since a vector is completely determined by its magnitude and direction, we can now say that $\mathbf{a} \times \mathbf{b}$ is the vector that is perpendicular to both **a** and **b**, whose orientation is determined by the right-hand rule, and whose length is $|\ \mathbf{a}\ |\ |\ \mathbf{b}\ | \sin \theta$. In fact, that is exactly how physicists *define* $\mathbf{a} \times \mathbf{b}$.

Characteristic of Cross Vector

- The cross product is not commutative. For example

$$\mathbf{i} \times (\mathbf{i} \times \mathbf{j}) = \mathbf{i} \times \mathbf{k} = -\mathbf{j} \text{ , whereas } (\mathbf{i} \times \mathbf{i}) \times \mathbf{j} = 0 \times \mathbf{j} = 0$$

- The associative law for multiplication does not usually hold; that is, in general,

$$(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} \neq \mathbf{a} \times (\mathbf{b} \times \mathbf{c})$$

- However, some of the usual laws of algebra *do* hold for cross products.

The Properties of Vector Products

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $(c\mathbf{a}) \times \mathbf{b} = c(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (c\mathbf{b})$
3. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
4. $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$
5. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$
6. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

Mixed Triple Product of Three Vectors

- Mixed triple product of three vectors,

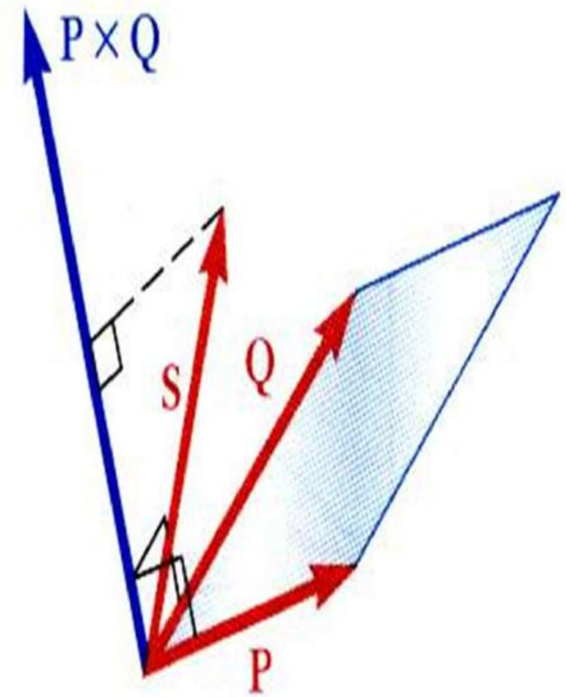
  $\vec{S} \bullet \left(\vec{P} \times \vec{Q}\right) = \text{scalar result}$

- The six mixed triple products formed from ***S***, ***P***, and ***Q*** have equal magnitudes but not the same sign,

$$\vec{S} \bullet \left(\vec{P} \times \vec{Q}\right) = \vec{P} \bullet \left(\vec{Q} \times \vec{S}\right) = \vec{Q} \bullet \left(\vec{S} \times \vec{P}\right)$$
$$= -\vec{S} \bullet \left(\vec{Q} \times P\right) = -\vec{P} \bullet \left(\vec{S} \times \vec{Q}\right) = -\vec{Q} \bullet \left(\vec{P} \times \vec{S}\right)$$

- Evaluating the mixed triple product,

$$\vec{S} \bullet \left(\vec{P} \times \vec{Q}\right) = S_x\left(P_y Q_z - P_z Q_y\right) + S_y\left(P_z Q_x - P_x Q_z\right) + S_z\left(P_x Q_y - P_y Q_x\right)$$
$$= \begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$



Basic Physics 1
Lecture Module

Rahadian N, S.Si. M.Si.

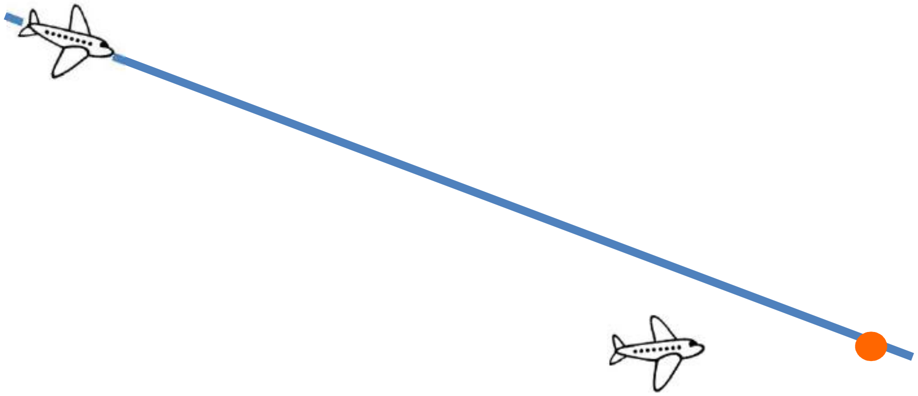
Motion Along a Straight Line

Motion Along a Straight Line

- Motion
- Position and displacement
- Average velocity and average speed
- Instantaneous velocity and speed
- Acceleration
- Derivation of the equation
- Free fall acceleration

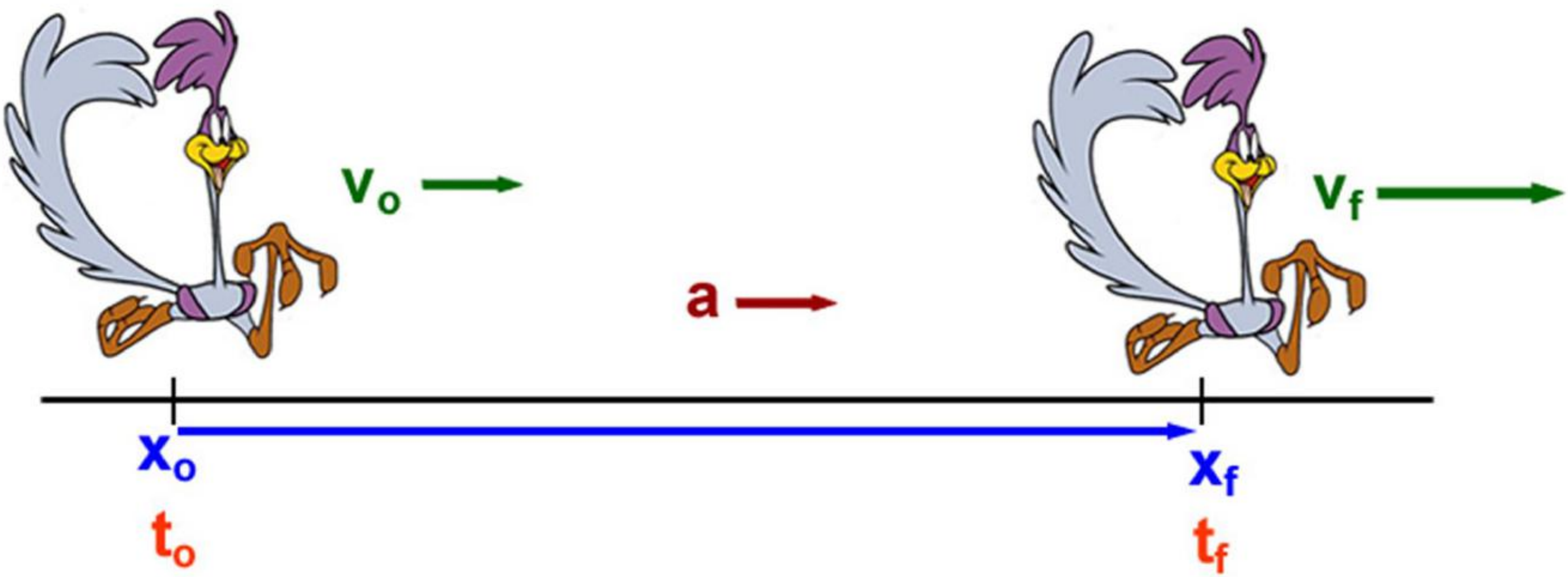
Motion

- Everything moves! Motion is one of the main topics in Physics I
- In the spirit of taking things apart for study, then putting them back together, we will first consider only motion along a straight line.
- Simplification: Consider a moving object as a particle, i.e. it moves like a particle—a "point object"



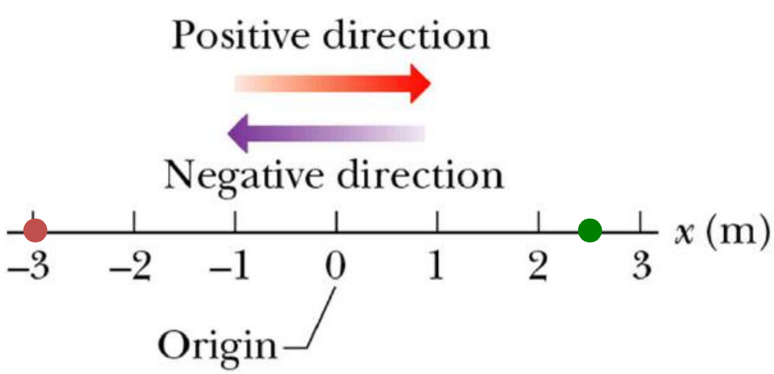
4 Basic Quantities in Kinematics

Displacement*, *Velocity*, *Time* and *Acceleration



One Dimensional Position x

- Motion can be defined as the change of position over time.
- How can we represent position along a straight line?
- Position definition:
  - Defines a starting point: origin ($x = 0$), x relative to origin
  - Direction: positive (right or up), negative (left or down)
  - It depends on time: $t = 0$ (start clock), $x(t=0)$ does not have to be zero.
- Position has units of [Length]: meters.

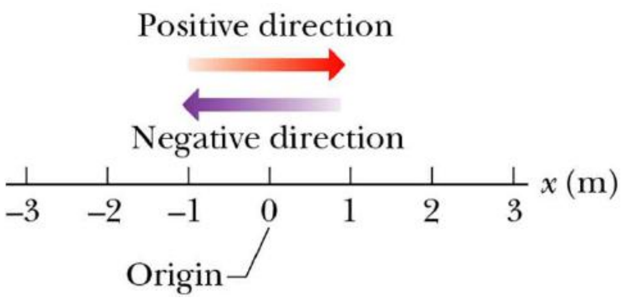


$x = +2.5$ m

$x = -3$ m

Vector and Scalar

- A vector quantity is characterized by having both a magnitude and a direction.
  - Displacement, Velocity, Acceleration, Force …
  - Denoted in boldface type $\mathbf{v}$, $\mathbf{a}$, $\mathbf{F}$ … or with an arrow over the top $\vec{v}$, $\vec{a}$, $\vec{F}$. …
- A scalar quantity has magnitude, but no direction.
  - Distance, Mass, Temperature, Time …
- For motion along a straight line, the direction is represented simply by + and – signs.
  - + sign: Right or Up.
  - – sign: Left or Down.
- 1-D motion can be thought of as a component of 2-D and 3-D motions.



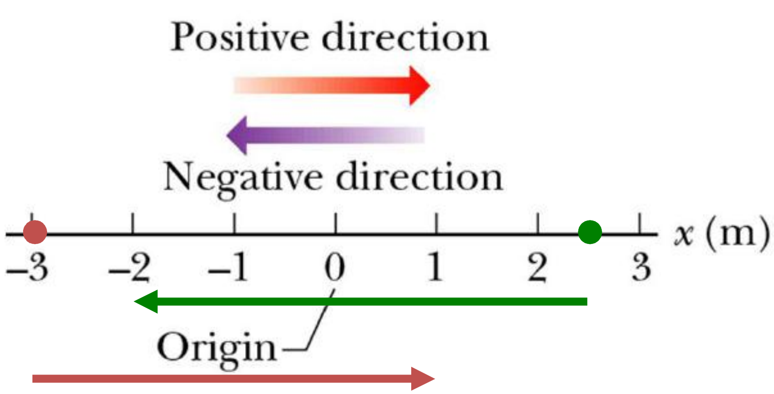
Quantities in Motion

- Any motion involves three concepts
  - Displacement
  - Velocity
  - Acceleration

- These concepts can be used to study objects in motion.

Displacement

- Displacement is a change of position in time.
- Displacement: $\boxed{\Delta x = x_f(t_f) - x_i(t_i)}$
  - f stands for final and i stands for initial.
- It is a vector quantity.
- It has both magnitude and direction: + or – sign
- It has units of [length]: meters.



$x_1(t_1) = +2.5\ \mathrm{m}$

$x_2(t_2) = -2.0\ \mathrm{m}$

$\Delta x = -2.0\ \mathrm{m} - 2.5\ \mathrm{m} = -4.5\ \mathrm{m}$

$x_1(t_1) = -3.0\ \mathrm{m}$

$x_2(t_2) = +1.0\ \mathrm{m}$

$\Delta x = +1.0\ \mathrm{m} + 3.0\ \mathrm{m} = +4.0\ \mathrm{m}$

Distance and Position-time graph

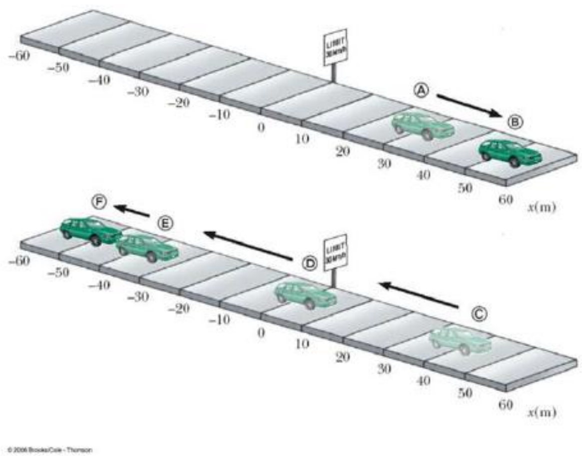
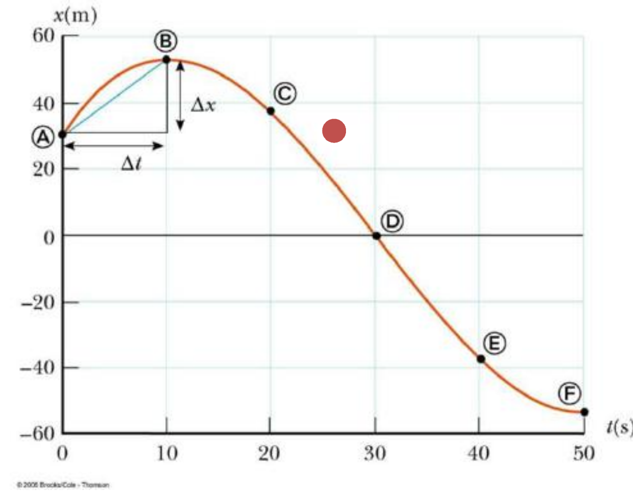


TABLE 2.1

Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	− 37
Ⓕ	50	− 53



- Displacement in space
  - From A to B: $\Delta x = x_B - x_A = 52\text{ m} - 30\text{ m} = 22\text{ m}$
  - From A to C: $\Delta x = x_c - x_A = 38\text{ m} - 30\text{ m} = 8\text{ m}$
- Distance is the length of a path followed by a particle
  - from A to B: $d = |x_B - x_A| = |52\text{ m} - 30\text{ m}| = 22\text{ m}$
  - from A to C: $d = |x_B - x_A| + |x_C - x_B| = 22\text{ m} + |38\text{ m} - 52\text{ m}| = 36\text{ m}$
- Displacement is not Distance.

Velocity

- Velocity is the rate of change of position.
- Velocity is a vector quantity.
- Velocity has both magnitude and direction.
- Velocity has a unit of [length/time]: meter/second.
- We will be concerned with three quantities, defined as:
  - Average velocity

displacement

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

distance

  - Average speed

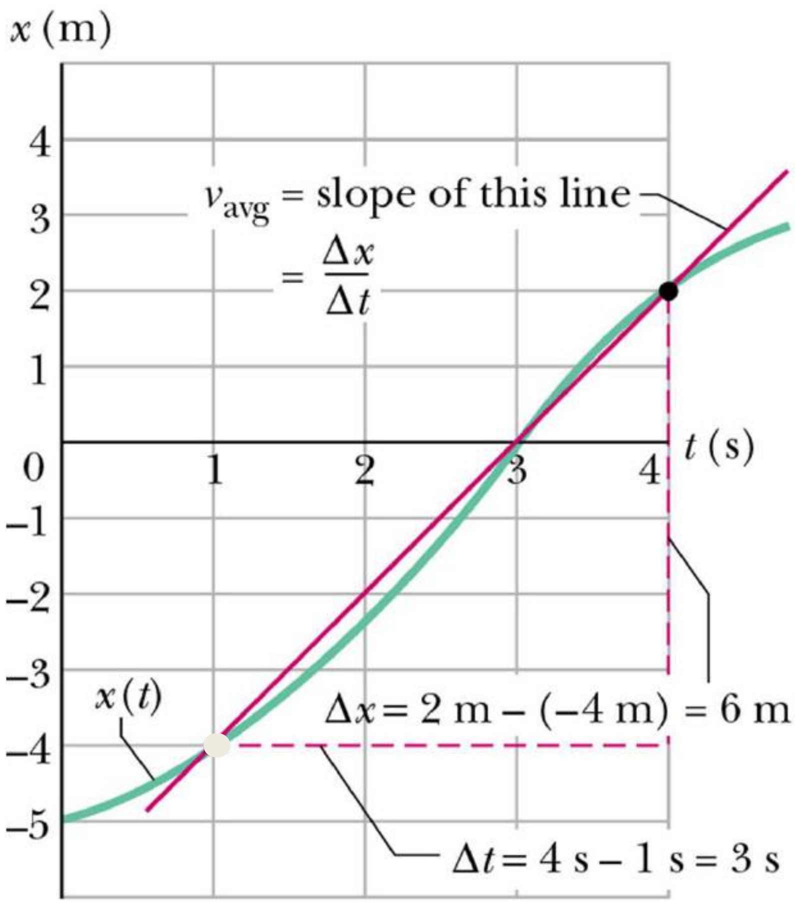
$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$

  - Instantaneous velocity

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

displacement

Average Velocity

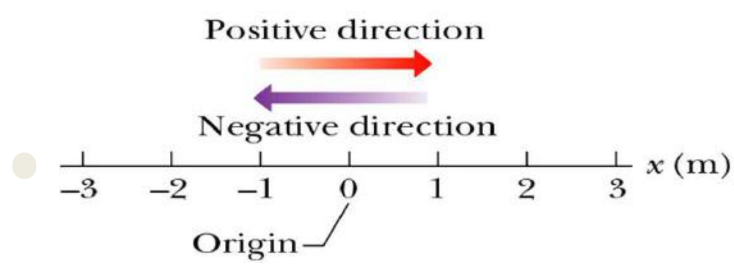


- Average velocity

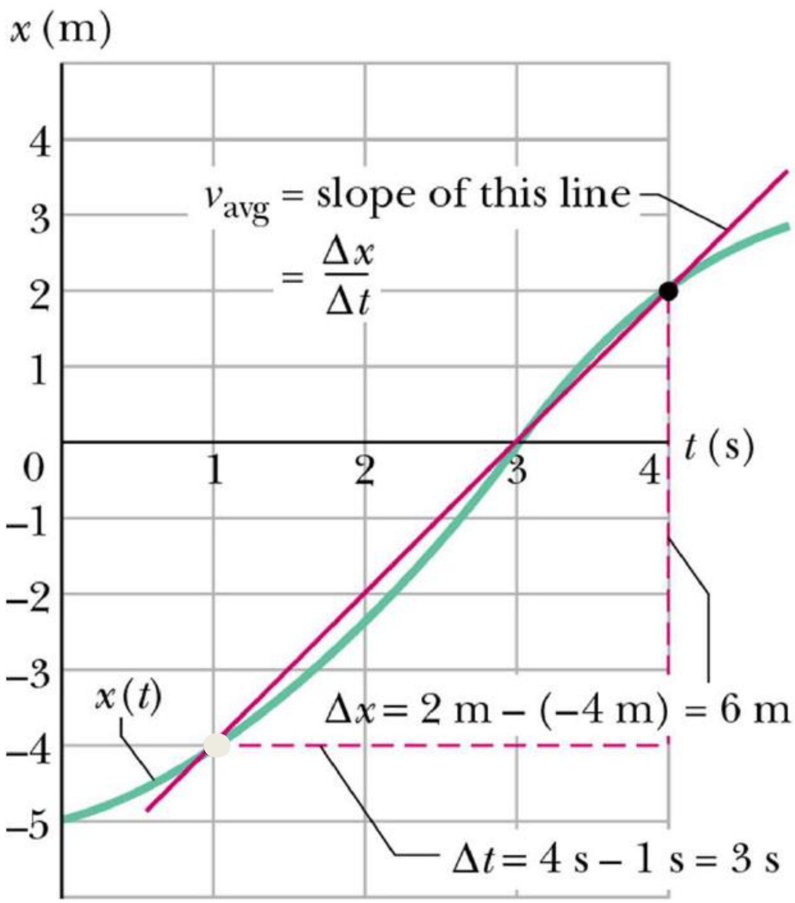
$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

is the slope of the line segment between end points on a graph.

- Dimensions: length/time (L/T) [m/s].
- SI unit: m/s.
- It is a vector (i.e. is signed), and displacement direction sets its sign.



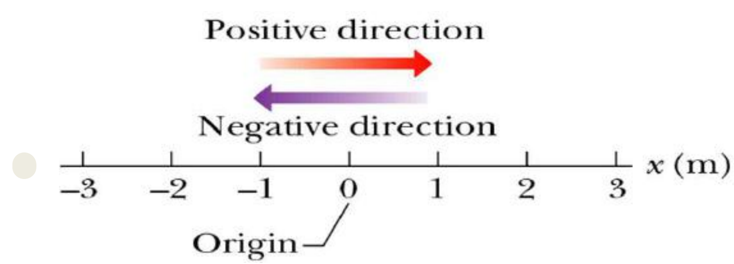
Average Speed



- Average speed

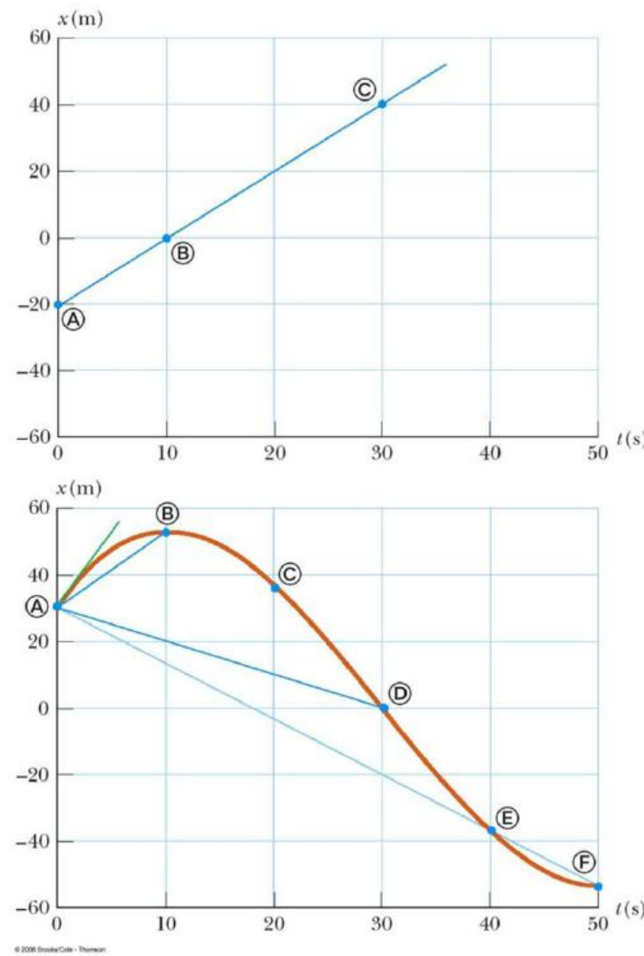
$$\boxed{s_{avg} = \frac{\text{total distance}}{\Delta t}}$$

- Dimension: length/time, [m/s].
- Scalar: No direction involved.
- Not necessarily close to V_{avg}:
  - S_{avg} = (6m + 6m)/(3s+3s) = 2 m/s
  - V_{avg} = (0 m)/(3s+3s) = 0 m/s



Graphical Interpretation of Velocity

- Velocity can be determined from a position-time graph
- Average velocity equals the slope of the line joining the initial and final positions. It is a vector quantity.
- An object moving with a constant velocity will have a graph that is a straight line.

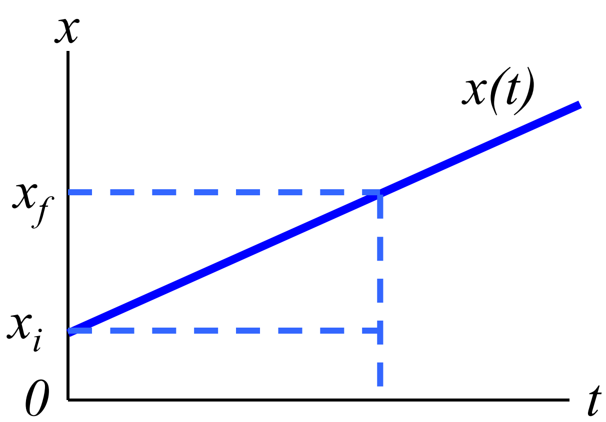


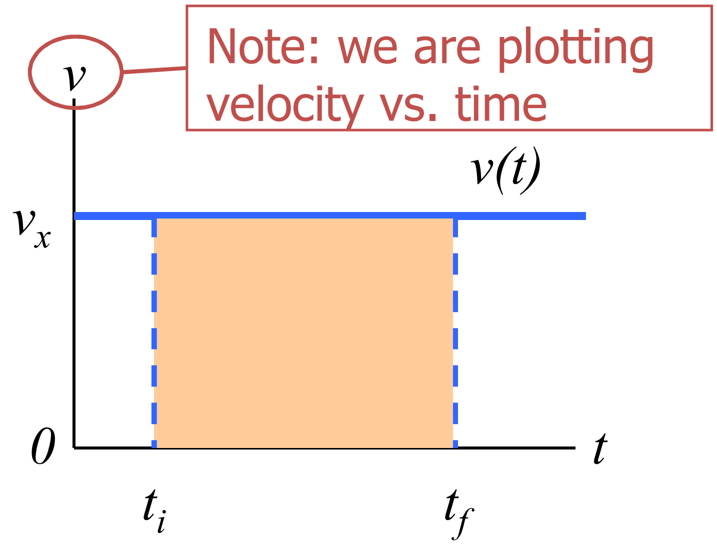
Instantaneous Velocity

- Instantaneous means “at some given instant”. The instantaneous velocity indicates what is happening at every point of time.
- Limiting process:
  - Chords approach the tangent as Δt => 0
  - Slope measure rate of change of position
- Instantaneous velocity: $$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$
- It is a vector quantity.
- Dimension: length/time (L/T), [m/s].
- It is the slope of the tangent line to x(t).
- Instantaneous velocity v(t) is a function of time.

Uniform Velocity

- Uniform velocity is the special case of constant velocity
- In this case, instantaneous velocities are always the same, all the instantaneous velocities will also equal the average velocity
- Begin with $v_x = \dfrac{\Delta x}{\Delta t} = \dfrac{x_f - x_i}{\Delta t}$ $\qquad x_f = x_i + v_x \Delta t$





Average Acceleration

- Changing velocity (non-uniform) means an acceleration is present.
- Acceleration is the rate of change of velocity.
- Acceleration is a vector quantity.
- Acceleration has both magnitude and direction.
- Acceleration has a dimensions of length/time^2: [m/s^2].
- Definition:
  - Average acceleration $$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$
  - Instantaneous acceleration $$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2 v}{dt^2}$$

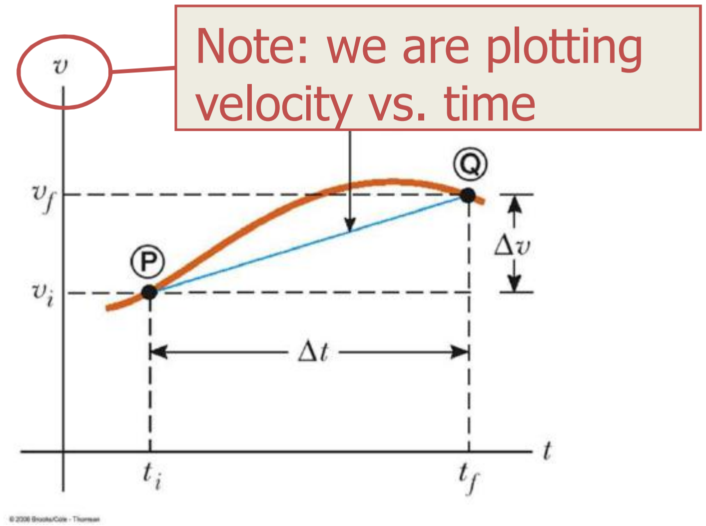
Average Acceleration

- Average acceleration

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

- Velocity as a function of time

$$v_f(t) = v_i + a_{avg}\Delta t$$



- It is tempting to call a negative acceleration a "deceleration," but note:
  - When the sign of the velocity and the acceleration are the same (either positive or negative), then the speed is increasing
  - When the sign of the velocity and the acceleration are in the opposite directions, the speed is decreasing

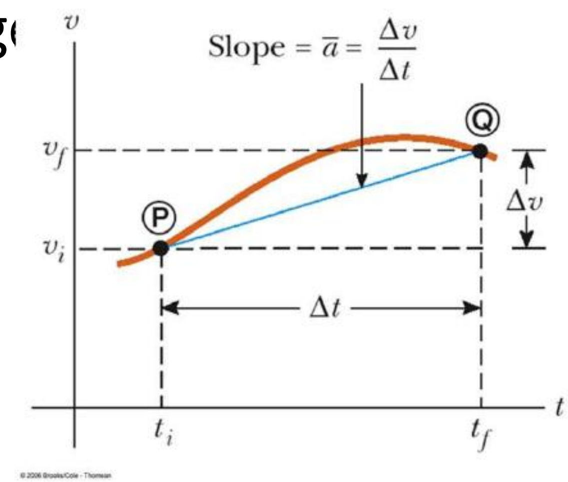
- Average acceleration is the slope of the line connecting the initial and final velocities on a velocity-time graph

Instantaneous and Uniform Acceleration

- The limit of the average acceleration as the time interval goes to zero

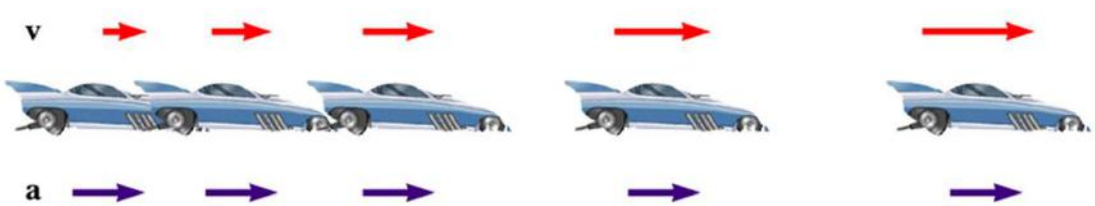
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2 v}{dt^2}$$

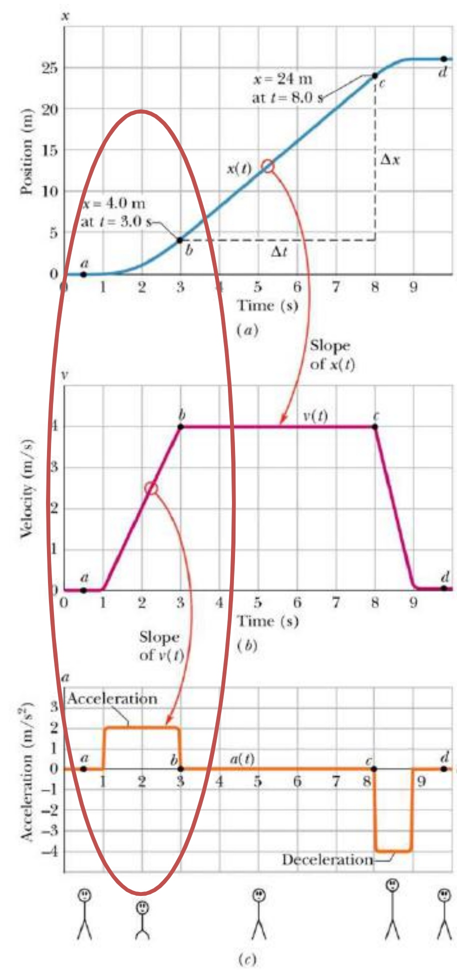
- When the instantaneous accelerations are always the same, the acceleration will be uniform. The instantaneous acceleration will be equal to the averag
- Instantaneous acceleration is the slope of the tangent to the curve of the velocity-time graph



Relationship between Acceleration and Velocity (First Stage)

- Velocity and acceleration are in the same direction
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer) $v_f(t) = v_i + at$
- Positive velocity and positive acceleration

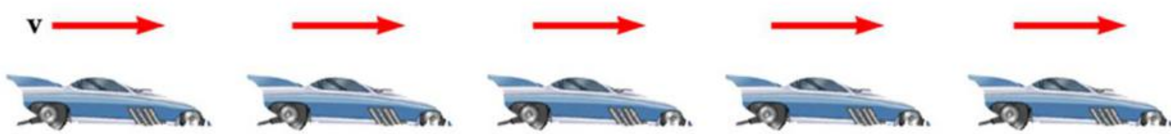


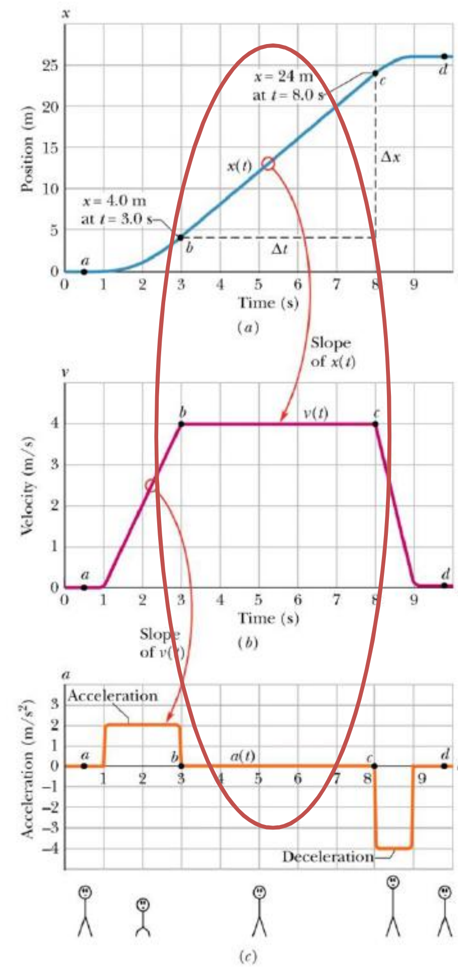


Relationship between Acceleration and Velocity (Second Stage)

- Uniform velocity (shown by red arrows maintaining the same size)
- Acceleration equals zero

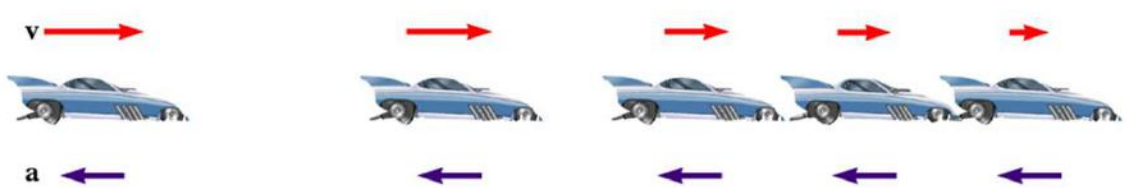
$$v_f(t) = v_i + at$$

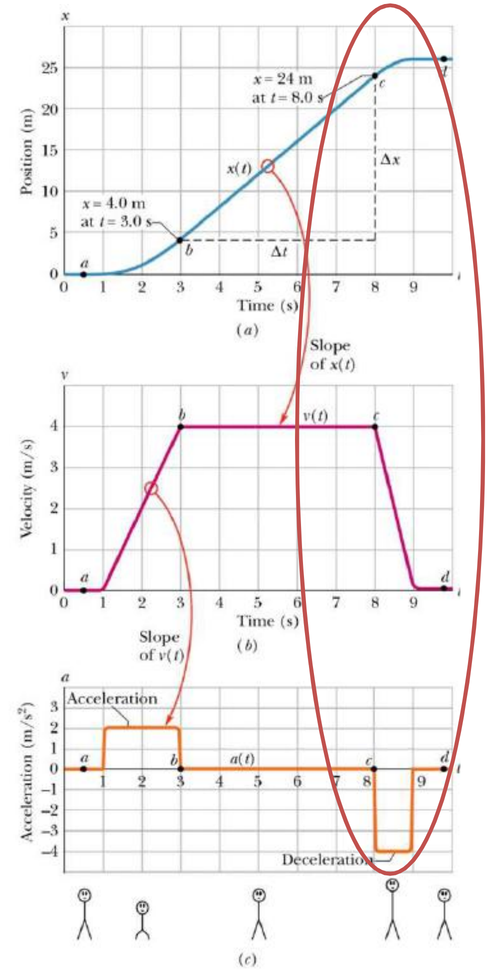




Relationship between Acceleration and Velocity (Third Stage)

- Acceleration and velocity are in opposite directions
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter) $v_f(t) = v_i + at$
- Velocity is positive and acceleration is negative





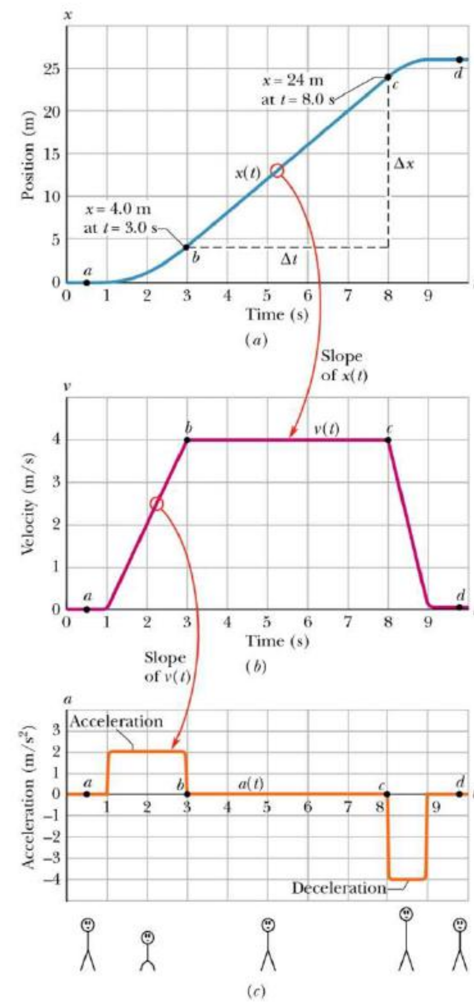
Kinematic Variables: x, v, a

- Position is a function of time: $x = x(t)$
- Velocity is the rate of change of position.
- Acceleration is the rate of change of velocity.

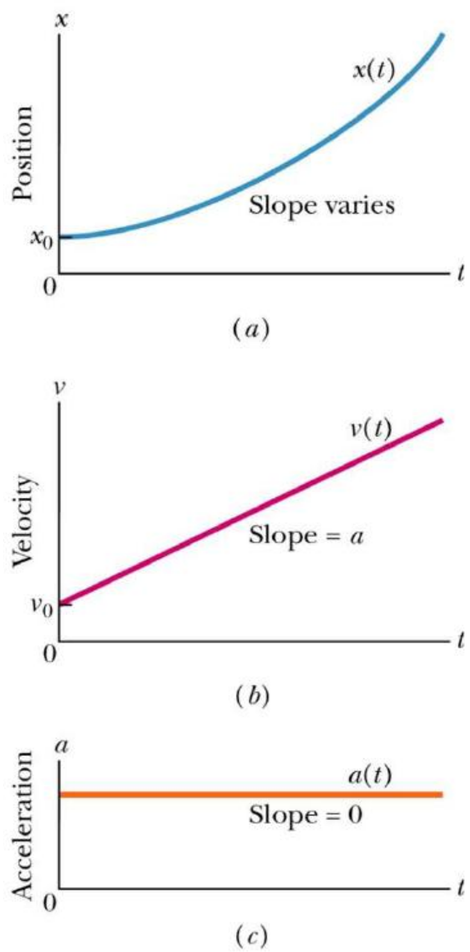
$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}} \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

- Position $\xrightarrow{\frac{d}{dt}}$ Velocity $\xrightarrow{\frac{d}{dt}}$ Acceleration
- Graphical relationship between x, v, and a

  This same plot can apply to an elevator that is initially stationary, then moves upward, and then stops. Plot v and a as a function of time.



Special Case: Motion with Uniform Acceleration (our typical case)



- Acceleration is a constant
- Kinematic Equations (which we will derive in a moment)

$$v = v_0 + at$$

$$\Delta x = \bar{v}t = \frac{1}{2}(v_0 + v)t$$

$$\Delta x = v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = {v_0}^2 + 2a\Delta x$$

Derivation of the Equation (1)

- Given initial conditions:
  - $a(t) = \text{constant} = a,\ v(t=0) = v_0,\ x(t=0) = x_0$

- Start with definition of average acceleration:

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0} = \frac{v - v_0}{t - 0} = \frac{v - v_0}{t} = a$$

- We immediately get the first equation

$$\boxed{v = v_0 + at}$$

- Shows velocity as a function of acceleration and time
- Use when you don't know and aren't asked to find the displacement

Derivation of the Equation (2)

- Given initial conditions:
  - $a(t) = \text{constant} = a,\ v(t=0) = v_0,\ x(t=0) = x_0$
- Start with definition of average velocity:

$$v_{avg} = \frac{x - x_0}{t} = \frac{\Delta x}{t}$$

- Since velocity changes at a constant rate, we have

$$\boxed{\Delta x = v_{avg} t = \frac{1}{2}(v_0 + v)t}$$

- Gives displacement as a function of velocity and time
- Use when you don't know and aren't asked for the acceleration

Derivation of the Equation (3)

- Given initial conditions:
  - $a(t) = \text{constant} = a,\ v(t=0) = v_0,\ x(t=0) = x_0$
- Start with the two just-derived equations:

$$v = v_0 + at \qquad \Delta x = v_{avg} t = \frac{1}{2}(v_0 + v)t$$

- We have $\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(v_0 + v_0 + at)t$ $\quad \boxed{\Delta x = x - x_0 = v_0 t + \frac{1}{2}at^2}$
- Gives displacement as a function of all three quantities: time, initial velocity and acceleration
- Use when you don't know and aren't asked to find the final velocity

Derivation of the Equation (4)

- Given initial conditions:
  - $a(t) = \text{constant} = a$, $v(t=0) = v_0$, $x(t=0) = x_0$
- Rearrange the definition of average acceleration

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t} = a$$

the time

$$t = \frac{v - v_0}{a}$$

- Use it to eliminate t in the second equation:

$$\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2a}(v + v_0)(v - v_0) = \frac{v^2 - v_0^{\,2}}{2a}$$

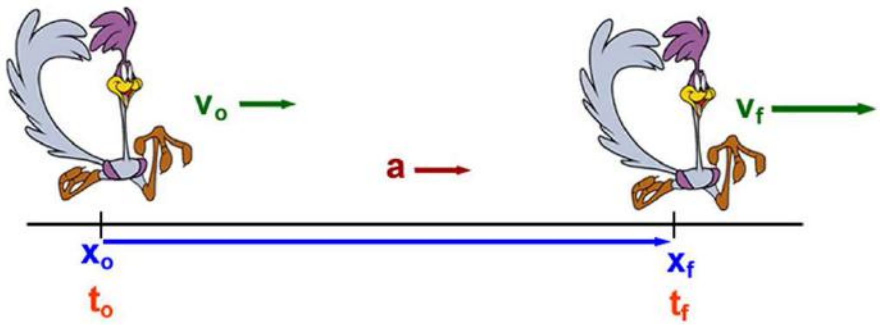
to get

$$\boxed{v^2 = v_0^{\,2} + 2a\Delta x = v_0^{\,2} + 2a(x - x_0)}$$

- Gives velocity as a function of acceleration and displacement
- Use when you don't know and aren't asked for the time

Problem-Solving Hints

- Read the problem
- Draw a diagram
  - Choose a coordinate system, label initial and final points, indicate a positive direction for velocities and accelerations



- Label all quantities, be sure all the units are consistent
  - Convert if necessary
- Choose the appropriate kinematic equation
- Solve for the unknowns
  - You may have to solve two equations for two unknowns
- Check your results

$$v = v_0 + at$$

$$\Delta x = v_0 t + \tfrac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

Example

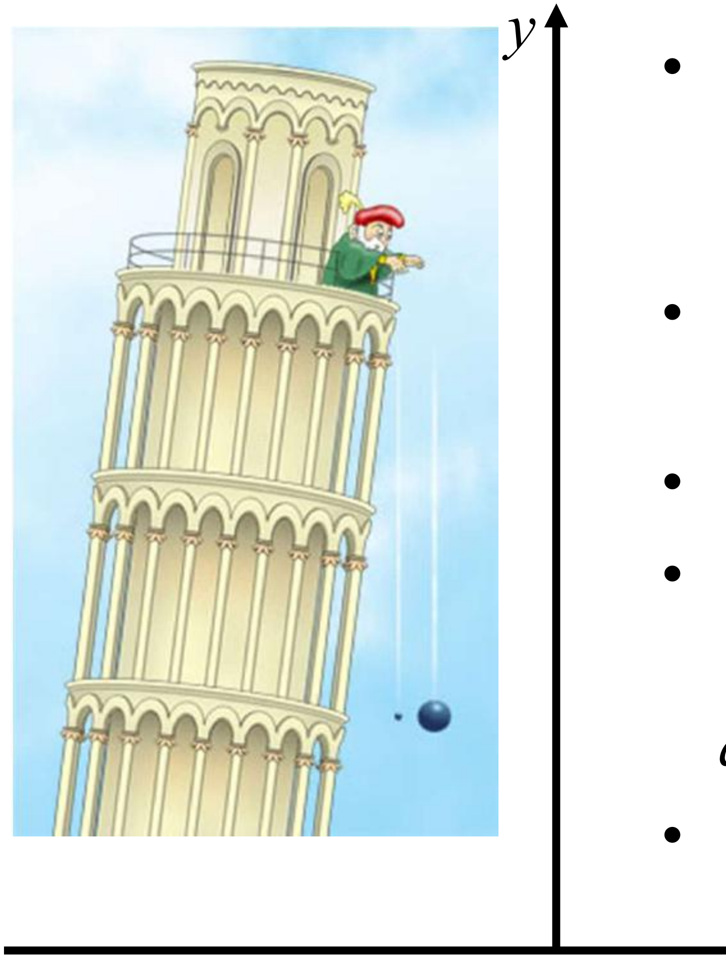
- An airplane has a lift-off speed of 30 m/s after a take-off run of 300 m, what minimum constant acceleration?

$v = v_0 + at$ $\Delta x = v_0 t + \frac{1}{2} a t^2$ $v^2 = v_0^{\;2} + 2a\Delta x$

- What is the corresponding take-off time?

$v = v_0 + at$ $\Delta x = v_0 t + \frac{1}{2} a t^2$ $v^2 = v_0^{\;2} + 2a\Delta x$

Free Fall Acceleration



- Earth gravity provides a constant acceleration. Most important case of constant acceleration.
- Free-fall acceleration is independent of mass.
- Magnitude: $|a| = g = 9.8\ \mathrm{m/s^2}$
- Direction: always downward, so a_g is negative if we define “up” as positive,

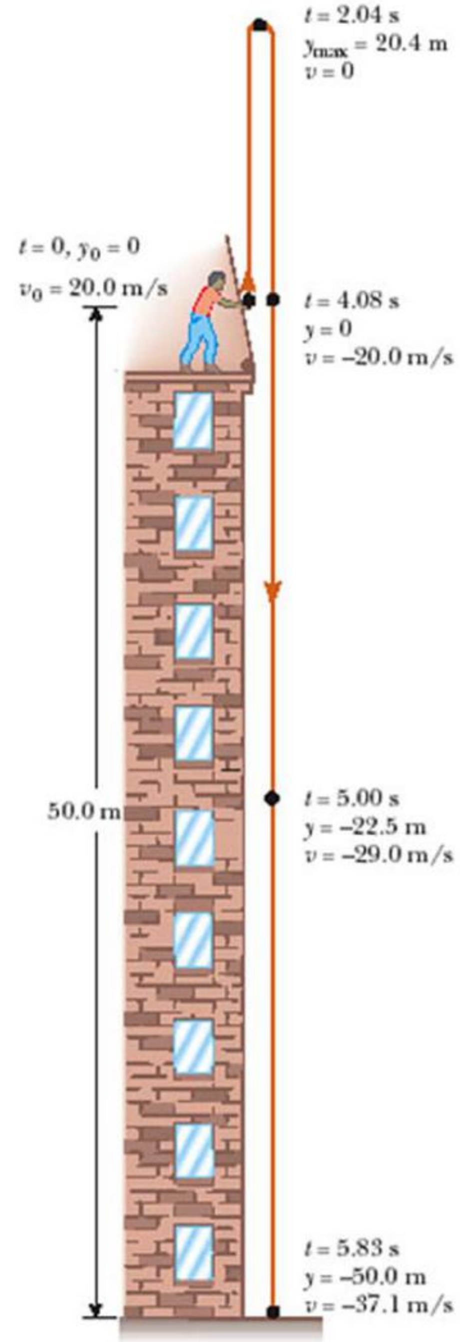
$$a = -g = -9.8\ \mathrm{m/s^2}$$

- Try to pick origin so that $x_i = 0$

Free Fall for Rookie

A stone is thrown from the top of a building with an initial velocity of 20.0 m/s straight upward, at an initial height of 50.0 m above the ground. The stone just misses the edge of the roof on the its way down. Determine

- (a) the time needed for the stone to reach its maximum height.
- (b) the maximum height.
- (c) the time needed for the stone to return to the height from which it was thrown and the velocity of the stone at that instant.
- (d) the time needed for the stone to reach the ground
- (e) the velocity and position of the stone at t = 5.00s



Review

- This is the simplest type of motion
- It lays the groundwork for more complex motion
- Kinematic variables in one dimension
  - Position $x(t)$ m L
  - Velocity $v(t)$ m/s L/T
  - Acceleration $a(t)$ m/s^2 L/T^2
  - All depend on time
  - All are vectors: magnitude and direction vector:
- Equations for motion with constant acceleration: missing quantities

	missing quantities
– $v = v_0 + at$	$x - x_0$
– $x - x_0 = v_0 t + \frac{1}{2} a t^2$	v
– $v^2 = v_0^2 + 2a(x - x_0)$	t
– $x - x_0 = \frac{1}{2}(v + v_0)t$	a
– $x - x_0 = vt - \frac{1}{2} a t^2$	v_0

Basic Physics 1
Lecture Module

Rahadian N, S.Si. M.Si.

Motion in Two Dimensions

Motion in Two Dimensions

- Reminder of vectors and vector algebra
- Displacement and position in 2-D
- Average and instantaneous velocity in 2-D
- Average and instantaneous acceleration in 2-D
- Projectile motion
- Uniform circular motion
- Relative velocity*

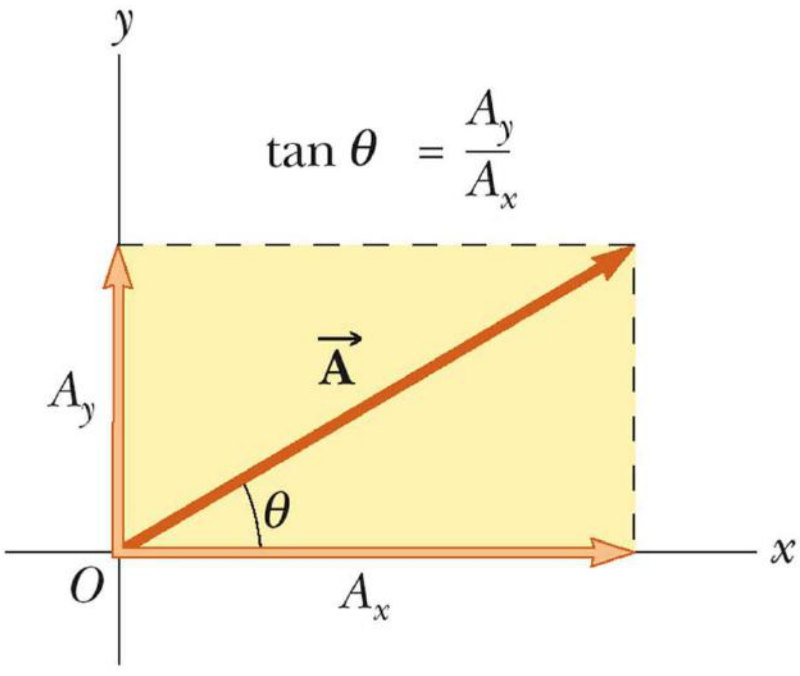
Vector and its components

- The components are the legs of the right triangle whose hypotenuse is *A*

$$\vec{A} = \vec{A}_x + \vec{A}_y$$

$$\begin{cases} A_x = A\cos(\theta) \\ A_y = A\sin(\theta) \end{cases}$$

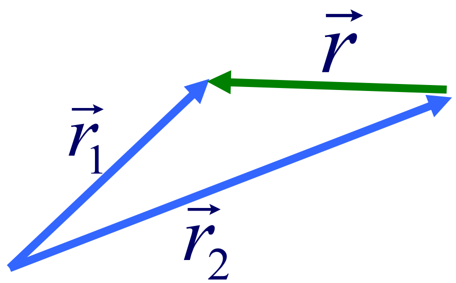
$$\begin{cases} \left|\vec{A}\right| = \sqrt{(A_x)^2 + (A_y)^2} \\ \tan(\theta) = \dfrac{A_y}{A_x} \quad \text{or} \quad \theta = \tan^{-1}\left(\dfrac{A_y}{A_x}\right) \end{cases}$$



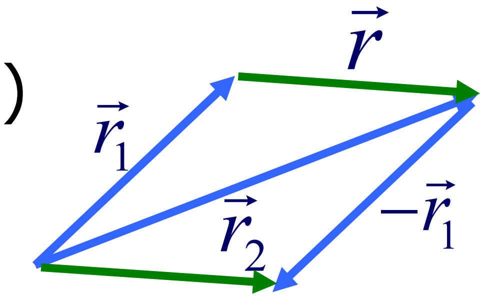
Vector Algebra

- Which diagram can represent $\vec{r} = \vec{r}_2 - \vec{r}_1$?

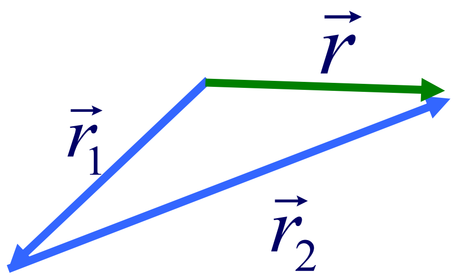
A)



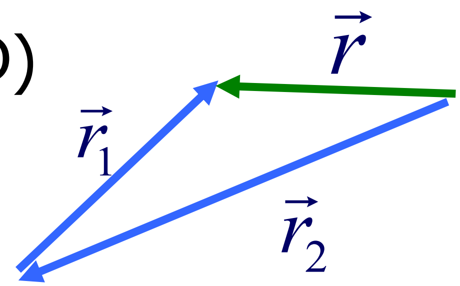
B)



C)

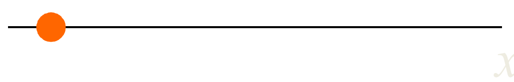


D)

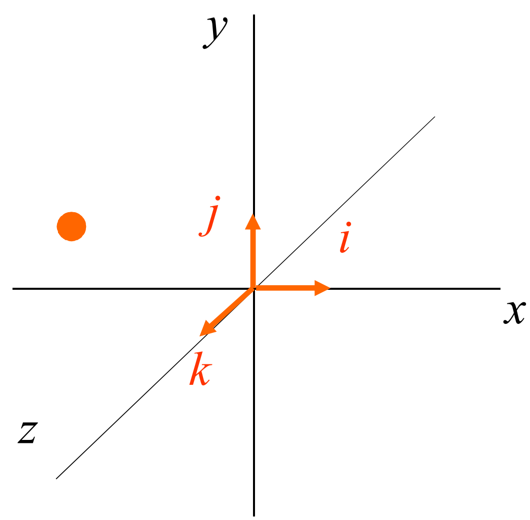


Motion in two dimensions

- Kinematic variables in one dimension
  - Position: $x(t)$ m
  - Velocity: $v(t)$ m/s
  - Acceleration: $a(t)$ m/s^2



- Kinematic variables in three dimensions
  - Position: $\vec{r}(t) = x\hat{i} + y\hat{j} + z\hat{k}$ m
  - Velocity: $\vec{v}(t) = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$ m/s
  - Acceleration: $\vec{a}(t) = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ m/s^2



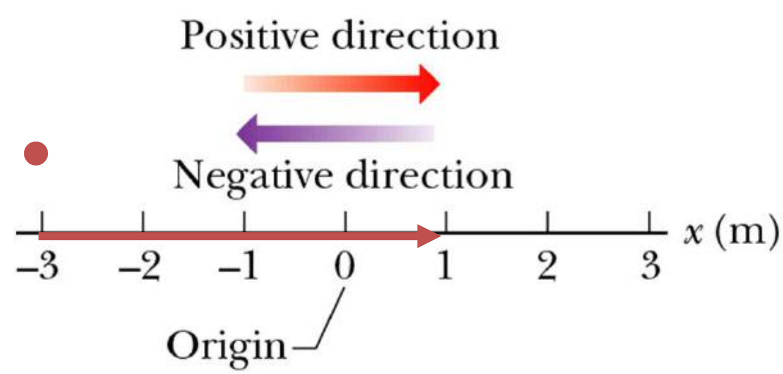
- All are vectors: have direction and magnitudes

Position and Displacement

- In one dimension

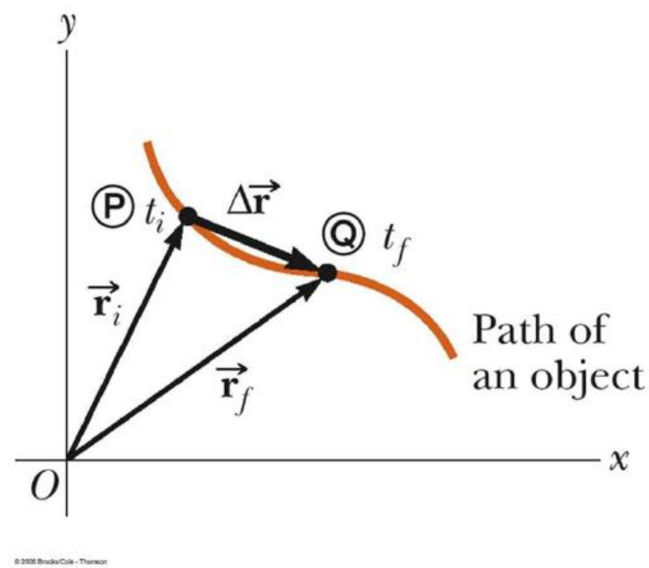
$$\Delta x = x_2(t_2) - x_1(t_1)$$

$x_1(t_1) = -3.0$ m, $x_2(t_2) = +1.0$ m
$\Delta x = +1.0$ m $+ 3.0$ m $= +4.0$ m



- In two dimensions
  - Position: the position of an object is described by its position vector $\vec{r}(t)$ --always points to particle from origin.
  - Displacement: $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$$\Delta\vec{r} = (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$$
$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$
$$= \Delta x\hat{i} + \Delta y\hat{j}$$



Average & Instantaneous Velocity

- Average velocity $\vec{v}_{avg} \equiv \dfrac{\Delta \vec{r}}{\Delta t}$

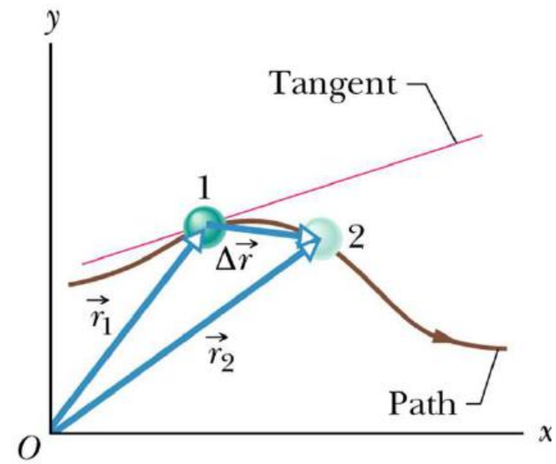
$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = v_{avg,x}\hat{i} + v_{avg,y}\hat{j}$$

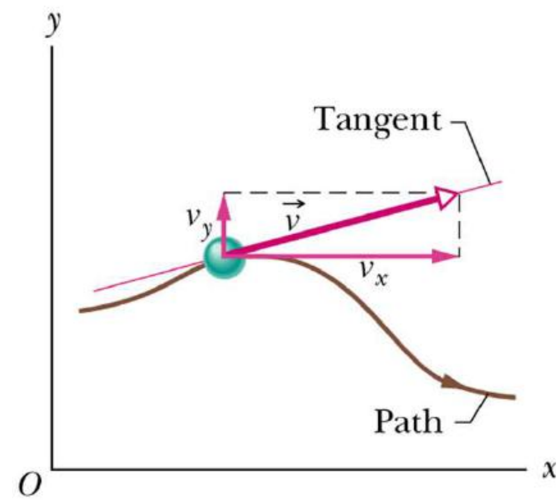
- Instantaneous velocity

$$\vec{v} \equiv \lim_{t\to 0} \vec{v}_{avg} = \lim_{t\to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

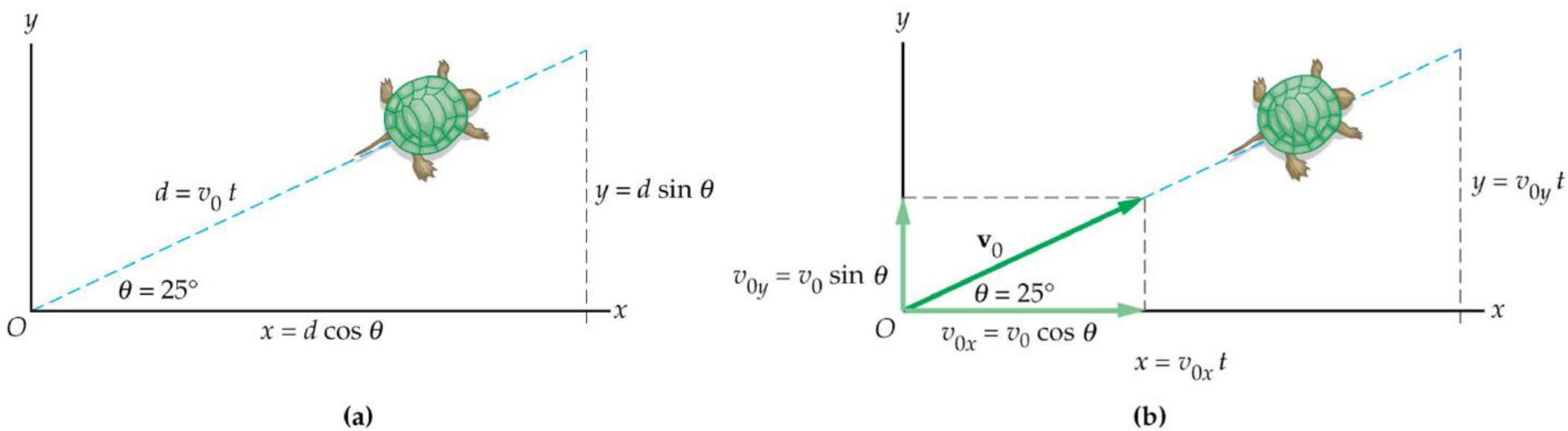
$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

- v is tangent to the path in x-y graph;





Motion of a Turtle



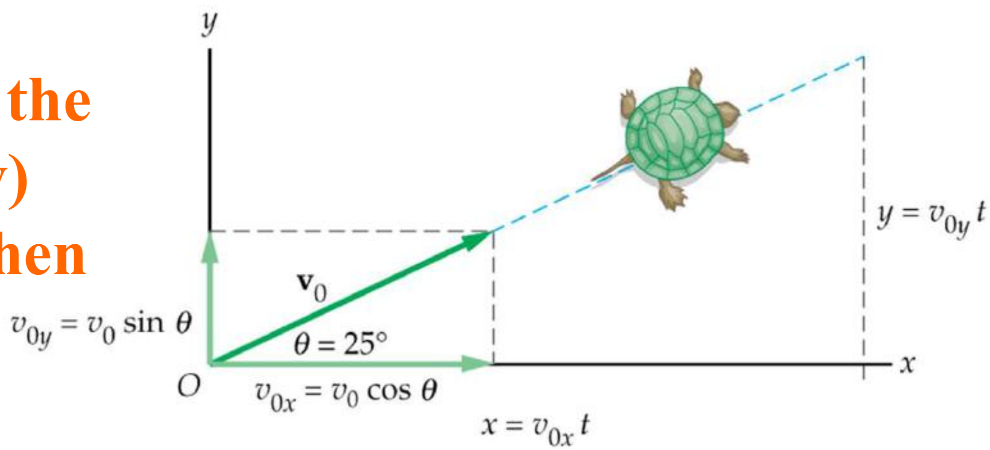
A turtle starts at the origin and moves with the speed of v_0=10 cm/s in the direction of 25° to the horizontal.

(a) Find the coordinates of a turtle 10 seconds later.

(b) How far did the turtle walk in 10 seconds?

Motion of a Turtle

Notice, you can solve the equations independently for the horizontal (x) and vertical (y) components of motion and then combine them!



(b)

$$\vec{v}_0 = \vec{v}_x + \vec{v}_y$$

- X components:

$$v_{0x} = v_0 \cos 25^\circ = 9.06 \text{ cm/s} \qquad \Delta x = v_{0x} t = 90.6 \text{ cm}$$

- Y components:

$$v_{0y} = v_0 \sin 25^\circ = 4.23 \text{ cm/s} \qquad \Delta y = v_{0y} t = 42.3 \text{ cm}$$

- Distance from the origin:

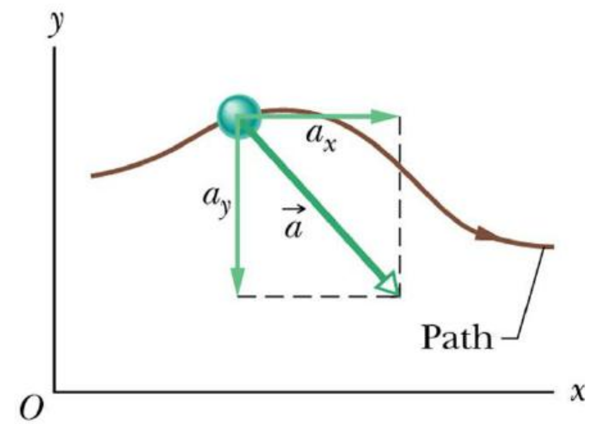
$$d = \sqrt{\Delta x^2 + \Delta y^2} = 100.0 \text{ cm}$$

Average & Instantaneous Acceleration

- Average acceleration

$$\vec{a}_{avg} \equiv \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a}_{avg} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} = a_{avg,x}\hat{i} + a_{avg,y}\hat{j}$$



- Instantaneous acceleration

$$\vec{a} \equiv \lim_{t\to 0}\vec{a}_{avg} = \lim_{t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$

- The magnitude of the velocity (the speed) can change
- The direction of the velocity can change, even though the magnitude is constant
- Both the magnitude and the direction can change

Motion in two dimensions

- Motions in each dimension are independent components
- Constant acceleration equations

$$\vec{v} = \vec{v}_0 + \vec{a}t \qquad \vec{r} - \vec{r} = \vec{v}_0 t + \tfrac{1}{2}\vec{a}t^2$$

- Constant acceleration equations hold in each dimension

$$v_x = v_{0x} + a_x t \qquad v_y = v_{0y} + a_y t$$

$$x - x_0 = v_{0x}t + \tfrac{1}{2}a_x t^2 \qquad y - y_0 = v_{0y}t + \tfrac{1}{2}a_y t^2$$

$$v_x{}^2 = v_{0x}{}^2 + 2a_x(x - x_0) \qquad v_y{}^2 = v_{0y}{}^2 + 2a_y(y - y_0)$$

  - $t = 0$ beginning of the process;
  - $\vec{a} = a_x\hat{i} + a_y\hat{j}$ where a_x and a_y are constant;
  - Initial velocity $\vec{v}_0 = v_{0x}\hat{i} + v_{0y}\hat{j}$ initial displacement $\vec{r}_0 = x_0\hat{i} + y_0\hat{j}$

Hints for solving problems

- Define coordinate system. Make sketch showing axes, origin.
- List known quantities. Find v_{0x}, v_{0y}, a_x, a_y, etc. Show initial conditions on sketch.
- List equations of motion to see which ones to use.
- Time t is the same for x and y directions.

  $x_0 = x(t = 0)$, $y_0 = y(t = 0)$, $v_{0x} = v_x(t = 0)$, $v_{0y} = v_y(t = 0)$.
- Have an axis point along the direction of **a** if it is constant.

$$v_x = v_{0x} + a_x t$$

$$x - x_0 = v_{0x} t + \tfrac{1}{2} a_x t^2$$

$$v_x^{\ 2} = v_{0x}^{\ 2} + 2a_x (x - x_0)$$

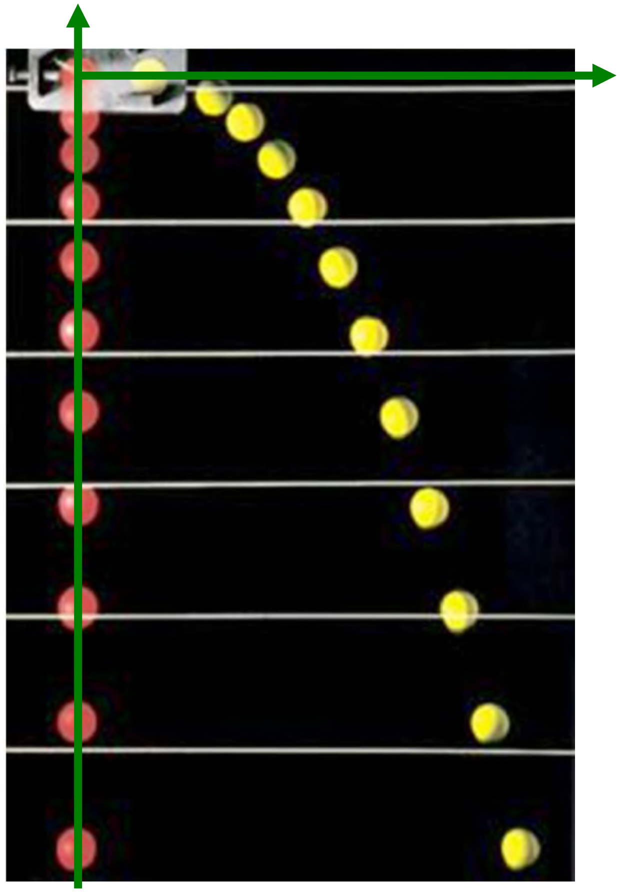
$$v_y = v_{0y} + a_y t$$

$$y - y_0 = v_{0y} t + \tfrac{1}{2} a_y t^2$$

$$v_y^{\ 2} = v_{0y}^{\ 2} + 2a_y (y - y_0)$$

Projectile Motion: Origin

- 2-D problem and define a coordinate system: x- horizontal, y- vertical (up +)
- Try to pick $x_0 = 0, y_0 = 0$ at $t = 0$
- Horizontal motion + Vertical motion
- Horizontal: $a_x = 0$, constant velocity motion
- Vertical: $a_y = -g = -9.8\ \mathrm{m/s^2}, v_{0y} = 0$
- Equations:



Horizontal	Vertical
$v_x = v_{0x} + a_x t$	$v_y = v_{0y} + a_y t$
$x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$	$y - y_0 = v_{0y} t + \frac{1}{2} a_y t^2$
$v_x^2 = v_{0x}^2 + 2a_x (x - x_0)$	$v_y^2 = v_{0y}^2 + 2a_y (y - y_0)$

Projectile Motion: Adjustment

- X and Y motions happen independently, so we can treat them separately

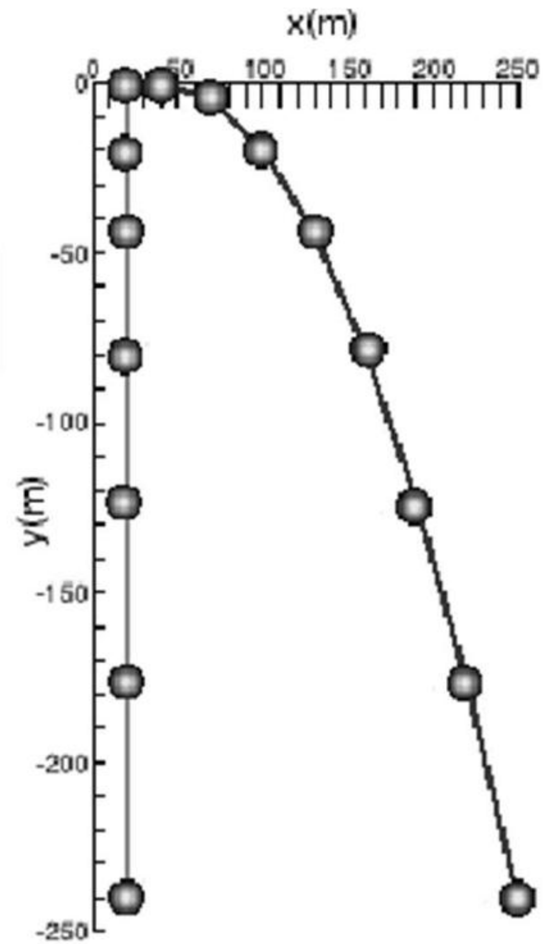
$$v_x = v_{0x}$$
$$x = x_0 + v_{0x}t$$

Horizontal

$$v_y = v_{0y} - gt$$
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$

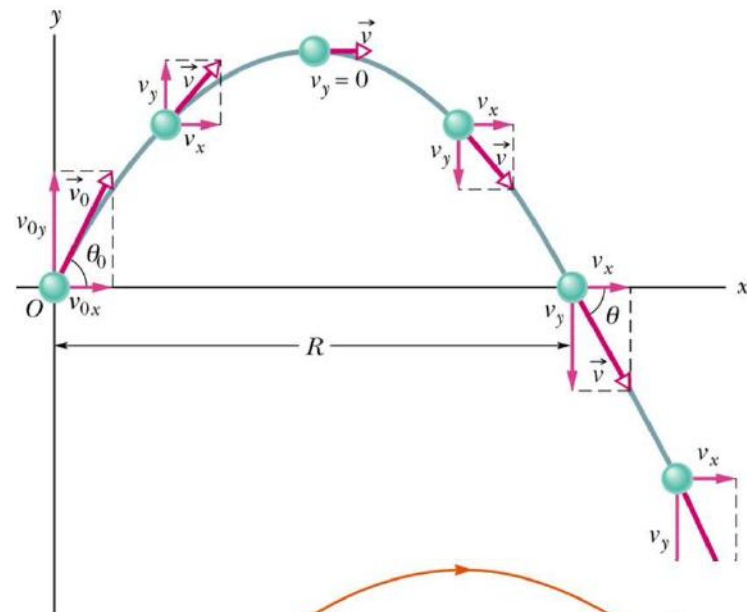
Vertical

- Try to pick $x_0 = 0, y_0 = 0$ at $t = 0$
- Horizontal motion + Vertical motion
- Horizontal: $a_x = 0$, constant velocity motion
- Vertical: $a_y = -g = -9.8\ \mathrm{m/s^2}$
- x and y are connected by time t
- y(x) is a parabola



Projectile Motion: Modification

- 2-D problem and define a coordinate system.
- Horizontal: $a_x = 0$ and vertical: $a_y = -g$.
- Try to pick $x_0 = 0, y_0 = 0$ at $t = 0$.
- Velocity initial conditions:
  - v_0 can have x, y components.
  - v_{0x} is constant usually. $v_{0x} = v_0 \cos\theta_0$
  - v_{0y} changes continuously. $v_{0x} = v_0 \sin\theta_0$
- Equations:



Horizontal

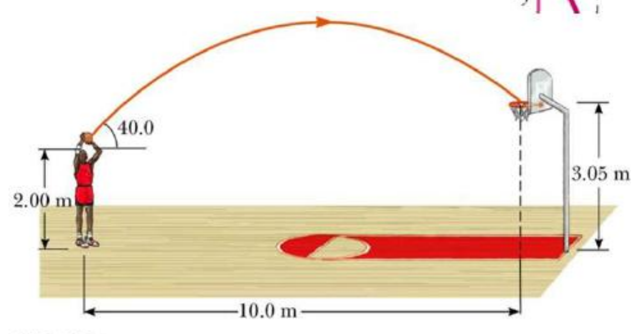
$$v_x = v_{0x}$$

$$x = x_0 + v_{0x}t$$

Vertical

$$v_y = v_{0y} - gt$$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$



Trajectory of Projectile Motion

- Initial conditions ($t = 0$): $x_0 = 0, y_0 = 0$
  $v_{0x} = v_0 \cos\theta_0$ and $v_{0y} = v_0 \sin\theta_0$
- Horizontal motion:

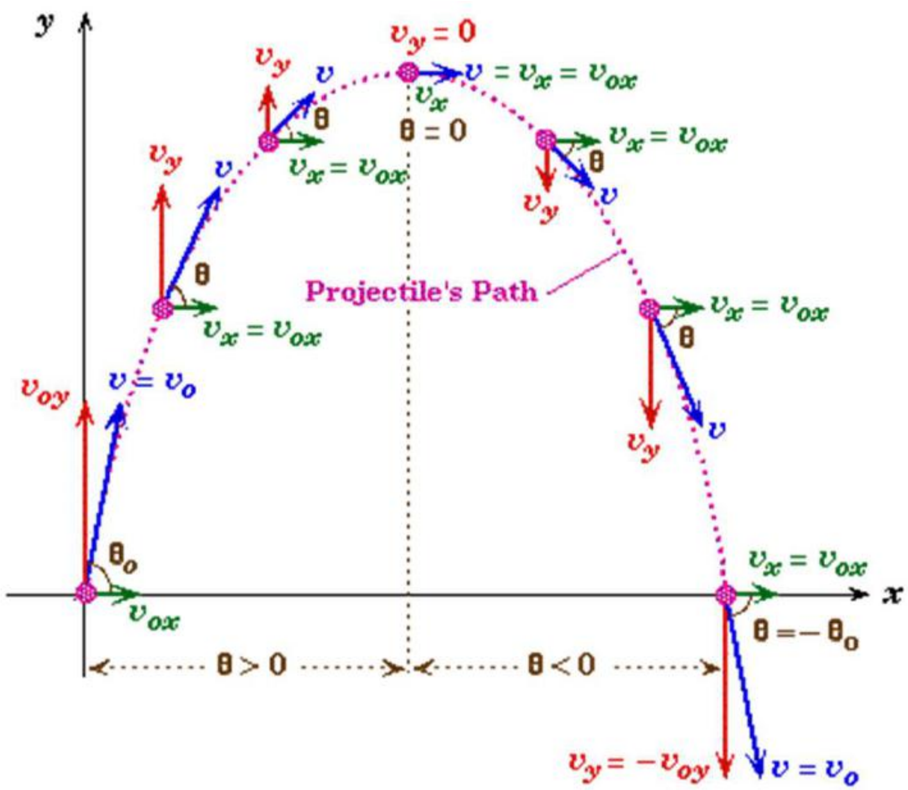
$$x = 0 + v_{0x}t \quad \Rightarrow \quad t = \frac{x}{v_{0x}}$$

- Vertical motion:

$$y = 0 + v_{0y}t - \tfrac{1}{2}gt^2$$

$$y = v_{0y}\left(\frac{x}{v_{0x}}\right) - \frac{g}{2}\left(\frac{x}{v_{0x}}\right)^2$$

$$\boxed{y = x\tan\theta_0 - \frac{g}{2{v_0}^2\cos^2\theta_0}x^2}$$



- Parabola;
  - $\theta_0 = 0$ and $\theta_0 = 90$?

What is R and h ?

- Initial conditions ($t = 0$): $x_0 = 0, y_0 = 0$
  $v_{0x} = v_0 \cos\theta_0$ and $v_{0x} = v_0 \sin\theta_0$, then

$$x = 0 + v_{0x} t \qquad 0 = 0 + v_{0y} t - \tfrac{1}{2} g t^2$$

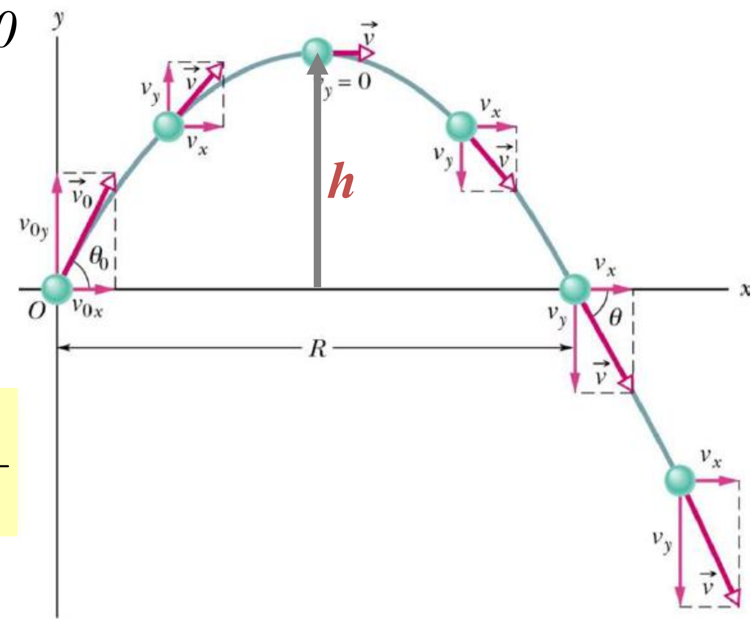
$$t = \frac{2v_{0y}}{g} = \frac{2v_0 \sin\theta_0}{g}$$

$$R = x - x_0 = v_{0x} t = \frac{2v_0 \cos\theta_0 v_0 \sin\theta_0}{g} = \frac{v_0^{\ 2} \sin 2\theta_0}{g}$$

$$h = y - y_0 = v_{0y} t_h - \tfrac{1}{2} g t_h^{\ 2} = v_{0y} \frac{t}{2} - \frac{g}{2}\left(\frac{t}{2}\right)^2$$

$$h = \frac{v_0^{\ 2} \sin^2 \theta_0}{2g}$$

$$v_y = v_{0y} - gt = v_{0y} - g\frac{2v_{0y}}{g} = -v_{0y}$$

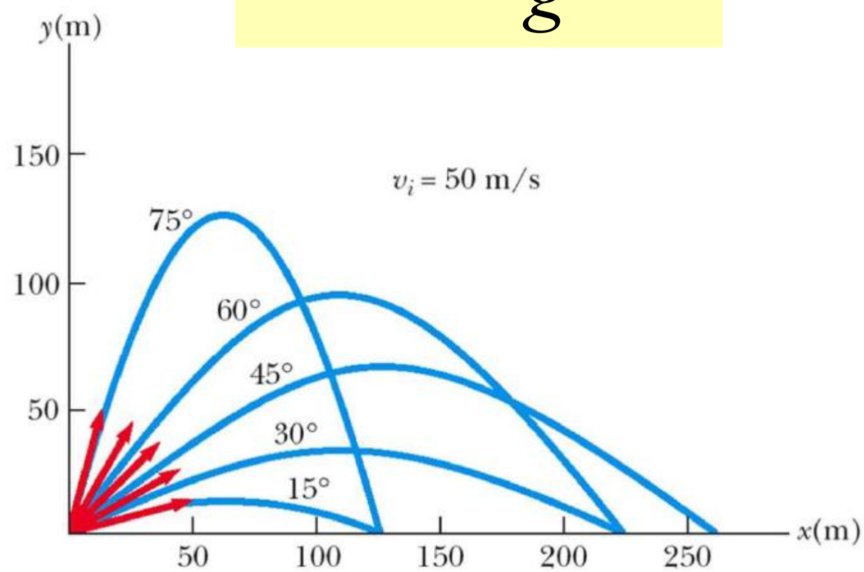


Horizontal	Vertical
$v_x = v_{0x}$	$v_y = v_{0y} - gt$
$x = x_0 + v_{0x} t$	$y = y_0 + v_{0y} t - \tfrac{1}{2} g t^2$

Projectile Motion at Various Initial Angles

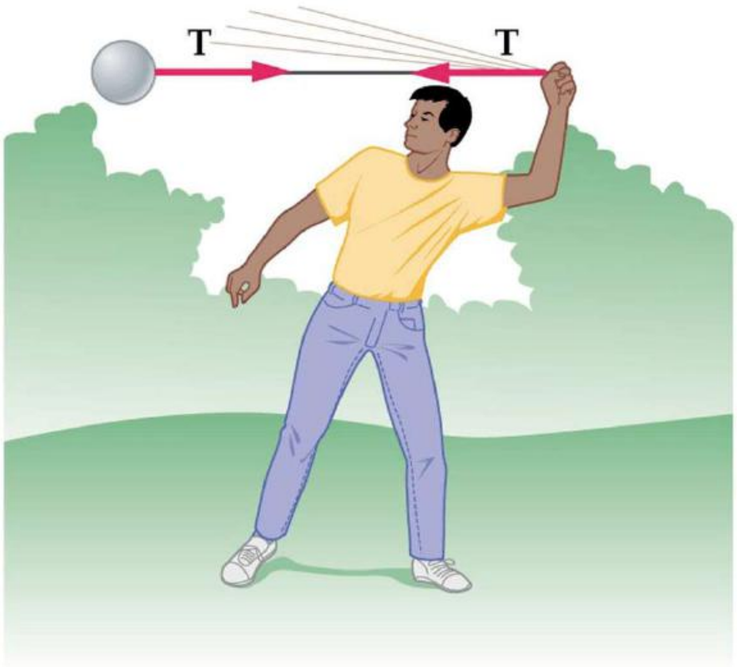
- Complementary values of the initial angle result in the same range
  - The heights will be different
- The maximum range occurs at a projection angle of 45°

$$R = \frac{v_0{}^2 \sin 2\phi}{g}$$

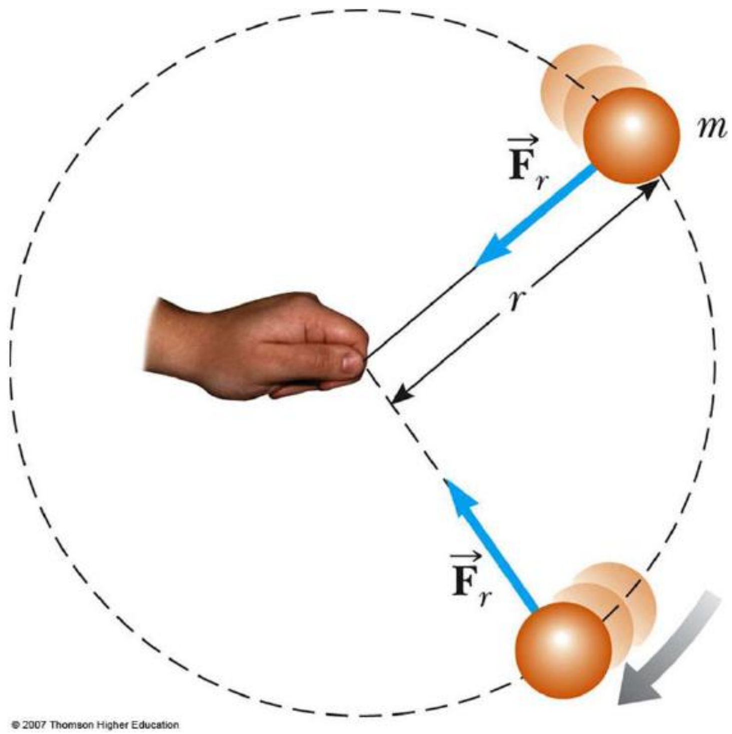


Circular Motion: Observations

- Object moving along a curved path with **constant speed**
  - Magnitude of velocity: same
  - Direction of velocity: changing
  - Velocity: changing
  - Acceleration is NOT zero!
  - **Net force acting on the object is NOT zero**
  - "**Centripetal force**"



$$\vec{F}_{net} = m\vec{a}$$



Uniform circular motion

Constant speed, or,
constant magnitude of velocity

Motion along a circle:
Changing direction of velocity

Uniform Circular Motion

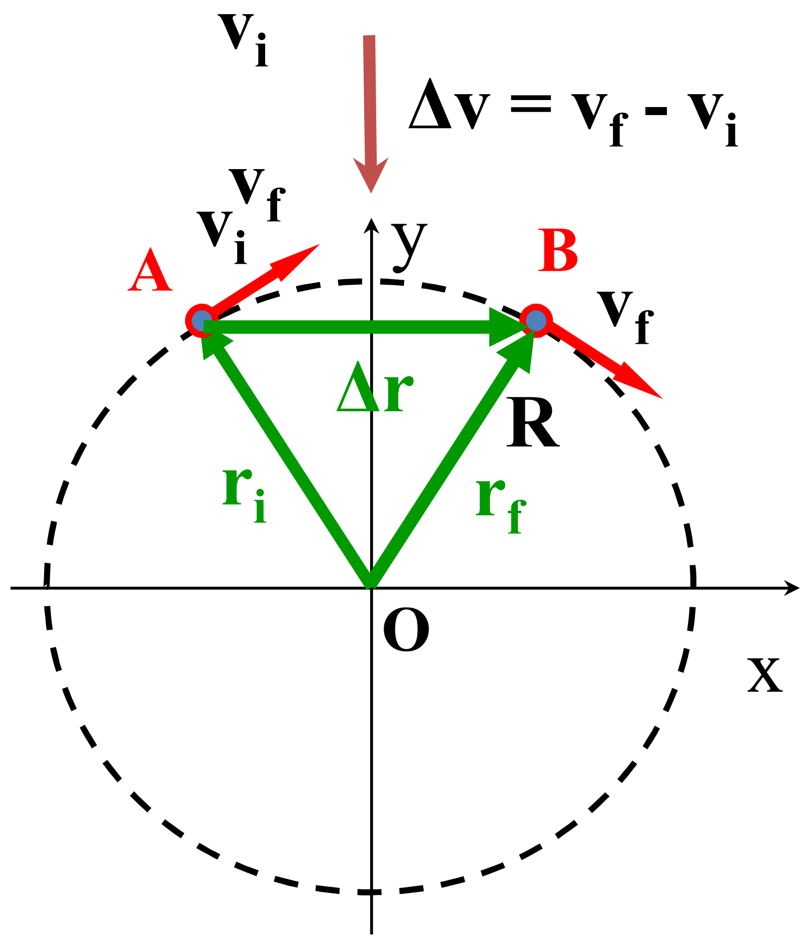
- Centripetal acceleration

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \qquad \text{so,} \quad \Delta v = \frac{v\Delta r}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{\Delta r}{\Delta t}\frac{v}{r} = \frac{v^2}{r}$$

$$a_r = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

- Direction: **Centripetal**

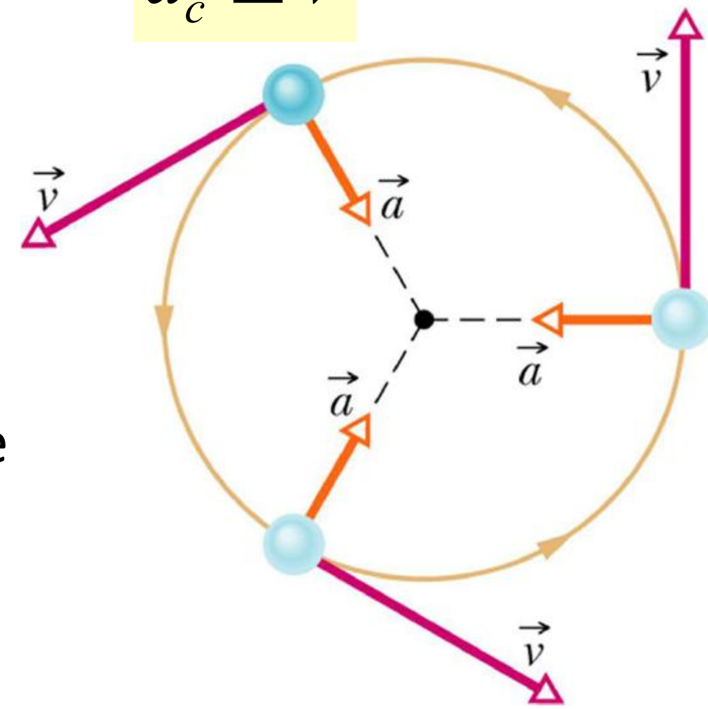


Uniform Circular Motion

- Velocity:
  - Magnitude: constant v
  - The direction of the velocity is tangent to the circle
- Acceleration:
  - Magnitude: $$a_c = \frac{v^2}{r}$$
  - directed toward the center of the circle of motion
- Period:
  - time interval required for one complete revolution of the particle

$$T = \frac{2\pi r}{v}$$

$$\vec{a}_c \perp \vec{v}$$



Review

- Position $\vec{r}(t) = x\hat{i} + y\hat{j}$
- Average velocity $$\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} = v_{avg,x}\hat{i} + v_{avg,y}\hat{j}$$
- Instantaneous velocity $v_x \equiv \frac{dx}{dt}$ $v_y \equiv \frac{dy}{dt}$
$$\vec{v}(t) = \lim_{t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$
- Acceleration $a_x \equiv \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$ $a_y \equiv \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$
$$\vec{a}(t) = \lim_{t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} = a_x\hat{i} + a_y\hat{j}$$
- $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ are not necessarily same direction.

- If a particle moves with constant acceleration a, motion equations are

$$\boxed{\vec{r}_f = \vec{r}_i + \vec{v}_i t + \tfrac{1}{2}\vec{a}t^2}$$

$$\vec{r}_f = x_f\hat{i} + y_f\hat{j} = (x_i + v_{xi}t + \tfrac{1}{2}a_{xi}t^2)\hat{i} + (y_i + v_{yi}t + \tfrac{1}{2}a_{yi}t^2)\hat{j}$$

$$\boxed{\vec{v} = \vec{v}_i + \vec{a}t}$$

$$\boxed{\vec{v}_f(t) = v_{fx}\hat{i} + v_{fy}\hat{j} = (v_{ix} + a_x t)\hat{i} + (v_{iy} + a_y t)\hat{j}}$$

- Projectile motion is one type of 2-D motion under constant acceleration, where $a_x = 0$, $a_y = -g$.

Basic Physics 1
Lecture Module

Rahadian N, S.Si. M.Si.

Rotational Motion

Rotational Motion

- Angular Position and Radians
- Angular Velocity
- Angular Acceleration
- Rigid Object under Constant Angular Acceleration
- Angular and Translational Quantities

Angle and Radian

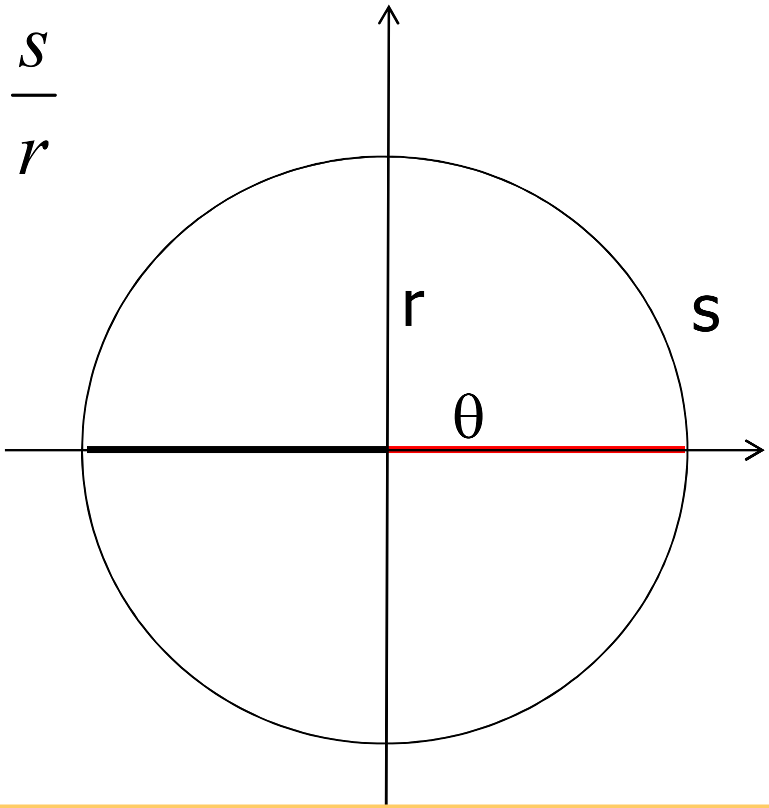
- What is the circumference *S* ?

$$s = (2\pi)r \qquad 2\pi = \frac{s}{r}$$

- θ can be defined as the arc length s along a circle divided by the radius r:

$$\theta = \frac{s}{r}$$

- θ is a pure number, but commonly is given the artificial unit, radian ("rad")



- Whenever using rotational equations, you must use angles expressed in radians

Conversions

- Comparing degrees and radians

$$2\pi(rad) = 360^\circ \qquad \pi(rad) = 180^\circ$$

- Converting from degrees to radians

$$\theta\left(rad\right) = \frac{\pi}{180^\circ}\,\theta\left(degrees\right)$$

- Converting from radians to degrees

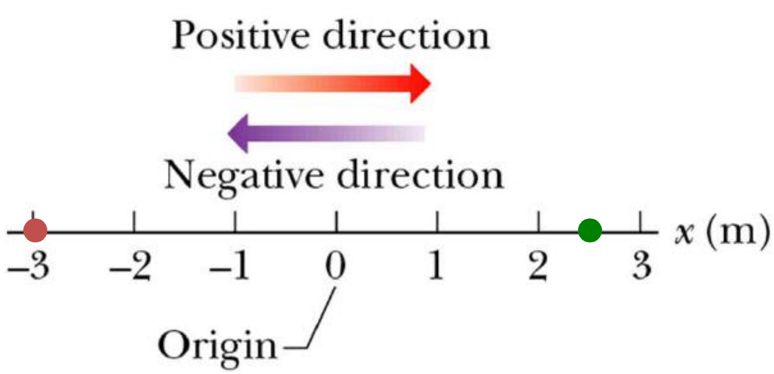
$$\theta(\deg rees) = \frac{180^\circ}{\pi}\theta(rad) \qquad 1\,rad = \frac{360^\circ}{2\pi} = 57.3^\circ$$

Rigid Object

- A rigid object is one that is nondeformable
  - The relative locations of all particles making up the object remain constant
  - All real objects are deformable to some extent, but the rigid object model is very useful in many situations where the deformation is negligible
- This simplification allows analysis of the motion of an extended object

Recall: 1-Dimensional Position x

- What is motion? Change of position over time.
- How can we represent position along a straight line?
- Position definition:
  - Defines a starting point: origin (x = 0), x relative to origin
  - Direction: positive (right or up), negative (left or down)
  - It depends on time: t = 0 (start clock), x(t=0) does not have to be zero.
- Position has units of [Length]: meters.

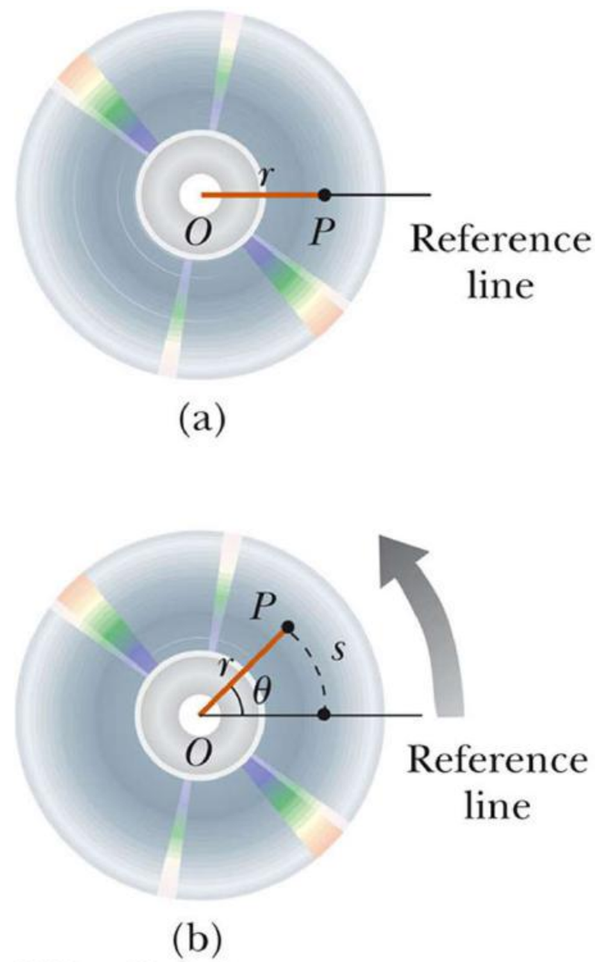


$x = + 2.5$ m

$x = - 3$ m

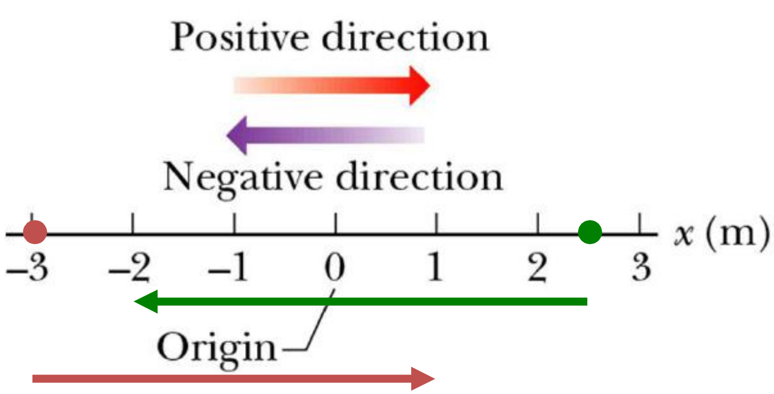
Angular Position

- Axis of rotation is the center of the disc
- Choose a fixed reference line
- Point P is at a fixed distance r from the origin
- As the particle moves, the only coordinate that changes is θ
- As the particle moves through θ, it moves though an arc length s.
- The angle θ, measured in radians, is called the angular position.



Recall: Displacement

- Displacement is a change of position in time.
- Displacement: $\boxed{\Delta x = x_f(t_f) - x_i(t_i)}$
  - f stands for final and i stands for initial.
- It is a vector quantity.
- It has both magnitude and direction: + or - sign
- It has units of [length]: meters.



$x_1(t_1) = +2.5$ m
$x_2(t_2) = -2.0$ m
$\Delta x = -2.0$ m $- 2.5$ m $= -4.5$ m

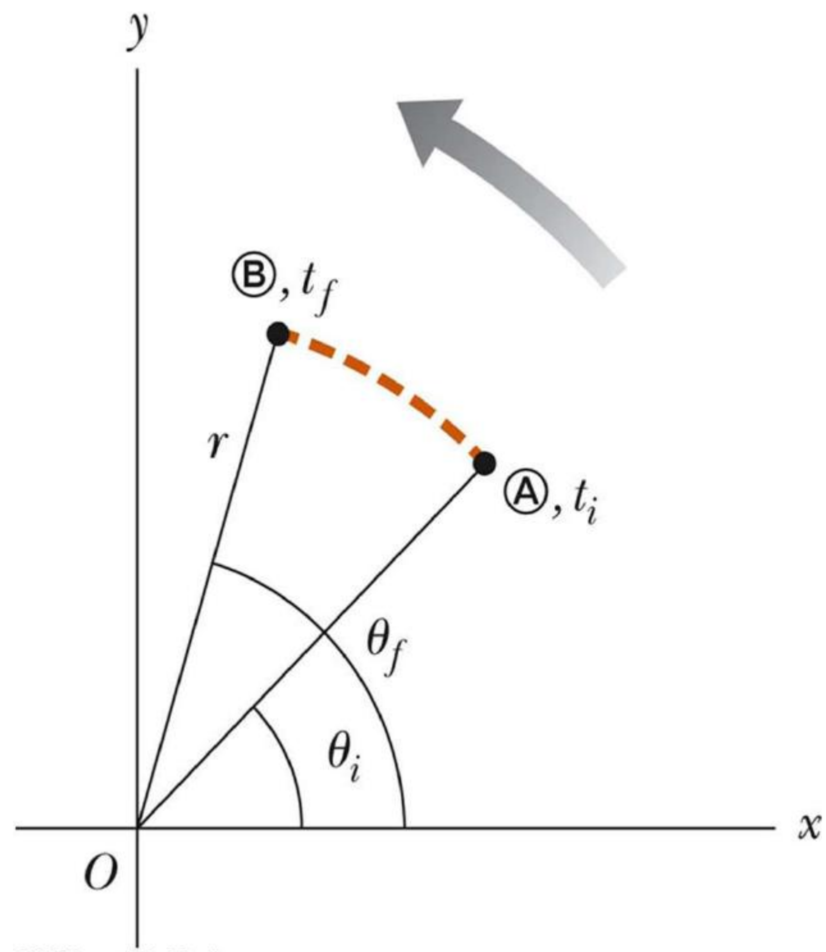
$x_1(t_1) = -3.0$ m
$x_2(t_2) = +1.0$ m
$\Delta x = +1.0$ m $+ 3.0$ m $= +4.0$ m

Angular Displacement

- The angular displacement is defined as the angle the object rotates through during some time interval

$$\Delta\theta = \theta_f - \theta_i$$

- SI unit: radian (rad)
- This is the angle that the reference line of length *r* sweeps out



Recall: Velocity

- Velocity is the rate of change of position.
- Velocity is a vector quantity.
- Velocity has both magnitude and direction.
- Velocity has a unit of [length/time]: meter/second.
- Definition:
  - Average velocity $$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$
  - Average speed $$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}$$
  - Instantaneous velocity $$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

Average and Instantaneous Angular Speed

- The *average* angular speed, ω_{avg}, of a rotating rigid object is the ratio of the angular displacement to the time interval

$$\omega_{avg} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

- The *instantaneous* angular speed is defined as the limit of the average speed as the time interval approaches zero

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

- SI unit: radian per second (rad/s)
- Angular speed positive if rotating in counterclockwise
- Angular speed will be negative if rotating in clockwise

Recall: Average Acceleration

- Changing velocity (non-uniform) means an acceleration is present.
- Acceleration is the rate of change of velocity.
- Acceleration is a vector quantity.
- Acceleration has both magnitude and direction.
- Acceleration has a unit of [length/time^2]: m/s^2.
- Definition:
  - Average acceleration

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

  - Instantaneous acceleration

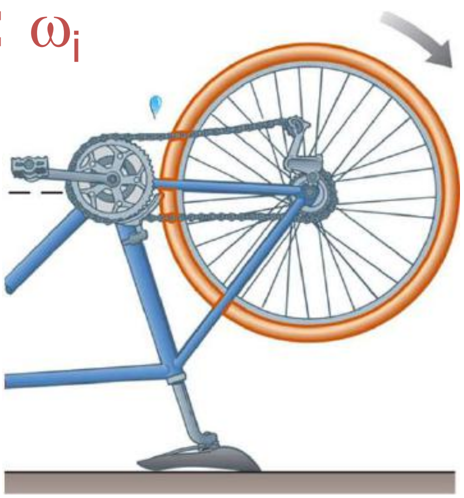
$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\frac{dx}{dt} = \frac{d^2 v}{dt^2}$$

Average Angular Acceleration

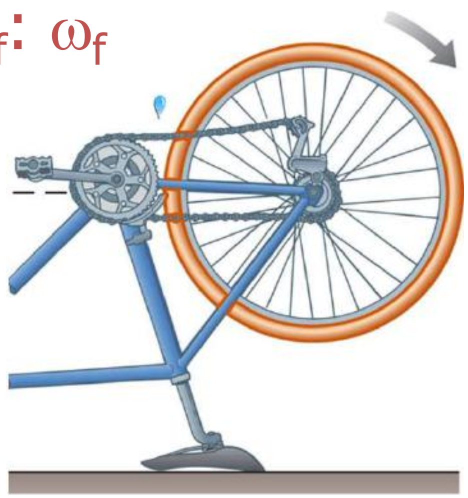
- The average angular acceleration, α, of an object is defined as the ratio of the change in the angular speed to the time it takes for the object to undergo the change:

$$\alpha_{avg} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

$t = t_i$: ω_i



$t = t_f$: ω_f



Instantaneous Angular Acceleration

- The instantaneous angular acceleration is defined as the limit of the average angular acceleration as the time goes to 0

$$\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

- SI Units of angular acceleration: rad/s^2
- Positive angular acceleration is counterclockwise (RH rule – curl your fingers in the direction of motion).
  - if an object rotating counterclockwise is speeding up
  - if an object rotating clockwise is slowing down
- Negative angular acceleration is clockwise.
  - if an object rotating counterclockwise is slowing down
  - if an object rotating clockwise is speeding up

Rotational Kinematics

- A number of parallels exist between the equations for rotational motion and those for linear motion.

$$v_{avg} = \frac{x_f - x_i}{t_f - t_i} = \frac{\Delta x}{\Delta t} \qquad \omega_{avg} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta \theta}{\Delta t}$$

- Under **constant angular acceleration**, we can describe the motion of the rigid object using a set of kinematic equations
  - These are similar to the kinematic equations for linear motion
  - The rotational equations have the same mathematical form as the linear equations

Analogy with Linear Kinematics

- Start with angular acceleration: $\alpha = \dfrac{d\omega}{dt}$
- Integrate once: $\omega_f = \int \alpha\, dt = \omega_i + \alpha t$ $\quad v_f = v_i + at$
- Integrate again: $\theta_f = \int \left(\omega_i + \alpha t\right) dt = \theta_i + \omega_i t + \tfrac{1}{2}\alpha t^2$

$$x_f = x_i + v_i t + \tfrac{1}{2}at^2$$

- Just substitute symbols, and all of the old equations apply: $x \Rightarrow \theta$

$$v \Rightarrow \omega$$

$$a \Rightarrow \alpha$$

Comparison Between Rotational and Linear Equations

Rotational Motion About a Fixed Axis	Translational Motion
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

A Rotating Wheel

- A wheel rotates with a constant angular acceleration of 3.5 rad/s^2. If the angular speed of the wheel is 2.0 rad/s at t = 0

(a) through what angle does the wheel rotate between t = 0 and t = 2.0 s? Given your answer in radians and in revolutions.

(b) What is the angular speed of the wheel at t = 2.0 s?

$$\omega_i = 2.0\, rad / s$$

$$\alpha = 3.5\, rad / s^2$$

$$t = 2.0\, s$$

$$\theta_f - \theta_i = ?$$

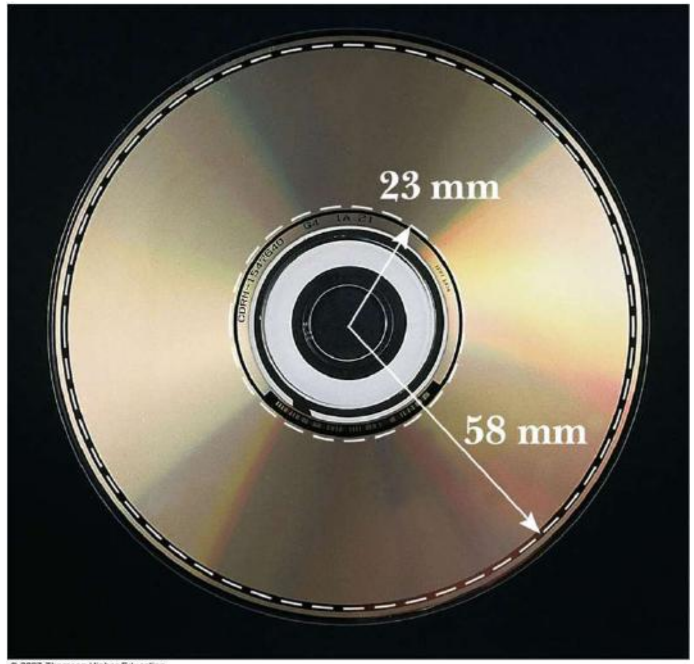
$$\omega_f = ?$$

Hints for Problem-Solving

- Similar to the techniques used in linear motion problems
  - With constant angular acceleration, the techniques are much like those with constant linear acceleration
- There are some differences to keep in mind
  - For rotational motion, define a rotational axis
    - The choice is arbitrary
    - Once you make the choice, it must be maintained
    - In some problems, the physical situation may suggest a natural axis
  - The object keeps returning to its original orientation, so you can find the number of revolutions made by the body

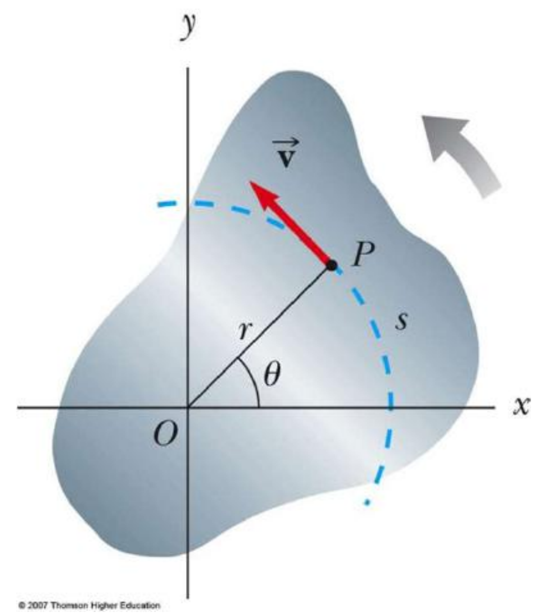
Relationship Between Angular and Linear Quantities

- Every point on the rotating object has the same angular motion
- Every point on the rotating object does **not** have the same linear motion
- Displacement $s = \theta r$
- Speeds $v = \omega r$
- Accelerations $a = \alpha r$



Speed Comparison

- The linear velocity is always tangent to the circular path
  - Called the tangential velocity



- The magnitude is defined by the tangential speed

$$\Delta\theta = \frac{\Delta s}{r}$$

$$\frac{\Delta\theta}{\Delta t} = \frac{\Delta s}{r\Delta t} = \frac{1}{r}\frac{\Delta s}{\Delta t} \qquad \omega = \frac{v}{r} \quad or \quad v = r\omega$$

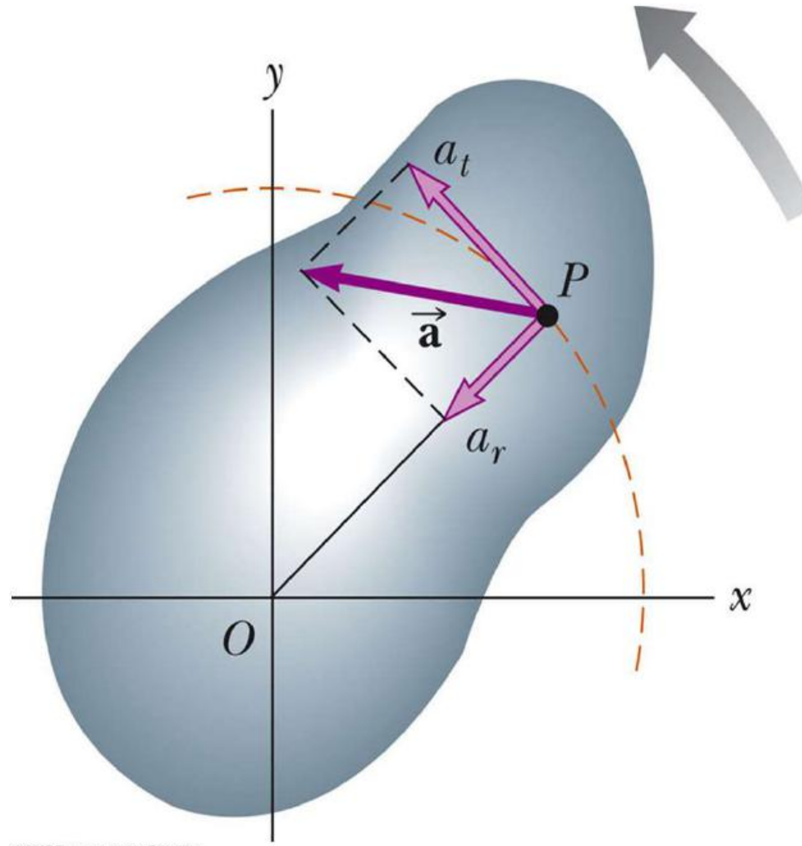
Acceleration Comparison

- The tangential acceleration is the derivative of the tangential velocity

$$\Delta v = r\Delta\omega$$

$$\frac{\Delta v}{\Delta t} = r\frac{\Delta\omega}{\Delta t} = r\alpha$$

$$a_t = r\alpha$$



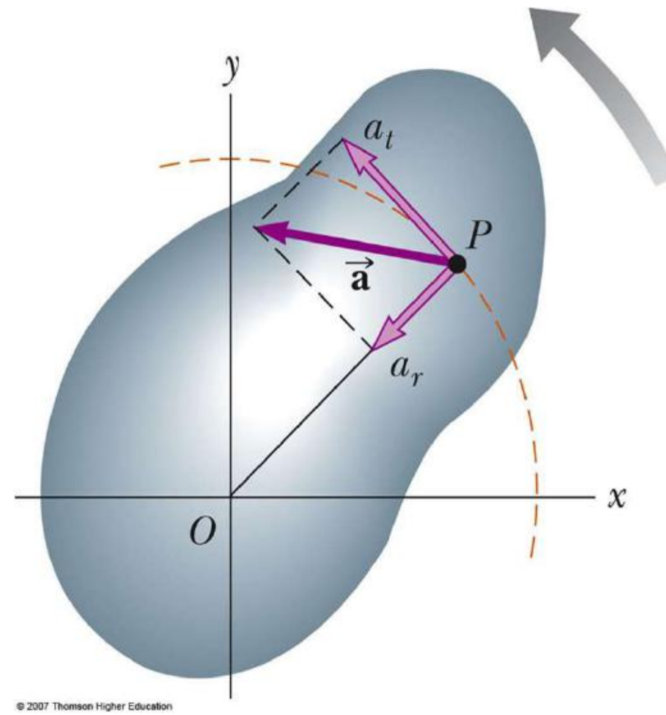
Speed and Acceleration Note

- All points on the rigid object will have the same angular speed, but not the same tangential speed
- All points on the rigid object will have the same angular acceleration, but not the same tangential acceleration
- The tangential quantities depend on r, and r is not the same for all points on the object

$$\omega = \frac{v}{r} \quad or \quad v = r\omega \qquad\qquad a_t = r\alpha$$

Centripetal Acceleration

- An object traveling in a circle, even though it moves with a constant speed, will have an acceleration
  - Therefore, each point on a rotating rigid object will experience a centripetal acceleration



$$a_r = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

Resultant Acceleration

- The tangential component of the acceleration is due to changing speed

- The centripetal component of the acceleration is due to changing direction

- Total acceleration can be found from these components

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{r^2\alpha^2 + r^2\omega^4} = r\sqrt{\alpha^2 + \omega^4}$$

Basic Physics 1
Lecture Module

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Newton's Laws and Applications

Newton’s Laws and Applications

- Newton’s first law
- Newton’s second law
- Newton’s third law
- Applications

Newton's Laws

Force is a vector
Unit of force in S.I.:

$$1N = 1\frac{kg\bullet m}{s^2}$$

I. If no net force acts on a body, then the body's velocity cannot change.

II. The net force on a body is equal to the product of the body's mass and acceleration.

III. When two bodies interact, the force on the bodies from each other are always equal in magnitude and opposite in direction.

Newton's First Law

An object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force

- An object at rest remains at rest as long as no **net force** acts on it
- An object moving with **constant velocity** continues to move with the same speed and in the same direction (the same velocity) as long as no net force acts on it *"Keep on doing what it is doing".*
- When forces are balanced, the acceleration of the object is zero
  - Object at rest: $v = 0$ and $a = 0$
  - Object in motion: $v \neq 0$ and $a = 0$
- The net force is defined as the vector sum of all the external forces exerted on the object. If the net force is zero, forces are balanced. When forces are balances, the object can be stationary, or move with constant velocity.

Mass and Inertia

- Every object continues in its state of rest, or uniform motion in a straight line, unless it is compelled to change that state by unbalanced forces impressed upon it

- **Inertia** is a property of objects to resist changes is motion. **Mass** is a measure of the amount of **inertia**.

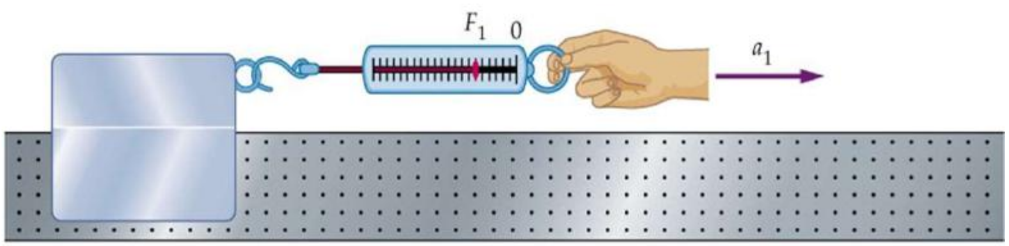
- **Mass** is a measure of the resistance of an object to changes in its velocity.

- Mass is an inherent property of an object

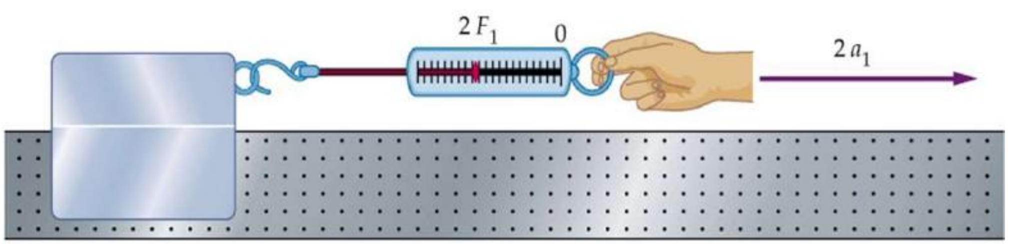
- Scalar quantity and SI unit: kg

Newton's Second Law

- The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass



$$\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}$$



$$\vec{F}_{net} = \sum \vec{F} = m\vec{a}$$

Units of Force

- Newton's second law:

$$\vec{F}_{net} = \sum \vec{F} = m\vec{a}$$

- SI unit of force is a Newton (N)

$$1\ \text{N} \equiv 1 \frac{\text{kg m}}{\text{s}^2}$$

- US Customary unit of force is a pound (lb)
  - 1 N = 0.225 lb
  - Weight, also measured in lbs. is a force (mass x acceleration). What is the acceleration in that case?

More about Newton's 2nd Law

- You must be certain about which body we are applying it to
- $\mathbf{F}_{\text{net}}$ must be the vector sum of all the forces that act on that body
- Only forces that act on that body are to be included in the vector sum
- Net force component along an axis gives rise to the acceleration along that same axis

$$F_{net,x} = ma_x \qquad F_{net,y} = ma_y$$

Gravitational Force

- Gravitational force is a vector
- Expressed by Newton's Law of Universal Gravitation:

$$F_g = G\frac{mM}{R^2}$$

  – G – gravitational constant
  – M – mass of the Earth
  – m – mass of an object
  – R – radius of the Earth
- Direction: pointing downward

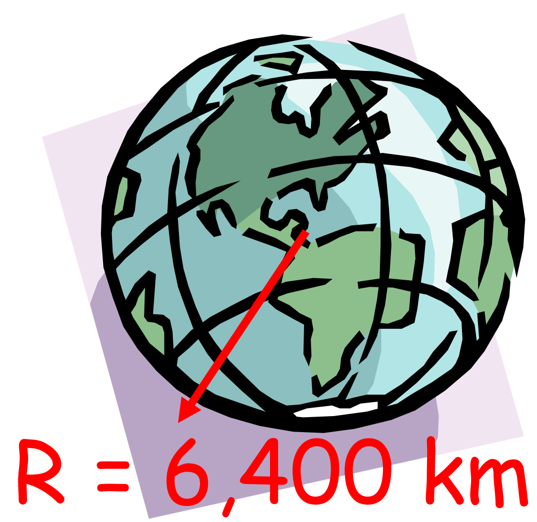
Weight

- The magnitude of the gravitational force acting on an object of mass m near the Earth's surface is called the weight w of the object: $w = mg$
- $\boldsymbol{g}$ can also be found from the Law of Universal Gravitation
- Weight has a unit of N

$$F_g = G\frac{mM}{R^2} \qquad w = F_g = mg$$

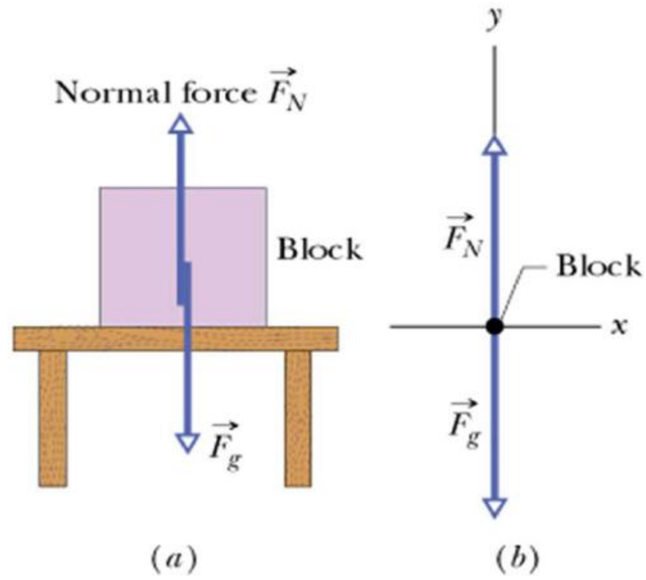
$$g = G\frac{M}{R^2} = 9.8\ \text{m/s}^2$$

- Weight depends upon location



Normal Force

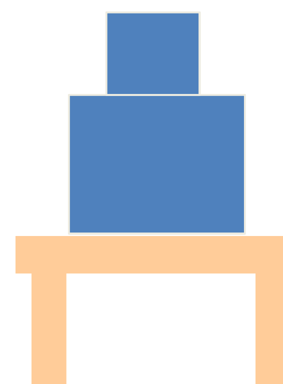
- Force from a solid surface which keeps object from falling through
- Direction: always perpendicular to the surface
- Magnitude: depends on situation



$$N - F_g = ma_y$$

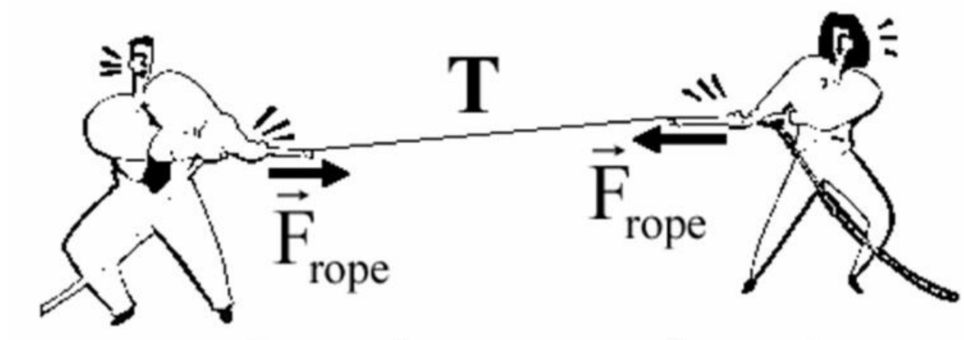
$$N - mg = ma_y$$

$$N = mg$$

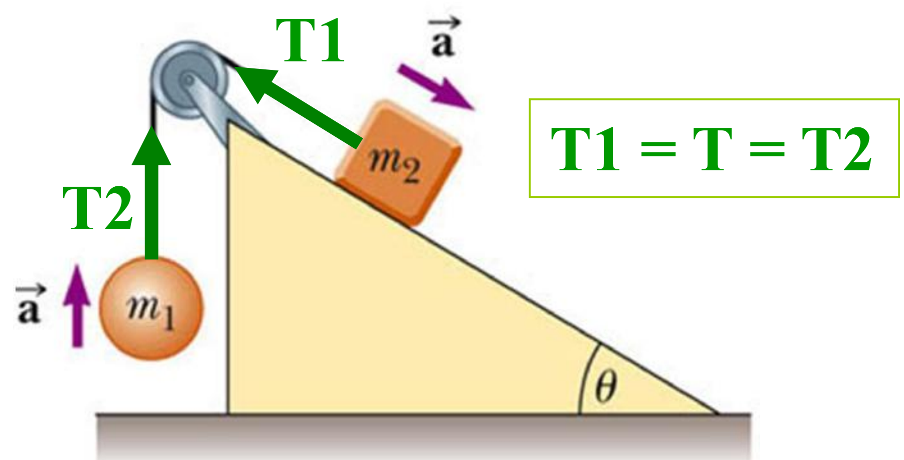


Tension Force

- A taut rope exerts forces on whatever holds its ends
- Direction: always along the cord (rope, cable, string ……) and away from the object
- Magnitude: depend on situation

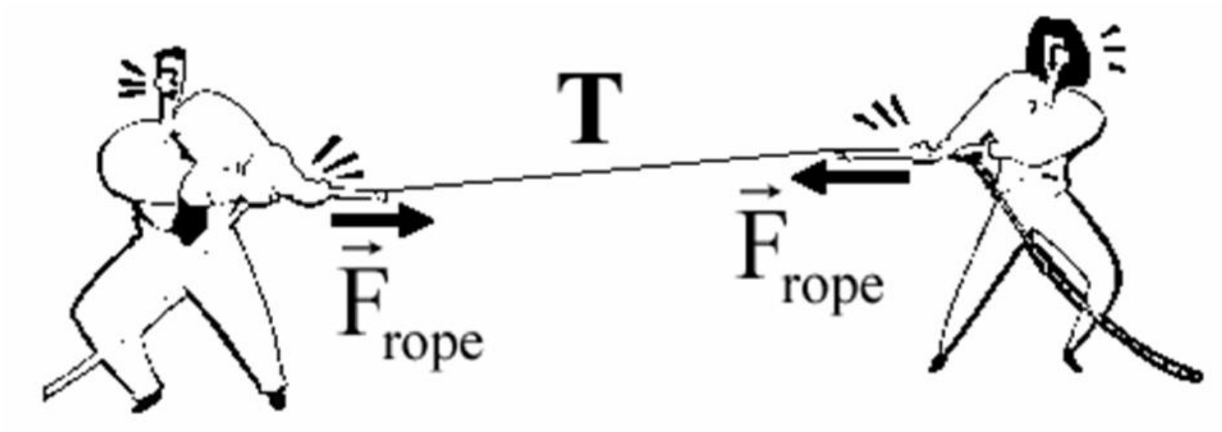


$$\left|\vec{F}_{\text{on A}}\right| = T = \left|\vec{F}_{\text{on B}}\right|$$



Newton's Third Law

- If object 1 and object 2 interact, the force exerted by object 1 on object 2 is equal in magnitude but opposite in direction to the force exerted by object 2 on object 1



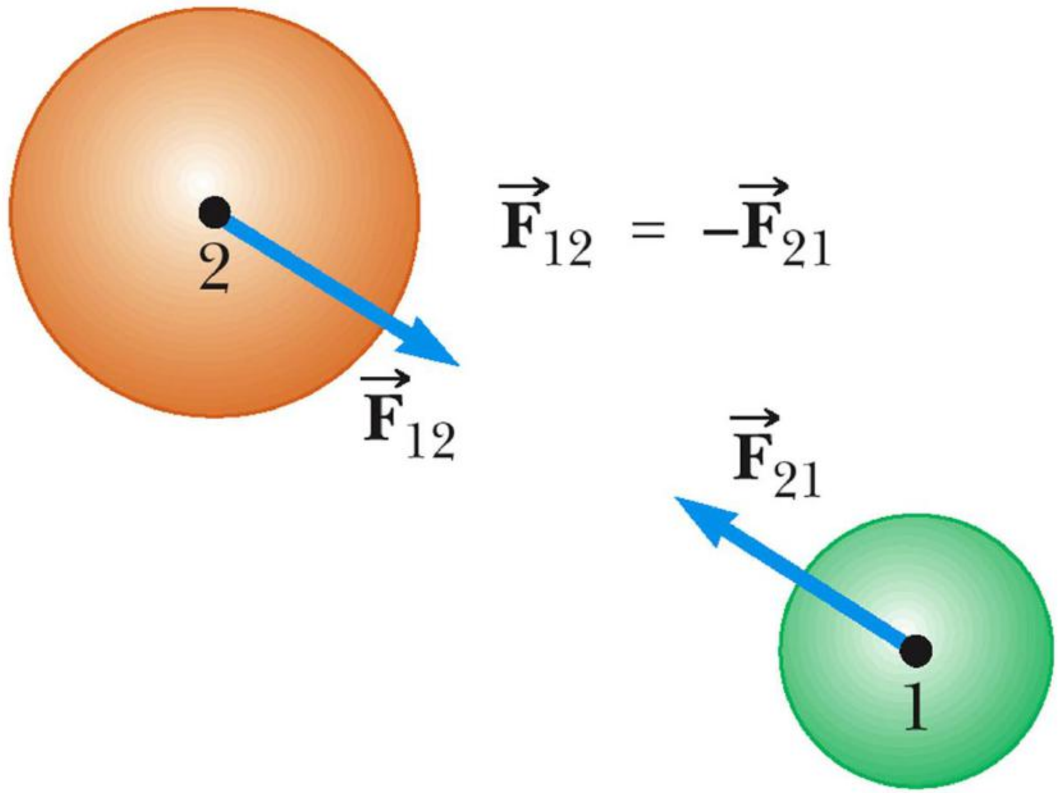
$$\left|\vec{F}_{\text{on A}}\right| = T = \left|\vec{F}_{\text{on B}}\right|$$

$$\vec{F}_{on\ A} = -\vec{F}_{on\ B}$$

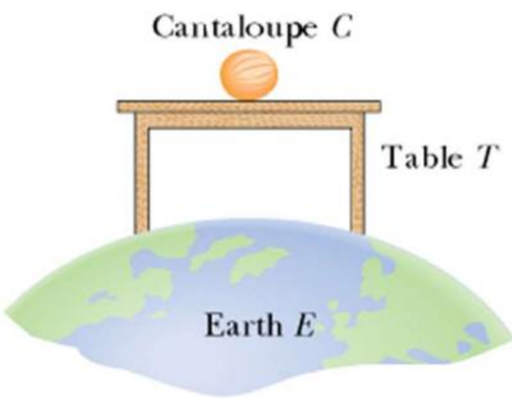
- Equivalent to saying a single isolated force cannot exist

Newton's Third Law cont.

- F_{12} may be called the *action* force and F_{21} the *reaction* force
  - Actually, either force can be the action or the reaction force
- The action and reaction forces act on **different** objects

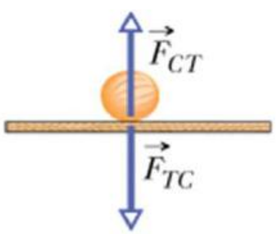


Some Action-Reaction Pairs



(a)

(c)



(d)

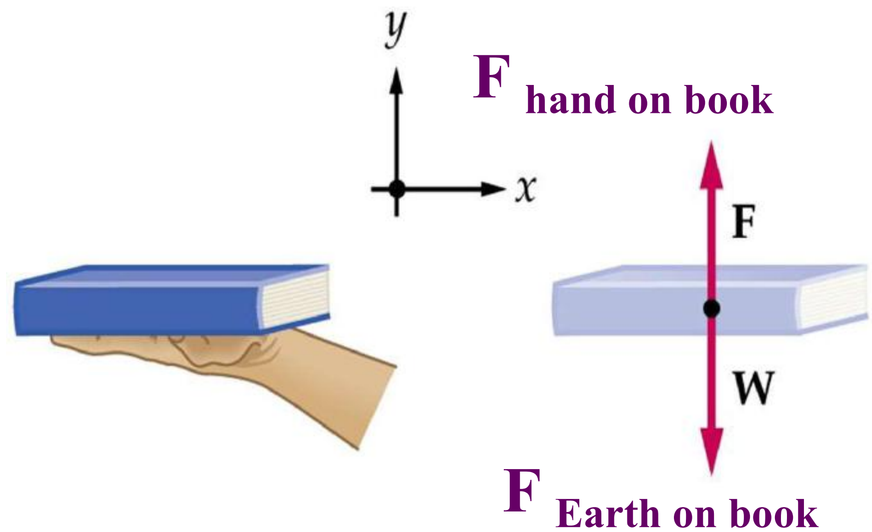
$$F_g = mg = m\frac{GM}{R^2}$$

$$F_g = G\frac{mM}{R^2}$$

$$F_g = Ma = M\frac{Gm}{R^2}$$

Free Body Diagram

- The most important step in solving problems involving Newton's Laws is to draw the free body diagram
- Be sure to include only the forces acting on the object of interest
- Include any field forces acting on the object
- Do not assume the normal force equals the weight



(a) Physical picture

(b) Free-body diagram

Hints for Problem-Solving

- **Read** the problem carefully at least once
- **Draw** a picture of the system, identify the object of primary interest, and indicate forces with arrows
- **Label** each force in the picture in a way that will bring to mind what physical quantity the label stands for (e.g., T for tension)
- **Draw** a free-body diagram of the object of interest, based on the labeled picture. If additional objects are involved, draw separate free-body diagram for them
- **Choose a convenient coordinate system** for each object
- **Apply Newton's second law.** The x- and y-components of Newton second law should be taken from the vector equation and written individually. This often results in two equations and two unknowns
- **Solve** for the desired unknown quantity, and substitute the numbers

$$F_{net,x} = ma_x \qquad F_{net,y} = ma_y$$

Objects in Equilibrium

- Objects that are either at rest or moving with constant velocity are said to be in **equilibrium**
- Acceleration of an object can be modeled as zero
- Mathematically, the net force acting on the object is zero

$$\vec{a} = 0 \qquad\qquad \sum \vec{F} = 0$$

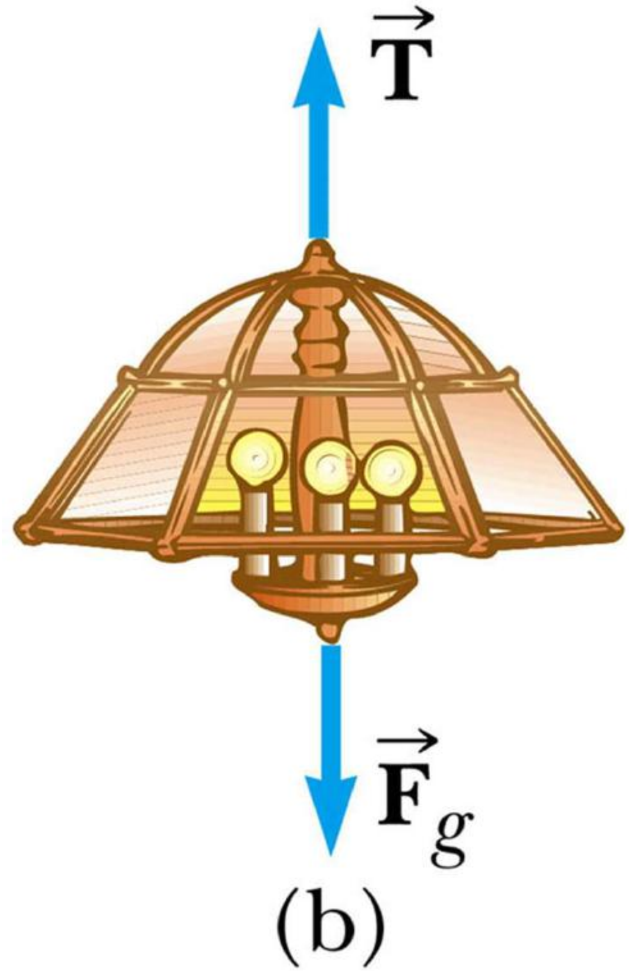
- Equivalent to the set of component equations given by

$$\sum F_x = 0 \qquad\qquad \sum F_y = 0$$

Equilibrium, Example 1

- A lamp is suspended from a chain of negligible mass
- The forces acting on the lamp are
  - the downward force of gravity
  - the upward tension in the chain
- Applying equilibrium gives

$$\sum F_y = 0 \rightarrow T - F_g = 0 \rightarrow T = F_g$$



Equilibrium, Example 2

- A traffic light weighing 100 N hangs from a vertical cable tied to two other cables that are fastened to a support. The upper cables make angles of $37°$ and $53°$ with the horizontal. Find the tension in each of the three cables.

- Conceptualize the traffic light
  - Assume cables don't break
  - Nothing is moving
- Categorize as an equilibrium problem
  - No movement, so acceleration is zero
  - Model as an object in equilibrium

$$\sum F_x = 0 \qquad \sum F_y = 0$$

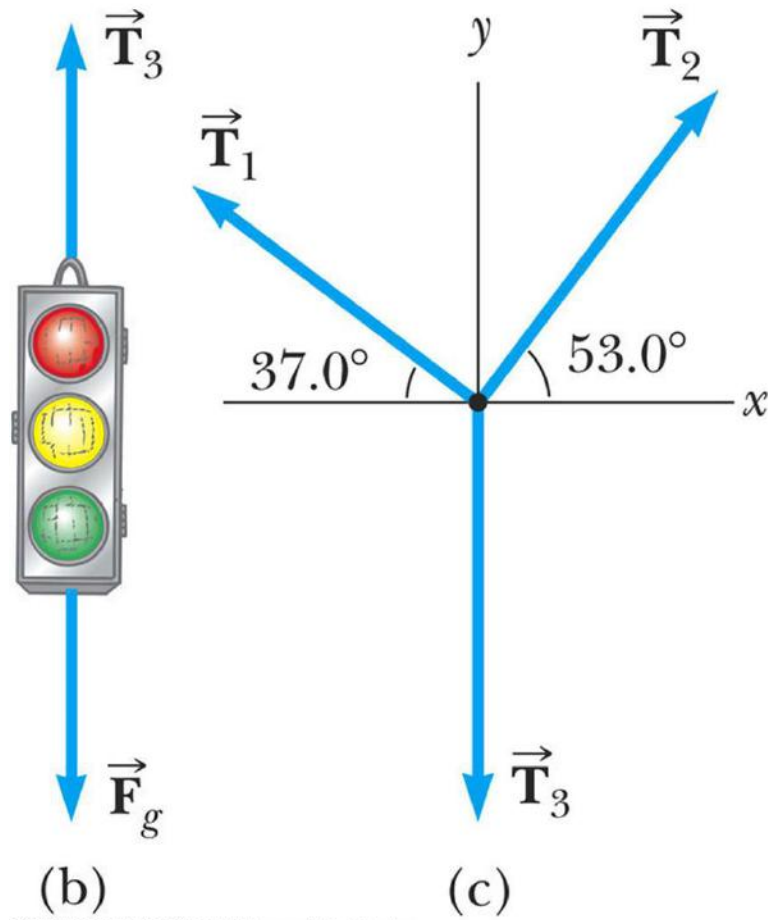
Equilibrium, Example 2

- Need 2 free-body diagrams
  - Apply equilibrium equation to light

$$\sum F_y = 0 \rightarrow T_3 - F_g = 0$$
$$T_3 = F_g = 100N$$

  - Apply equilibrium equations to knot

$$\sum F_x = T_{1x} + T_{2x} = -T_1 \cos 37^\circ + T_2 \cos 53^\circ = 0$$
$$\sum F_y = T_{1y} + T_{2y} + T_{3y}$$
$$= T_1 \sin 37^\circ + T_2 \sin 53^\circ - 100N = 0$$
$$T_2 = T_1 \left(\frac{\cos 37^\circ}{\cos 53^\circ} \right) = 1.33 T_1$$
$$T_1 = 60N \qquad T_2 = 1.33 T_1 = 80N$$



Accelerating Objects

- If an object that can be modeled as a particle experiences an acceleration, there must be a nonzero net force acting on it
- Draw a free-body diagram
- Apply Newton's Second Law in component form

$$\sum \vec{F} = m\vec{a}$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$

Accelerating Objects, Example 1

- A man weighs himself with a scale in an elevator. While the elevator is at rest, he measures a weight of 800 N.
  - What weight does the scale read if the elevator accelerates upward at 2.0 m/s^2?   a = 2.0 m/s^2
  - What weight does the scale read if the elevator accelerates downward at 2.0 m/s^2? a = - 2.0 m/s^2

❑ Upward: $\sum F_y = N - mg = ma$

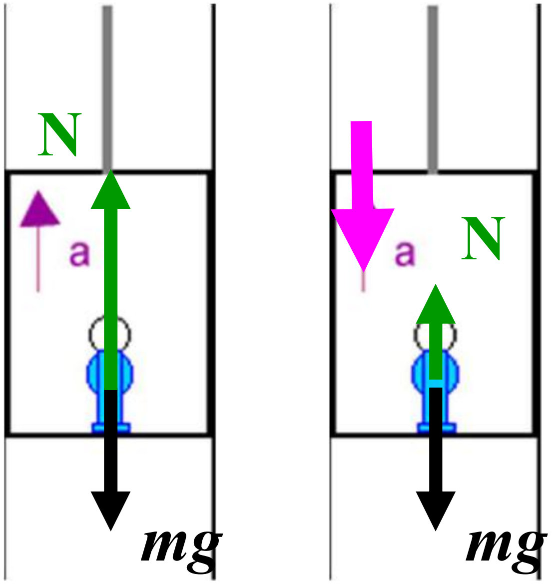
$$N = mg + ma = m(g + a)$$

$$m = \frac{w}{g} = \frac{800\,\mathrm{N}}{9.8\,\mathrm{m/s^2}} = 80\,\mathrm{N}$$

$$\boxed{N > mg}$$

❑ Downward: $N = 80(-2.0 + 9.8) = 624\,\mathrm{N}$

$$\boxed{N < mg}$$



Basic Physics 1
Lecture Module

Rahadian N, S.Si. M.Si.

Forces

Forces

- Dynamics
- Force
- Friction
- Applications
- Dynamics: uniform circular motion

Dynamics

- Describes the relationship between the motion of objects in our everyday world and the forces acting on them.
- Dynamics analysis
- Language of Dynamics
  - **Mass:** The measure of how difficult it is to change object's velocity (sluggishness or inertia of the object)
  - **Force:** The measure of interaction between **two objects** (pull or push). It is a vector quantity – it has a ***magnitude and direction***

Dynamics Analysis

- Determine the reaction forces on pins, etc. as a consequence of a specified motion.
- Determine the input force of torque required to achieve a specified motion, or determine the motion as a consequence of a specified set of forces and/or torques.
- *Inverse Kinematics: s*tart with the motion and determine the forces, or
- *Direct kinematics:* start with the forces and determine the motion.

Mass

- Every object continues in its state of rest, or uniform motion in a straight line, unless it is compelled to change that state by unbalanced forces impressed upon it

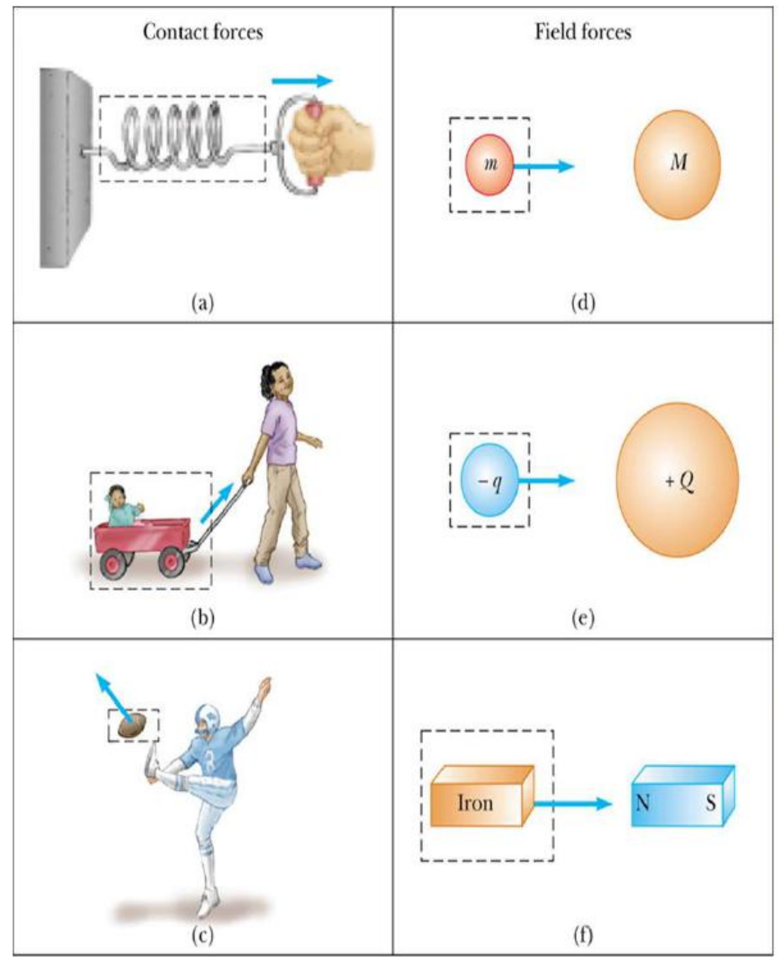
- **Mass** is a measure of the resistance of an object to changes in its velocity.

- Mass is an inherent property of an object

- Scalar quantity and SI unit: kg

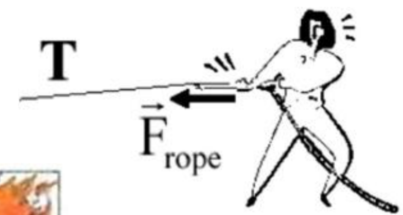
Force

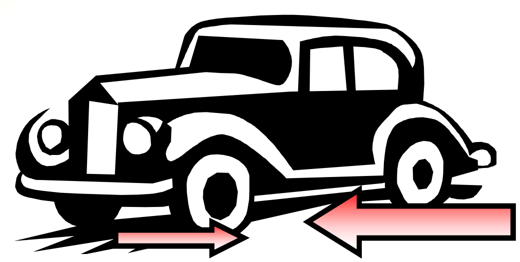
- The measure of interaction between **two objects** (pull or push)
- Vector quantity: has ***magnitude and direction***
- May be a **contact force** or a **field force**
  – Contact forces result from physical contact between two objects
  – Field forces act between disconnected objects
    • Also called “action at a distance”



Types of Force

- Gravitational Force
- Archimedes Force
- Friction Force
- Tension Force
- Spring Force
- Normal Force
- **Etc**



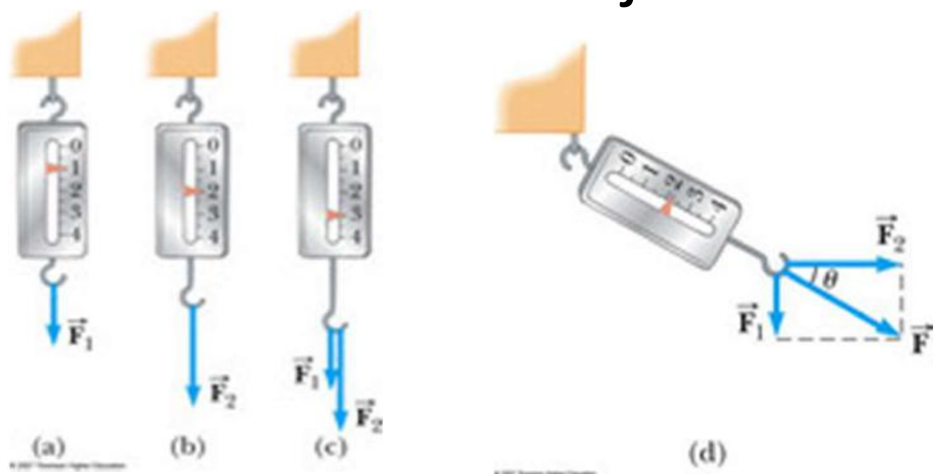


Vector Nature of Force

- Vector force: has ***magnitude and direction***
- Net Force: a resultant force acting on object

$$\vec{F}_{net} = \sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots\ldots$$

- You must use the rules of vector addition to obtain the net force on an object



$$|\vec{F}| = \sqrt{F_1^2 + F_2^2} = 2.24\,\mathrm{N}$$

$$\theta = \tan^{-1}\left(\frac{F_1}{F_2}\right) = -26.6^\circ$$

Units of Force

- Newton's second law:

$$\vec{F}_{net} = \sum \vec{F} = m\vec{a}$$

- SI unit of force is a Newton (N)

$$1\ \mathrm{N} \equiv 1\ \frac{\mathrm{kg\ m}}{\mathrm{s}^2}$$

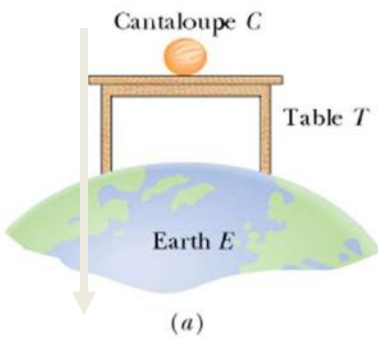
- US Customary unit of force is a pound (lb)
  - 1 N = 0.225 lb
  - Weight, also measured in lbs. is a force (mass x acceleration). What is the acceleration in that case?

Gravitational Force: *mg*

- Gravitational force is a vector
- The magnitude of the gravitational force acting on an object of mass *m* near the Earth's surface is called the weight *w* of the object

$$w = mg \qquad\qquad F_g = G\frac{mM}{R^2}$$

- Direction: vertically downward

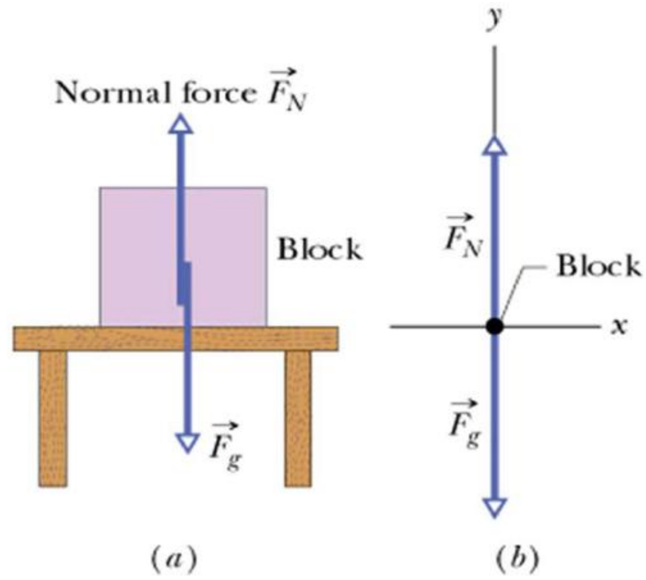


m: Mass

g = 9.8 m/s^2

Normal Force: $\boldsymbol{N}$

- Force from a solid surface which keeps object from falling through
- Direction: always perpendicular to the surface
- Magnitude: not necessary to be $\boldsymbol{mg}$



$$N - F_g = ma_y$$

$$N - mg = ma_y$$

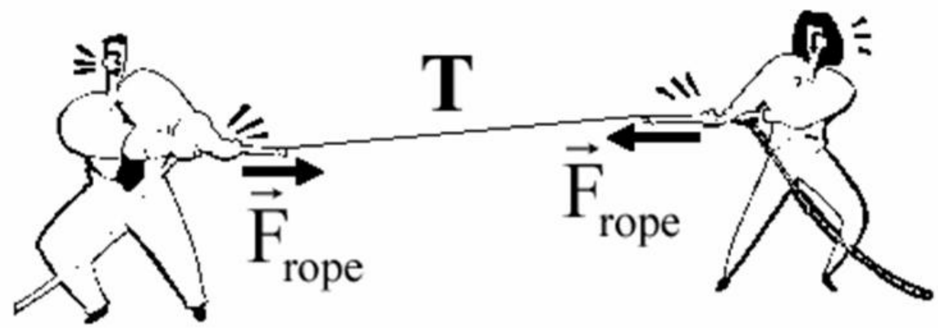
$$N = mg$$

Tension Force: T

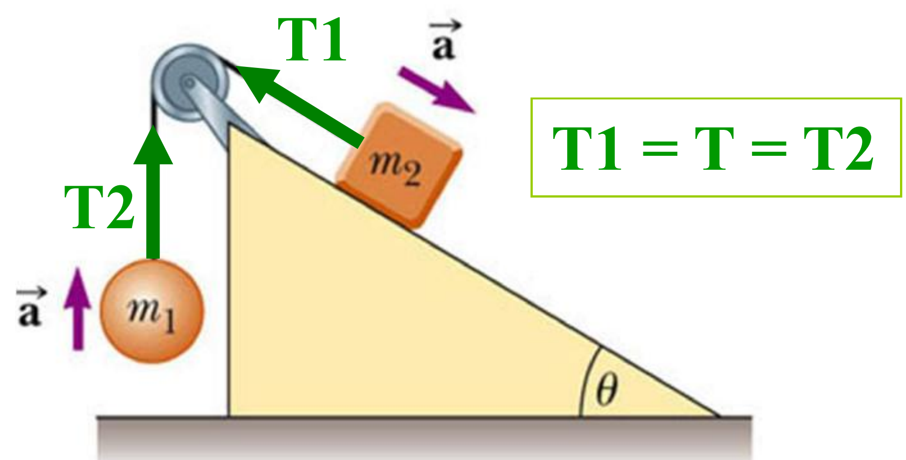
- A taut rope exerts forces on whatever holds its ends

- Direction: always along the cord (rope, cable, string ……) and away from the object

- Magnitude: depend on situation

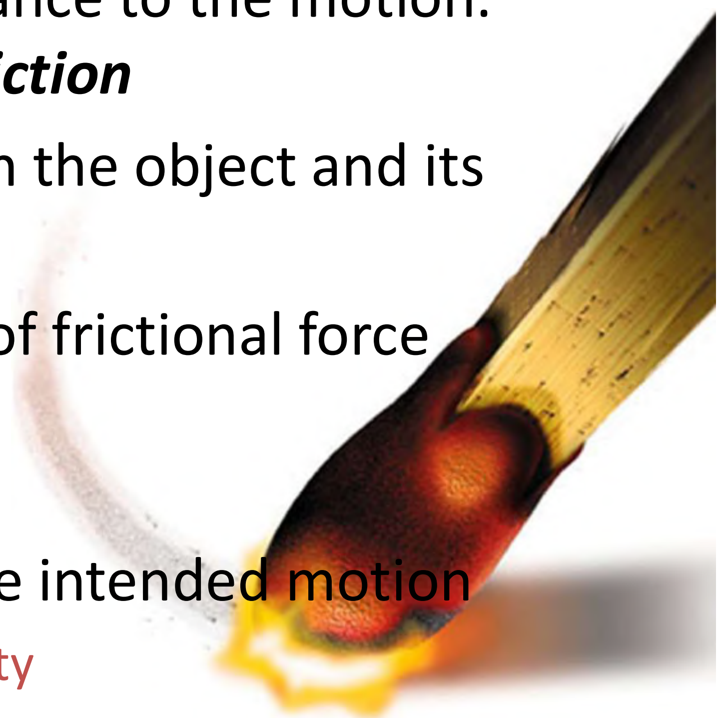


$$\left|\vec{F}_{\text{on A}}\right| = T = \left|\vec{F}_{\text{on B}}\right|$$



Friction (f)

- When an object is in motion on a surface or through a viscous medium, there will be a resistance to the motion. This resistance is called the ***force of friction***
- This is due to the interactions between the object and its environment
- We will be concerned with two types of frictional force
  - Force of static friction: f_s
  - Force of kinetic friction: f_k
- Direction: opposite the direction of the intended motion
  - If moving: in direction opposite the velocity
  - If stationary, in direction of the vector sum of other forces



Forces of Friction *Magnitude*

- Magnitude: Friction is proportional to the normal force
  - Static friction: $F_f = F \leq \mu_s N$
  - Kinetic friction: $F_f = \mu_k N$
  - μ is the **coefficient of friction**
- The coefficients of friction are nearly independent of the area of contact (why?)

Coefficients of Friction

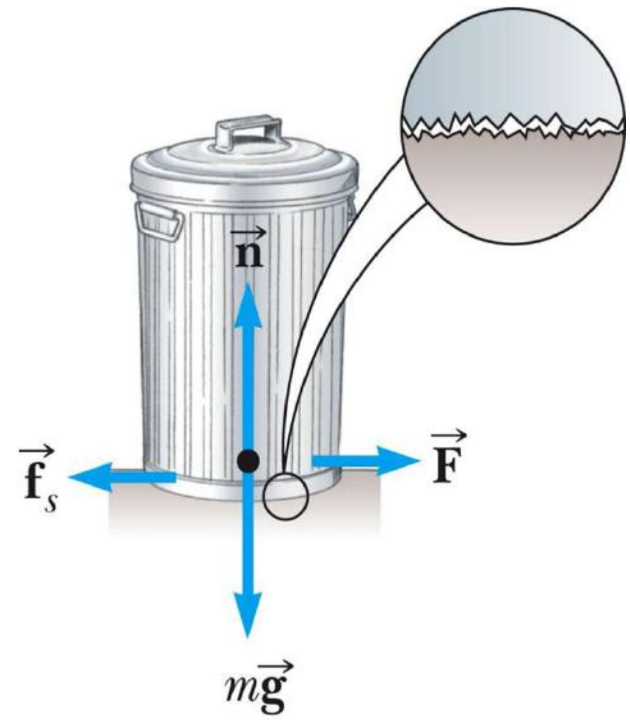
	μ_s	μ_k
Rubber on concrete	1.0	0.8
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Glass on glass	0.94	0.4
Copper on steel	0.53	0.36
Wood on wood	0.25–0.5	0.2
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	—	0.04
Metal on metal (lubricated)	0.15	0.06
Teflon on Teflon	0.04	0.04
Ice on ice	0.1	0.03
Synovial joints in humans	0.01	0.003

Note: All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

Static Friction

- Static friction acts to keep the object from moving
- If $\vec{\mathbf{F}}$ increases, so does $\vec{f}_s$
- If $\vec{\mathbf{F}}$ decreases, so does $\vec{f}_s$
- $f_s \le \mu_s N$

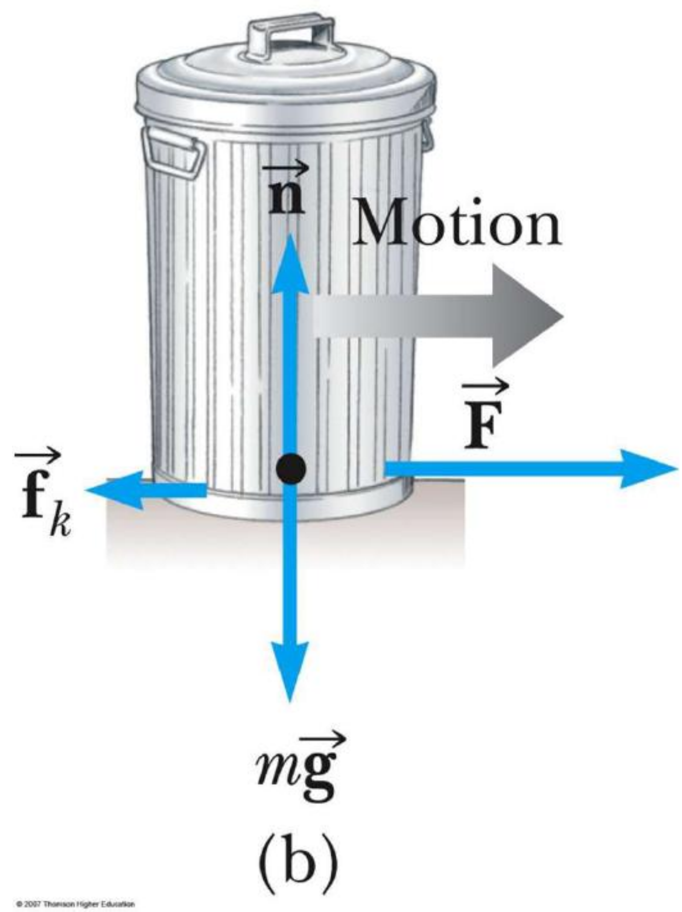
  Remember, the equality holds when the surfaces are on the verge of slipping



(a)

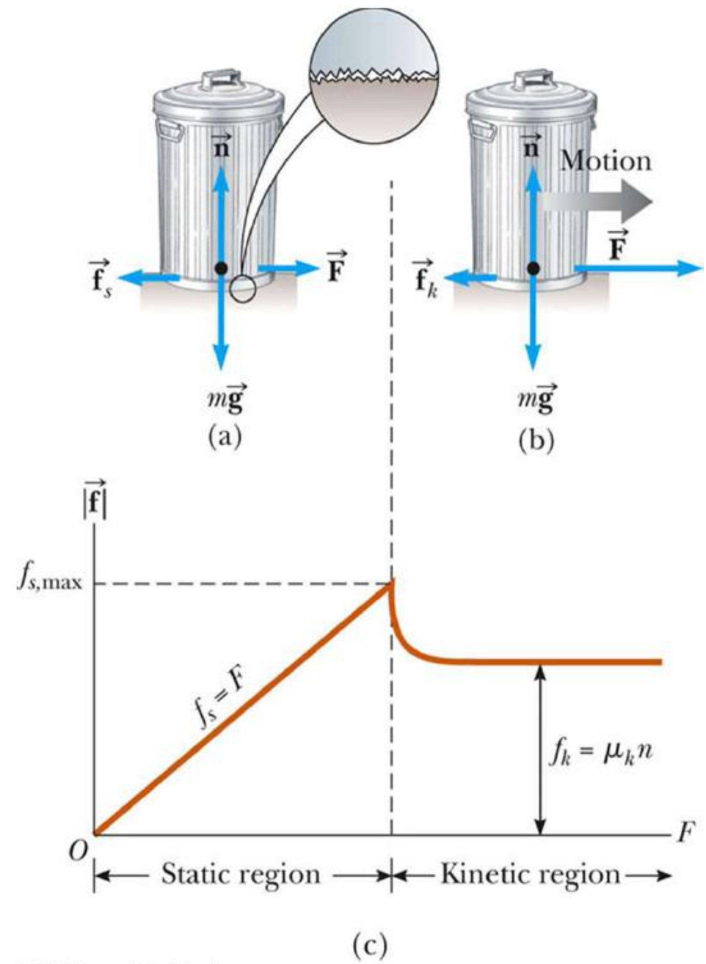
Kinetic Friction

- The force of kinetic friction acts when the object is in motion
- Although μ_k can vary with speed, we shall neglect any such variations
- $f_k = \mu_k N$



Explore Forces of Friction

- Vary the applied force
- Note the value of the frictional force
  - Compare the values
- Note what happens when the can starts to move



Hints for Problem-Solving

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- **Draw** a picture of the system, identify the object of primary interest, and indicate forces with arrows
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Objects in Equilibrium

- Objects that are either at rest or moving with constant velocity are said to be in **equilibrium**
- Acceleration of an object can be modeled as zero:
- Mathematically, the net force acting on the object is zero

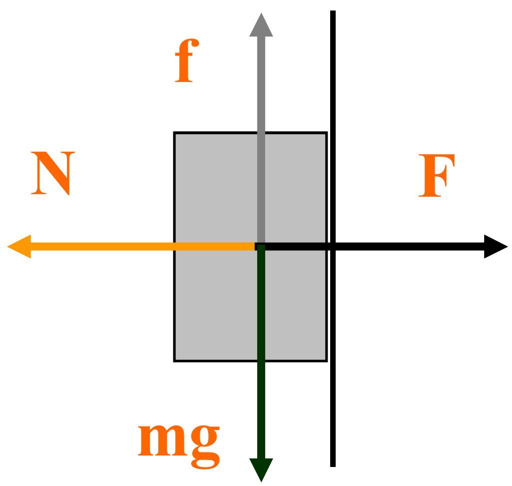
$$\vec{a} = 0 \qquad \sum \vec{F} = 0$$

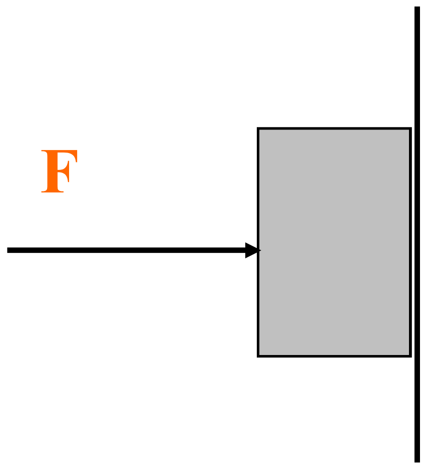
- Equivalent to the set of component equations given by

$$\sum F_x = 0 \qquad \sum F_y = 0$$

Equilibrium, Example 1

- What is the smallest value of the force F such that the 2.0-kg block will not slide down the wall? The coefficient of static friction between the block and the wall is 0.2. ?





Accelerating Objects

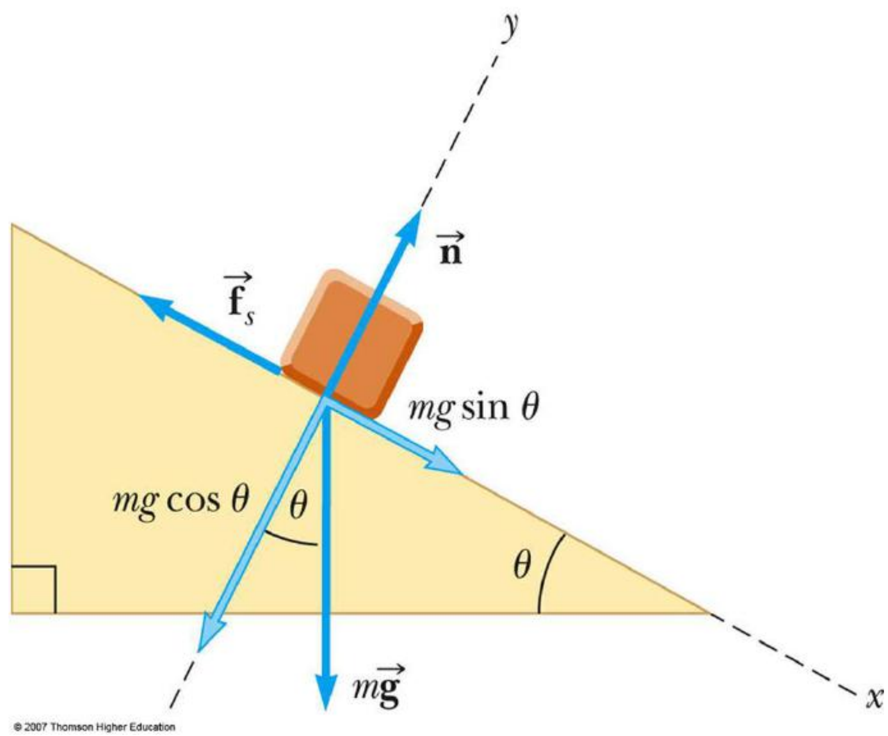
- If an object that can be modeled as a particle experiences an acceleration, there must be a nonzero net force acting on it
- Draw a free-body diagram
- Apply Newton's Second Law in component form

$$\sum \vec{F} = m\vec{a}$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$

Inclined Plane

- Suppose a block with a mass of 2.50 kg is resting on a ramp. If the coefficient of static friction between the block and ramp is 0.350, what maximum angle can the ramp make with the horizontal before the block starts to slip down?



Inclined Plane

- Newton 2nd law:

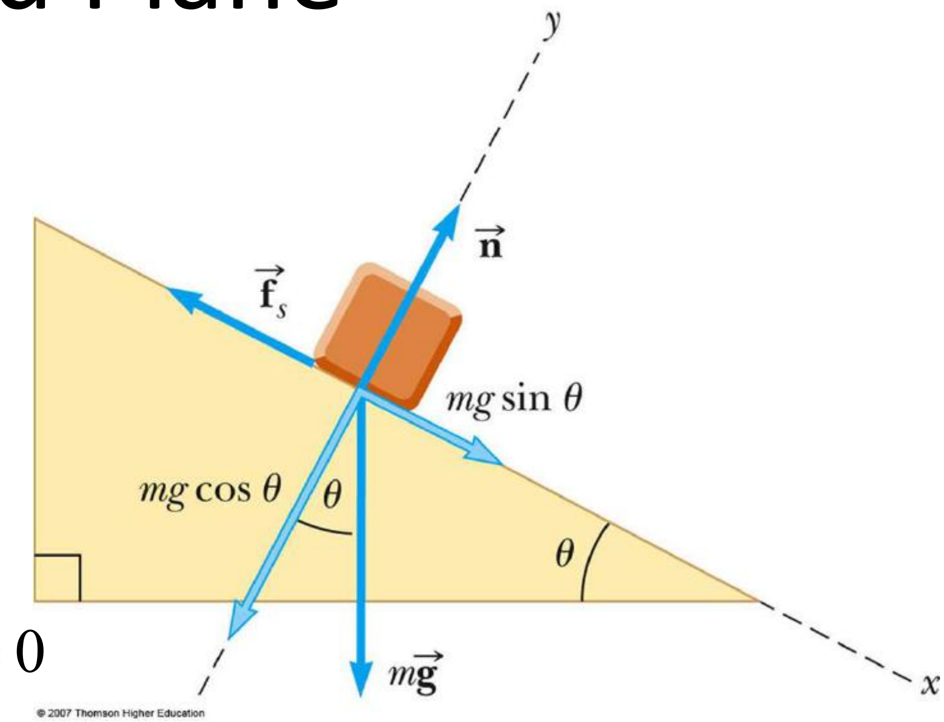
$$\sum F_x = mg\sin\theta - \mu_s N = 0$$
$$\sum F_y = N - mg\cos\theta = 0$$

- Then $N = mg\cos\theta$

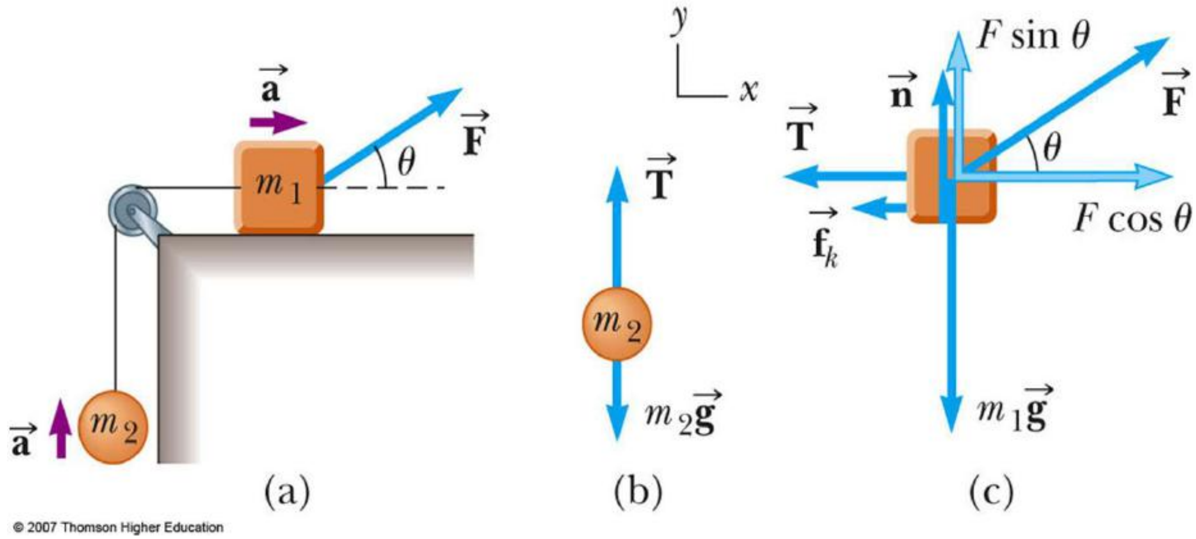
$$\sum F_y = mg\sin\theta - \mu_s mg\cos\theta = 0$$

- So $\tan\theta = \mu_s = 0.350$

$$\theta = \tan^{-1}(0.350) = 19.3^\circ$$



Multiple Objects



- A block of mass m1 on a rough, horizontal surface is connected to a ball of mass m2 by a lightweight cord over a lightweight, frictionless pulley as shown in figure. A force of magnitude F at an angle ϑ with the horizontal is applied to the block as shown and the block slides to the right. The coefficient of kinetic friction between the block and surface is μ_k. Find the magnitude of acceleration of the two objects.

Multiple Objects

- m_1: $\sum F_x = F\cos\theta - f_k - T = m_1 a_x = m_1 a$

  $\sum F_y = N + F\sin\theta - m_1 g = 0$

- m_2: $\sum F_y = T - m_2 g = m_2 a_y = m_2 a$

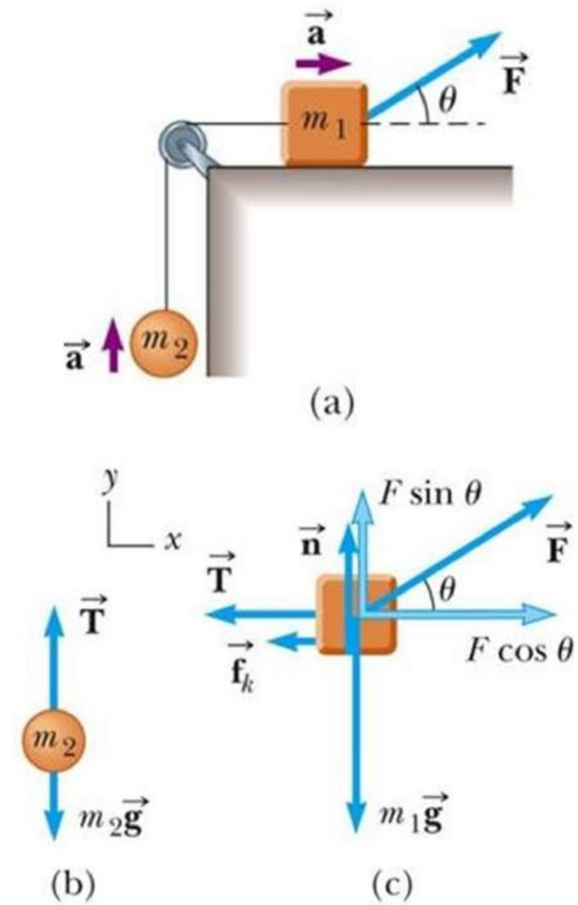
$$T = m_2(a+g)$$

$$N = m_1 g - F\sin\theta$$

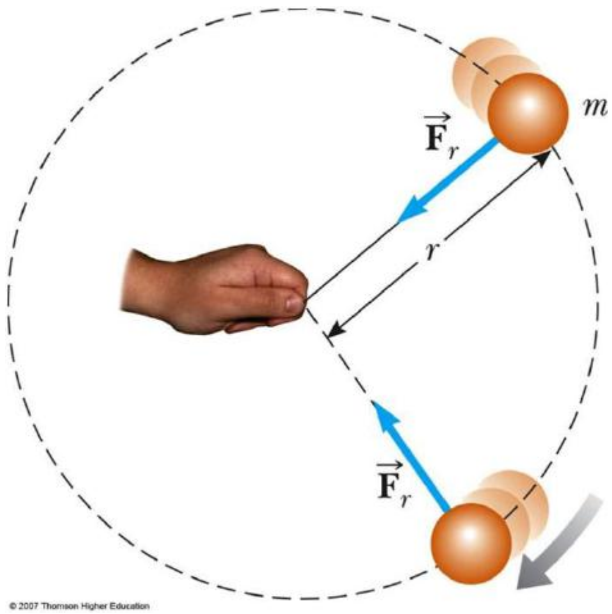
$$f_k = \mu_k N = \mu_k(m_1 g - F\sin\theta)$$

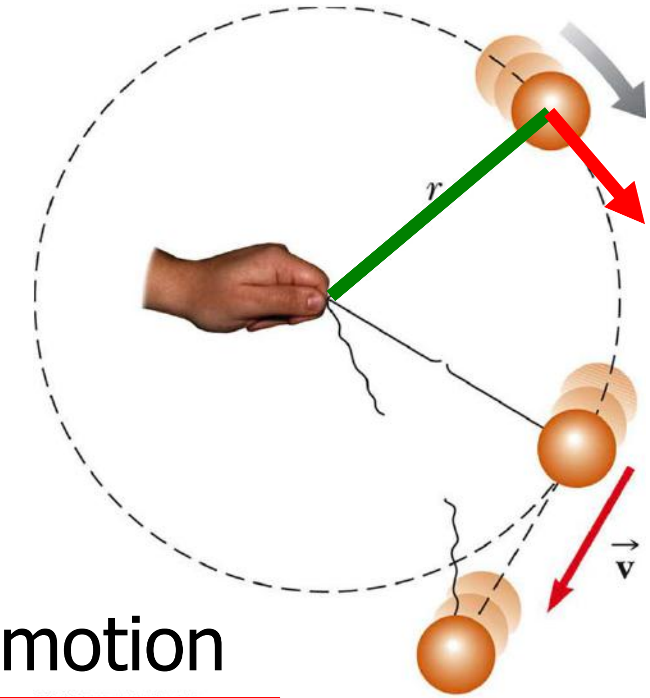
$$F\cos\theta - \mu_k(m_1 g - F\sin\theta) - m_2(a+g) = m_1 a$$

$$a = \frac{F(\cos\theta + \mu_k\sin\theta) - (m_2 + \mu_k m_1)g}{m_1 + m_2}$$



Dynamics: Uniform Circular Motion





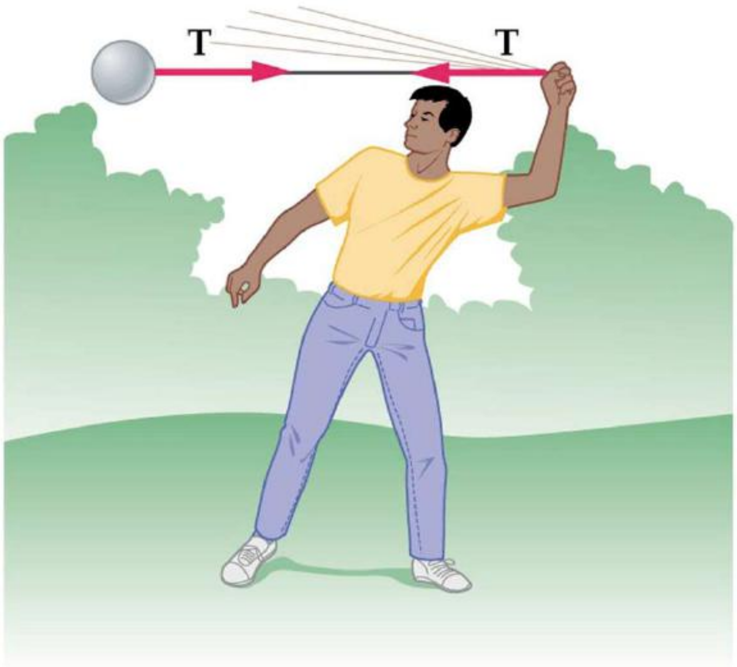
Uniform circular motion

Constant speed, or,
constant magnitude of velocity

Motion along a circle:
Changing direction of velocity

Uniform Circular Motion: Observations

- Object moving along a curved path with **constant speed**
  - Magnitude of velocity: same
  - Direction of velocity: changing
  - Velocity $\vec{v}$: changing
  - Acceleration is NOT zero!
  - **Net force acting on an object is NOT zero**
  - "**Centripetal force**"



$$\vec{F}_{net} = m\vec{a}$$

Uniform Circular Motion

- Magnitude:

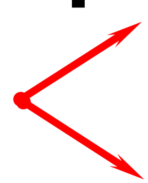
$$\boxed{\Delta\vec{r} = \vec{r}_f - \vec{r}_i} \qquad \boxed{\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}}$$

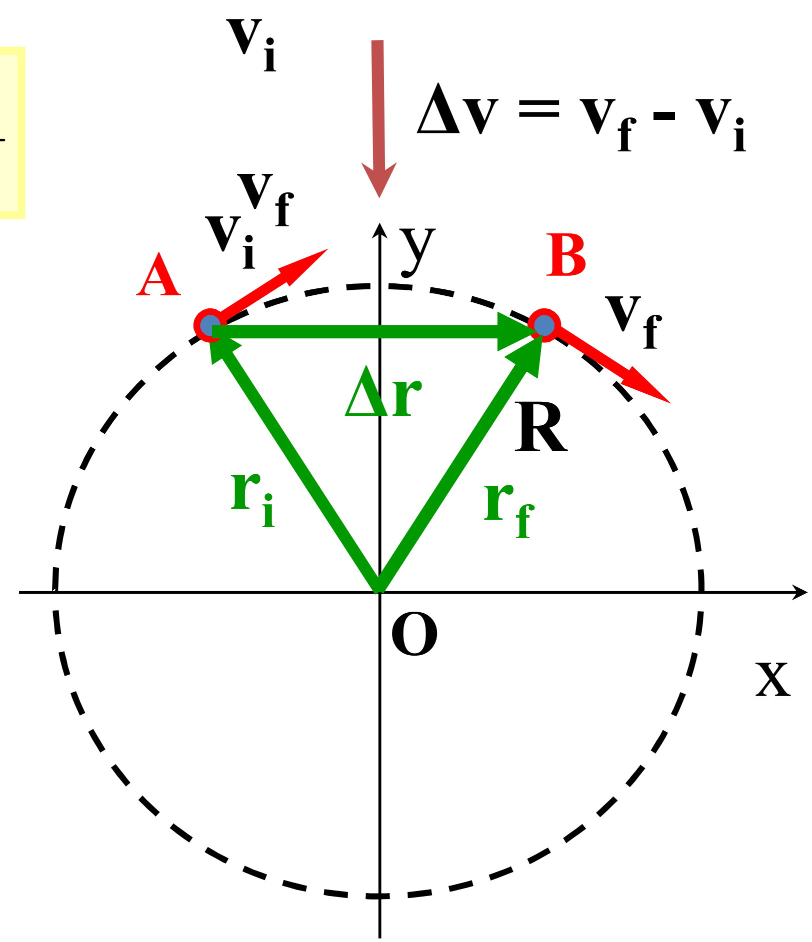
$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \qquad \text{so,} \quad \Delta v = \frac{v\Delta r}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{\Delta r}{\Delta t}\frac{v}{r} = \frac{v^2}{r}$$

$$a_r = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

- Direction: **Centripetal**





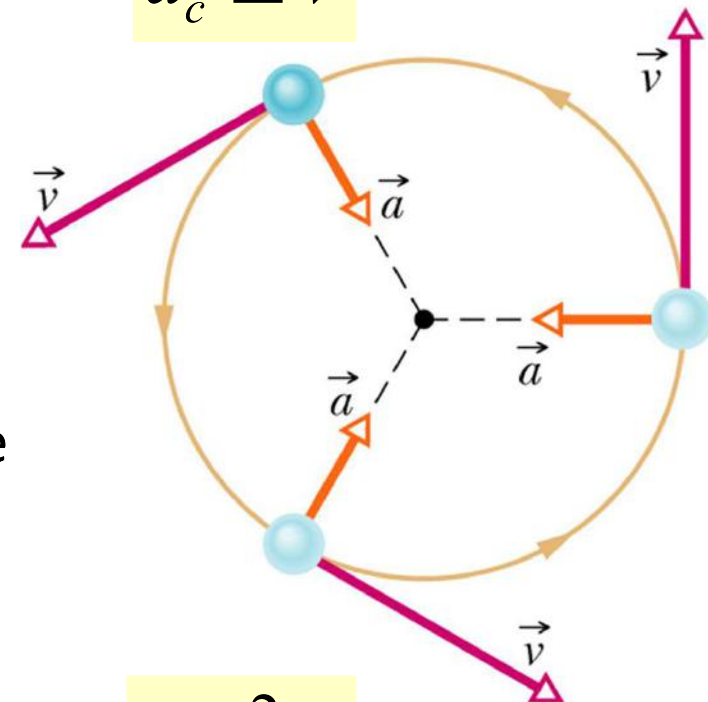
Uniform Circular Motion

- Velocity:
  - Magnitude: constant v
  - The direction of the velocity is tangent to the circle

$$\vec{a}_c \perp \vec{v}$$

- Acceleration:
  - Magnitude: $a_c = \dfrac{v^2}{r}$
  - directed toward the center of the circle of motion
- Period:
  - time interval required for one complete revolution of the particle

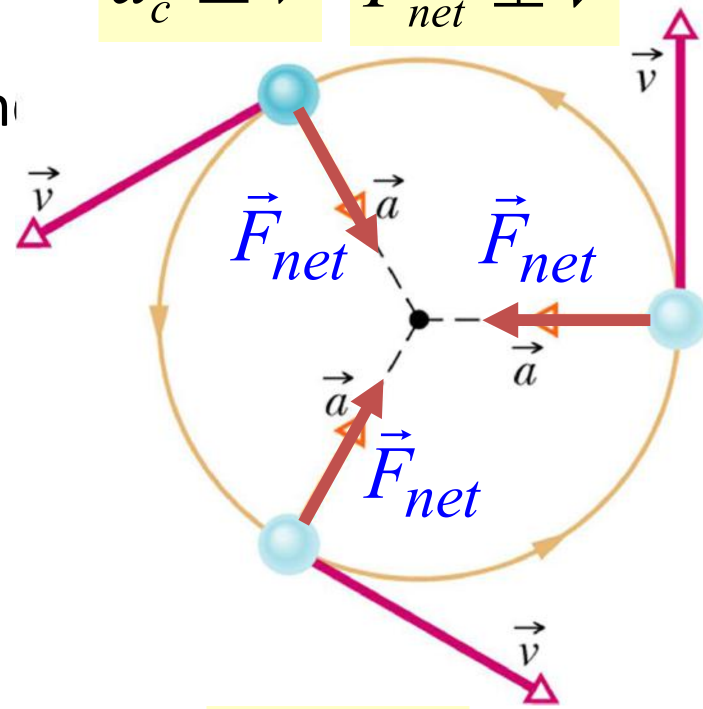
$$T = \frac{2\pi r}{v}$$



Centripetal Force

- Acceleration:
  - Magnitude: $$a_c = \frac{v^2}{r}$$
  - Direction: toward the center of th circle of motion
- Force:
  - Start from Newton's 2nd Law
    $$\vec{F}_{net} = m\vec{a}$$
  - Magnitude:
    $$F_{net} = ma_c = \frac{mv^2}{r}$$
  - Direction: toward the center of the circle of motion

$$\vec{a}_c \perp \vec{v} \qquad \vec{F}_{net} \perp \vec{v}$$



$$\vec{a}_c \parallel \vec{F}_{net}$$

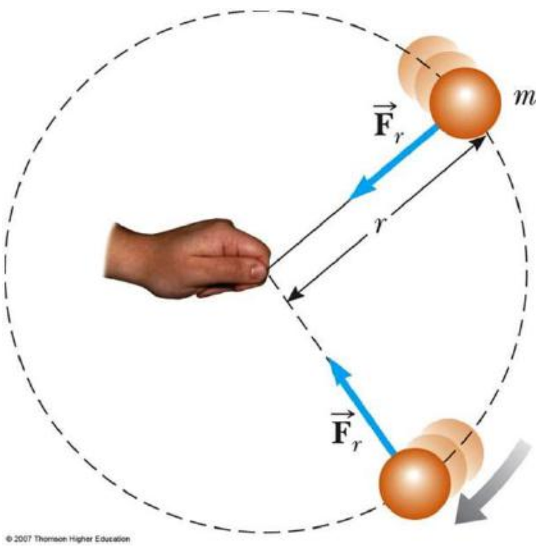
What provides Centripetal Force ?

- Centripetal force is not a new kind of force
- Centripetal force refers to any force that keeps an object following a circular path

$$F_c = ma_c = \frac{mv^2}{r}$$

- Centripetal force is a combination of
  - Gravitational force $m\mathbf{g}$: downward to the ground
  - Normal force $\mathbf{N}$: perpendicular to the surface
  - Tension force $\mathbf{T}$: along the cord and away from object
  - Static friction force: $f_s^{max} = \mu_s N$

What provides Centripetal Force ?

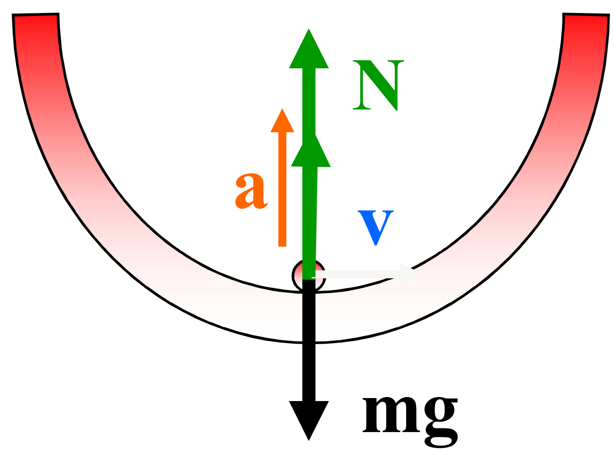


$$F_{net} = T = ma$$

$$T = \frac{mv^2}{r}$$

$$F_{net} = N - mg = ma$$

$$N = mg + m\frac{v^2}{r}$$

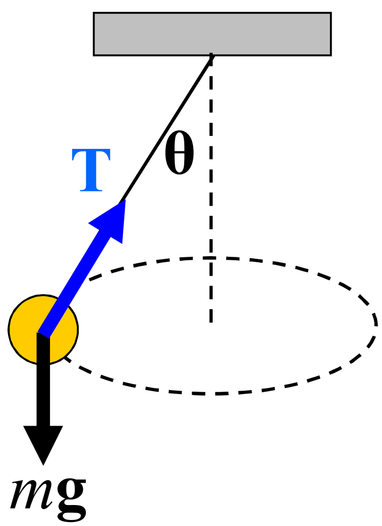


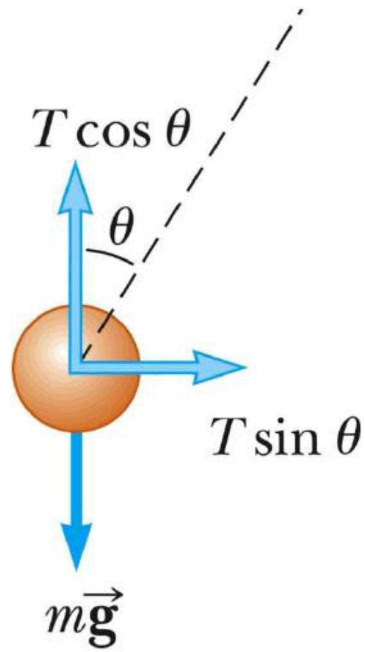
Problem Solving Strategy

- **Draw a free body diagram**, showing and labeling all the forces acting on the object(s)
- **Choose a coordinate system** that has one axis perpendicular to the circular path and the other axis tangent to the circular path
- **Find the net force toward the center** of the circular path (this is the force that causes the centripetal acceleration, F_C)
- **Use Newton's second law**
  - The directions will be radial, normal, and tangential
  - The acceleration in the radial direction will be the centripetal acceleration
- **Solve for the unknown(s)**

The Conical Pendulum

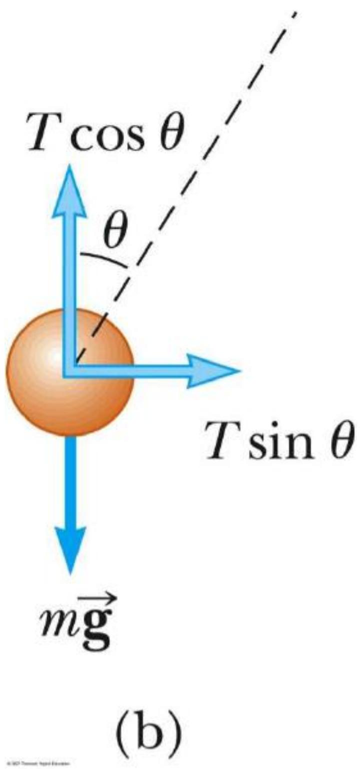
- A small ball of mass m = 5 kg is suspended from a string of length L = 5 m. The ball revolves with constant speed v in a horizontal circle of radius r = 2 m. Find an expression for v and a.





The Conical Pendulum

Find v and a



$$m = 5\,kg \quad L = 5\,m \quad r = 2\,m$$

$$\sum F_y = T\cos\theta - mg = 0$$

$$T\cos\theta = mg$$

$$\sum F_x = T\sin\theta = \frac{mv^2}{r}$$

$$\sin\theta = \frac{r}{L} = 0.4$$

$$\tan\theta = \frac{r}{\sqrt{L^2 - r^2}} = 0.44$$

$$T\sin\theta = \frac{mv^2}{r}$$

$$T\cos\theta = mg$$

$$\tan\theta = \frac{v^2}{gr}$$

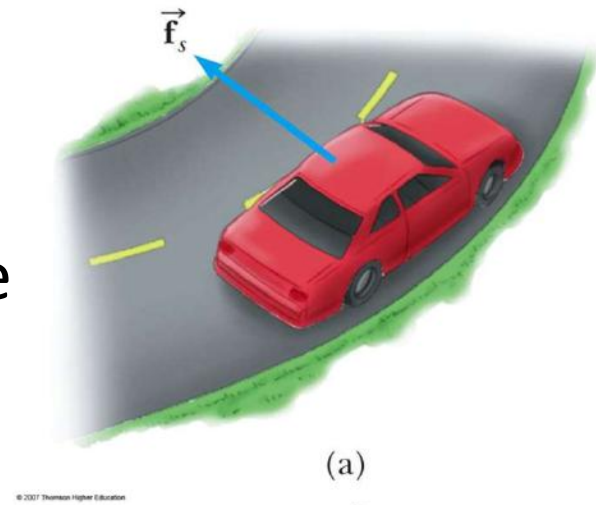
$$v = \sqrt{rg\tan\theta}$$

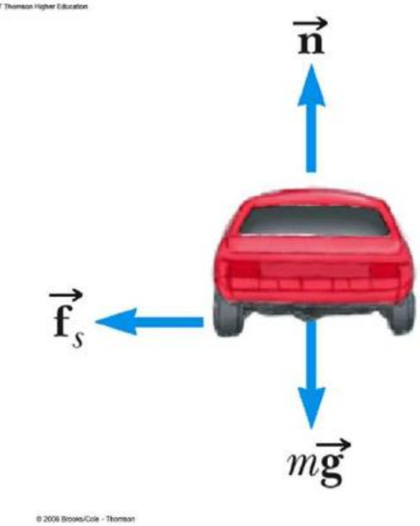
$$v = \sqrt{Lg\sin\theta\tan\theta} = 2.9\text{ m/s}$$

$$a = \frac{v^2}{r} = g\tan\theta = 4.3\text{ m/s}^2$$

Level Curves

- A 1500 kg car moving on a flat, horizontal road negotiates a curve as shown. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.





Level Curves

- The force of static friction directed toward the center of the curve keeps the car moving in a circular path.

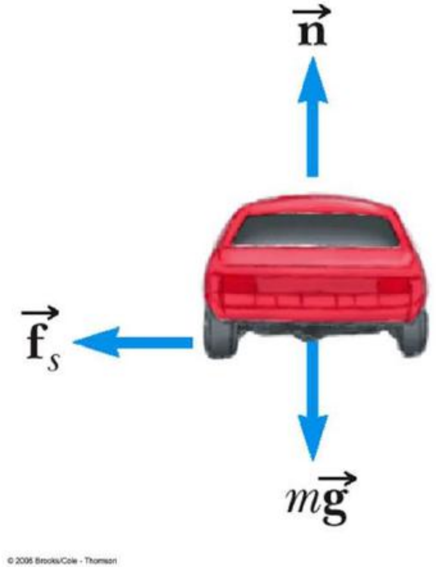
$$f_{s,\max} = \mu_s N = m\frac{v_{\max}^2}{r}$$

$$\sum F_y = N - mg = 0$$

$$N = mg$$

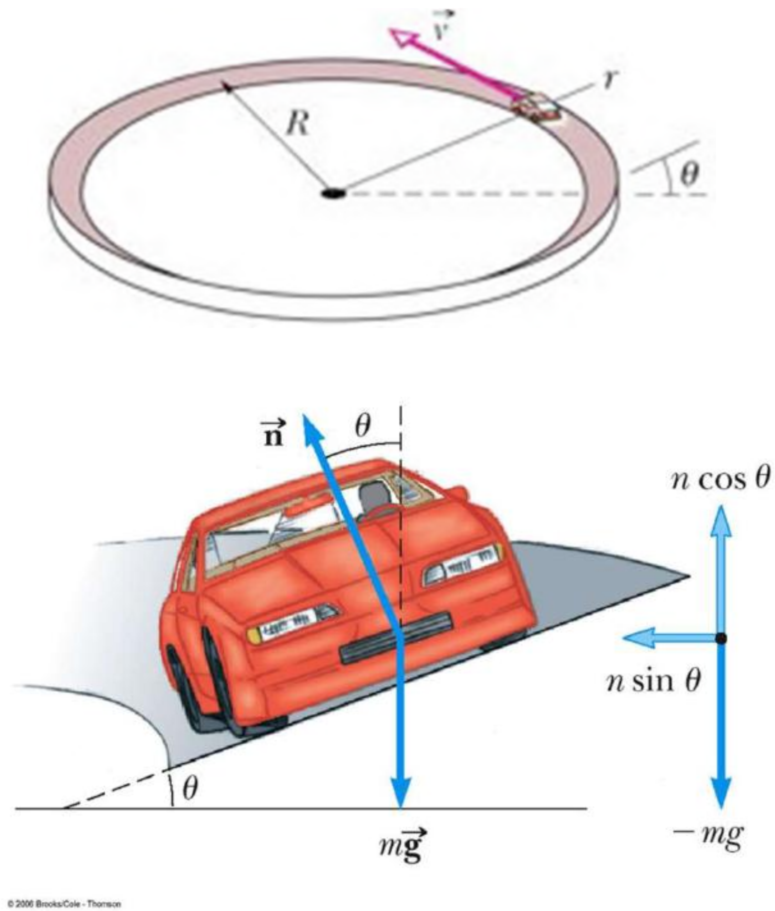
$$v_{\max} = \sqrt{\frac{\mu_s N r}{m}} = \sqrt{\frac{\mu_s m g r}{m}} = \sqrt{\mu_s g r}$$

$$= \sqrt{(0.523)(9.8m/s^2)(35.0m)} = 13.4m/s$$



Banked Curves

- A car moving at the designated speed can negotiate the curve. Such a ramp is usually banked, which means that the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be 13.4 m/s and the radius of the curve is 35.0 m. At what angle should the curve be banked?



Banked Curves

$$v = 13.4\ \text{m/s} \qquad r = 35.0\ \text{m}$$

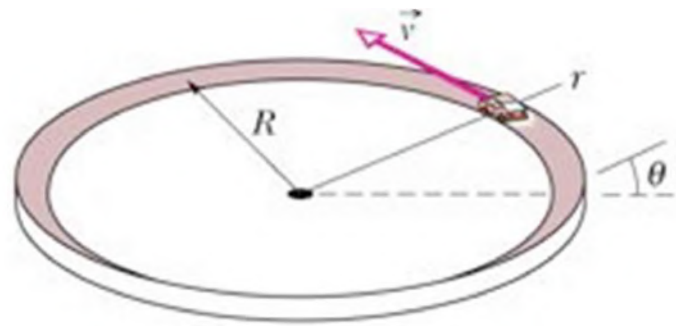
$$\sum F_r = n\sin\theta = ma_c = \frac{mv^2}{r}$$

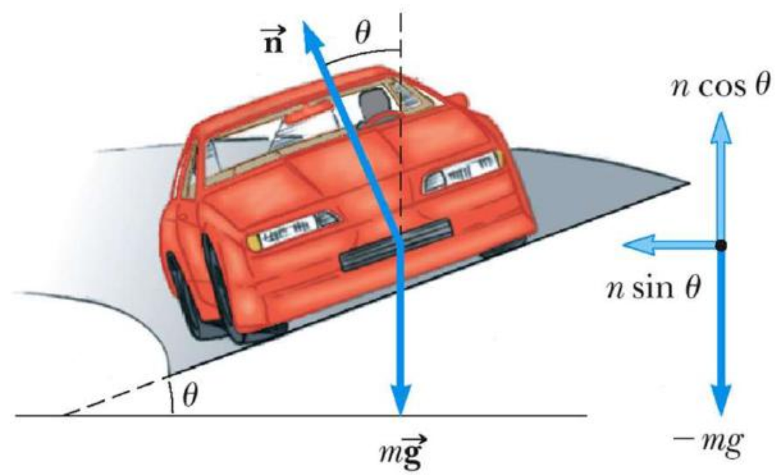
$$\sum F_y = n\cos\theta - mg = 0$$

$$n\cos\theta = mg$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{13.4\ \text{m/s}}{(35.0\ \text{m})(9.8\ \text{m/s}^2)}\right) = 27.6^\circ$$





Basic Physics 1
Lecture Module

Rahadian N, S.Si. M.Si.

Momentum and Collisions

Momentum and Collisions

- Conservation of Energy
- Momentum
- Impulse
- Conservation of Momentum
- 1-D Collisions & 2-D Collisions
- The Center of Mass
- Motion of a System of Particles

Conservation of Energy

- $\Delta E = \Delta K + \Delta U = 0$ if conservative forces are the only forces that do work on the system.
- The total amount of energy in the system is constant.

$$\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2$$

- $\Delta E = \Delta K + \Delta U = -f_k d$ if friction forces are doing work on the system.
- The total amount of energy in the system is still constant, but the change in mechanical energy goes into "internal energy" or heat.

$$-f_k d = \left(\frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}kx_f^2\right) - \left(\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}kx_i^2\right)$$

Linear Momentum

- This is a new fundamental quantity, like force, energy. It is a vector quantity (points in same direction as velocity).
- The linear momentum **p** of an object of mass *m* moving with a velocity **v** is defined to be the product of the mass and velocity:

$$\vec{p} = m\vec{v}$$

- The terms momentum and linear momentum will be used interchangeably in the text
- Momentum depend on an object's mass and velocity

Linear Momentum, cont'd

- Linear momentum is a vector quantity $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
  - Its direction is the same as the direction of the velocity
- The dimensions of momentum are ML/T
- The SI units of momentum are kg m / s
- Momentum can be expressed in component form:

$$p_x = mv_x \quad p_y = mv_y \quad p_z = mv_z$$

Newton's Law and Momentum

- Newton's Second Law can be used to relate the momentum of an object to the resultant force acting on it

$$\vec{F}_{net} = m\vec{a} = m\frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t}$$

- The change in an object's momentum divided by the elapsed time equals the constant net force acting on the object

$$\frac{\Delta\vec{p}}{\Delta t} = \frac{\text{change in momentum}}{\text{time interval}} = \vec{F}_{net}$$

Impulse

- When a single, constant force acts on the object, there is an **impulse** delivered to the object

$$\vec{I} = \vec{F}\Delta t$$ is defined as the *impulse*

  - The equality is true even if the force is not constant
  - Vector quantity, the direction is the same as the direction of the force

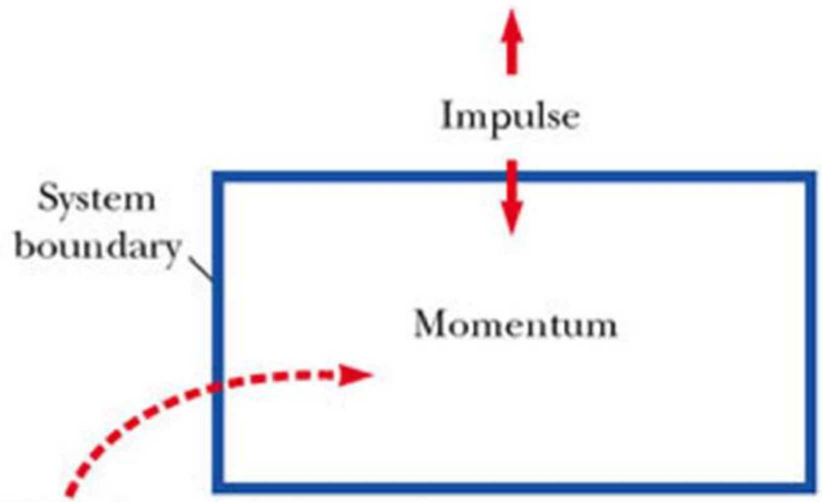
$$\frac{\Delta\vec{p}}{\Delta t} = \frac{\text{change in momentum}}{\text{time interval}} = \vec{F}_{net}$$

Impulse-Momentum Theorem

- The theorem states that the impulse acting on a system is equal to the change in momentum of the system

$$\Delta\vec{p} = \vec{F}_{net}\Delta t = \vec{I}$$

$$\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$$



The change in the total momentum of the system is equal to the total impulse on the system.

Calculating the Change of Momentum

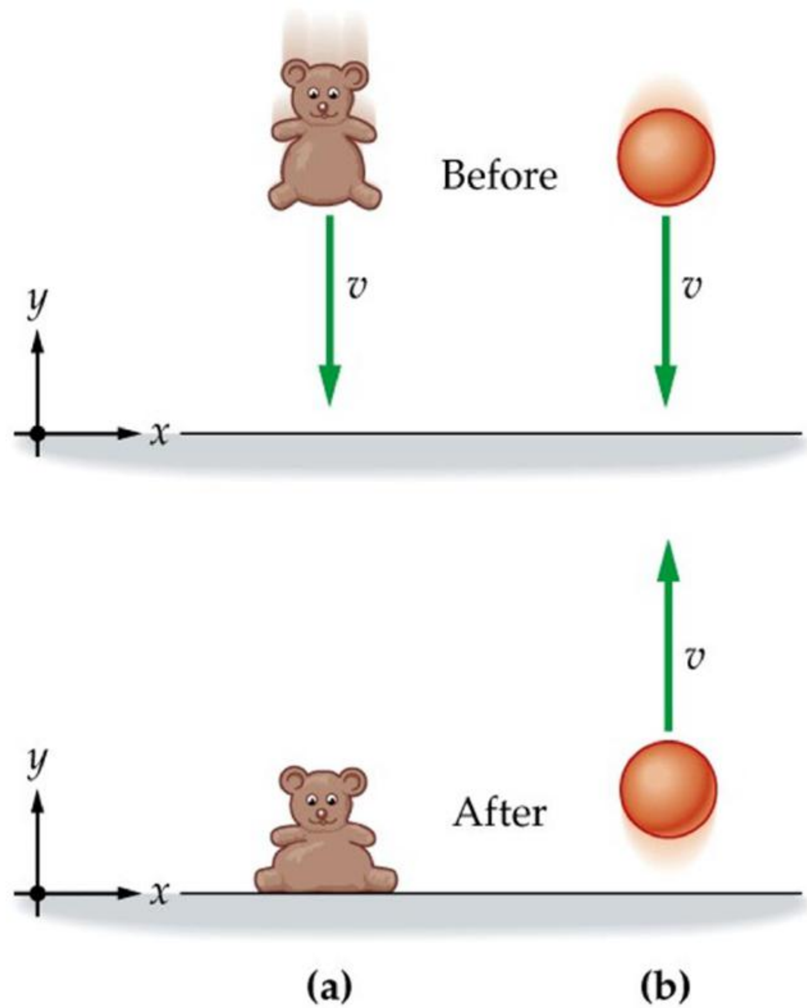
$$\boxed{\begin{aligned}\Delta\vec{p} &= \vec{p}_{after} - \vec{p}_{before} \\ &= mv_{after} - mv_{before} \\ &= m(v_{after} - v_{before})\end{aligned}}$$

For the teddy bear

$$\Delta p = m[0-(-v)] = mv$$

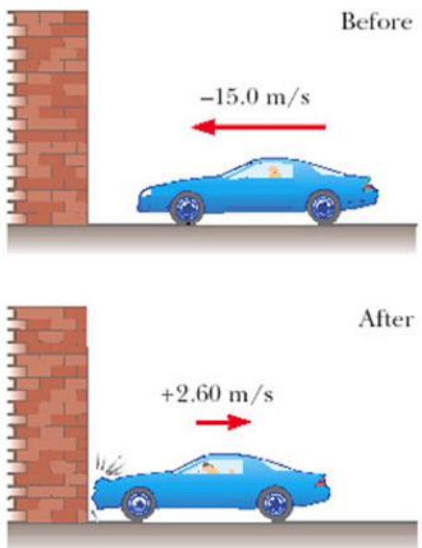
For the bouncing ball

$$\Delta p = m[v-(-v)] = 2mv$$



How Good Are the Bumpers?

In a crash test, a car of mass 1.5×10^3 kg collides with a wall and rebounds as in figure. The initial and final velocities of the car are v_i=-15 m/s and v_f = 2.6 m/s, respectively. If the collision lasts for 0.15 s, find
(a) the impulse delivered to the car due to the collision
(b) the size and direction of the average force exerted on the car



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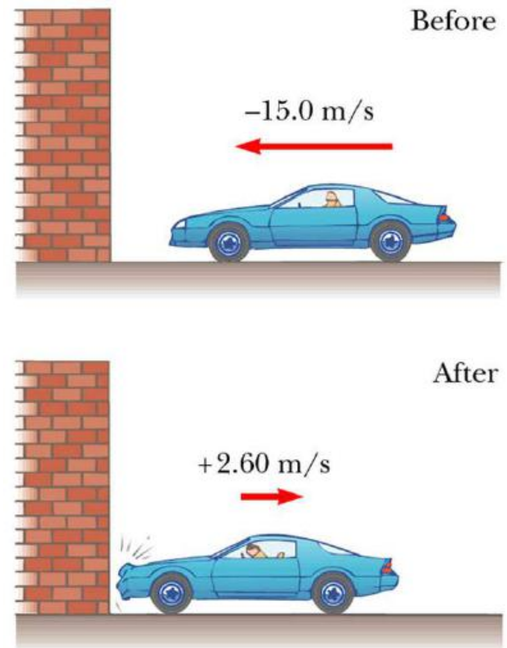
(a) the impulse delivered to the car due to the collision
(b) the size and direction of the average force exerted on the car

$$p_i = mv_i = (1.5\times10^3 kg)(-15 m/s) = -2.25\times10^4 kg\cdot m/s$$

$$p_f = mv_f = (1.5\times10^3 kg)(+2.6 m/s) = +0.39\times10^4 kg\cdot m/s$$

$$I = p_f - p_i = mv_f - mv_i$$
$$= (0.39\times10^4 kg\cdot m/s) - (-2.25\times10^4 kg\cdot m/s)$$
$$= 2.64\times10^4 kg\cdot m/s$$

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{I}{\Delta t} = \frac{2.64\times10^4 kg\cdot m/s}{0.15 s} = 1.76\times10^5 N$$



Impulse-Momentum Theorem

- A child bounces a 100 g superball on the sidewalk. The velocity of the superball changes from 10 m/s downward to 10 m/s upward. If the contact time with the sidewalk is 0.1s, what is the magnitude of the impulse imparted to the superball?

(A) 0
(B) 2 kg-m/s
(C) 20 kg-m/s
(D) 200 kg-m/s
(E) 2000 kg-m/s

$$\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i$$

Impulse-Momentum Theorem 2

- A child bounces a 100 g superball on the sidewalk. The velocity of the superball changes from 10 m/s downward to 10 m/s upward. If the contact time with the sidewalk is 0.1s, what is the magnitude of the force between the sidewalk and the superball?

  (A) 0

  (B) 2 N

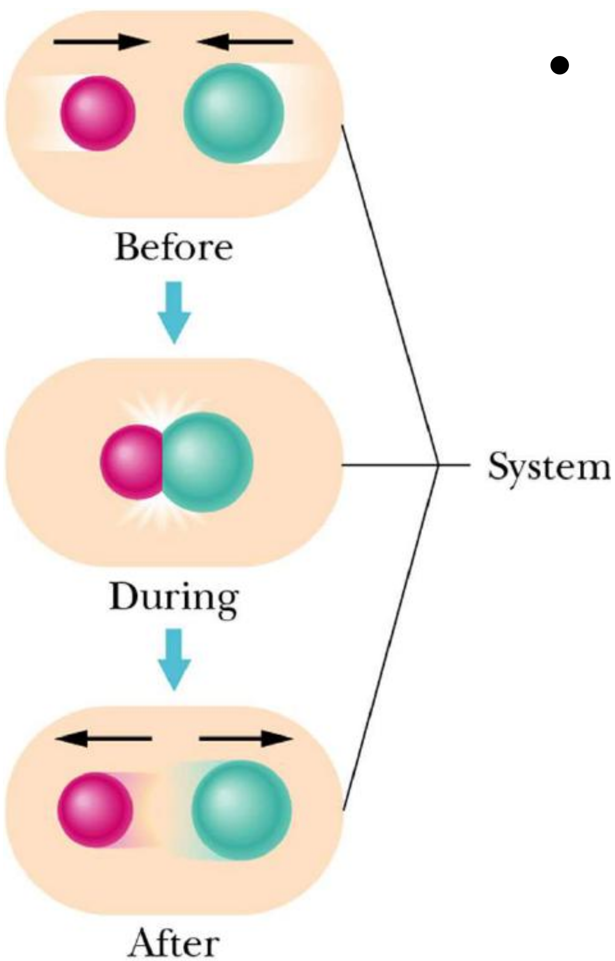
  (C) 20 N

  (D) 200 N

  (E) 2000 N

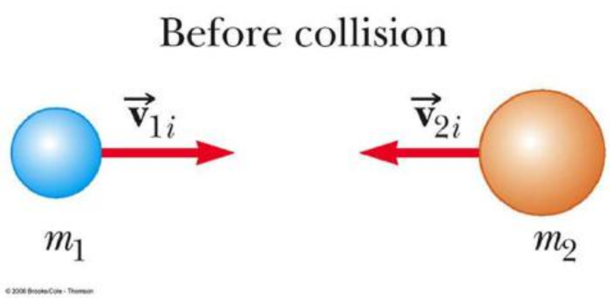
$$\vec{F} = \frac{\vec{I}}{\Delta t} = \frac{\Delta \vec{p}}{\Delta t} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}$$

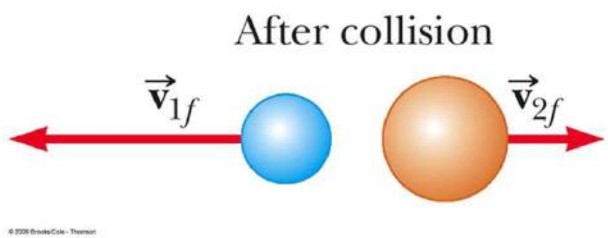
Conservation of Momentum

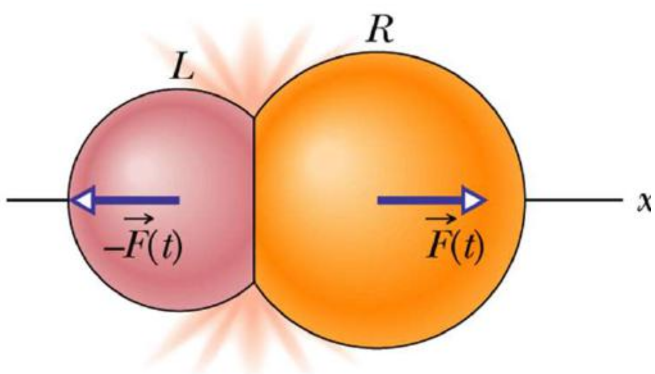


- In an isolated and closed system, the total momentum of the system remains constant in time.
  - Isolated system: no external forces
  - Closed system: no mass enters or leaves
  - The linear momentum of each colliding body may change
  - The total momentum *P* of the system cannot change.

Conservation of Momentum







- Start from impulse-momentum theorem

$$\vec{F}_{21}\Delta t = m_1\vec{v}_{1f} - m_1\vec{v}_{1i}$$

$$\vec{F}_{12}\Delta t = m_2\vec{v}_{2f} - m_2\vec{v}_{2i}$$

- Since $\quad \vec{F}_{21}\Delta t = -\vec{F}_{12}\Delta t$

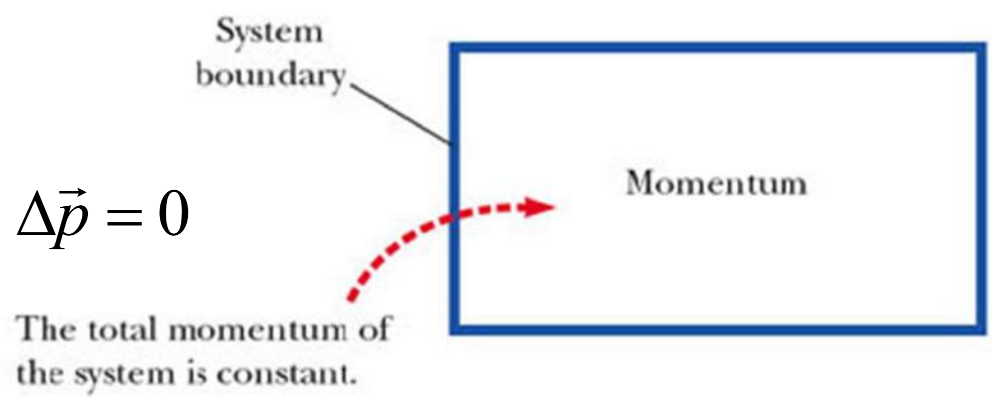
- Then $\quad m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = -(m_2\vec{v}_{2f} - m_2\vec{v}_{2i})$

- So $\quad m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$

Conservation of Momentum

- When no external forces act on a system consisting of two objects that collide with each other, the total momentum of the system remains constant in time

$$\vec{F}_{net}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$



- When $\vec{F}_{net} = 0$ then $\Delta\vec{p} = 0$
- For an isolated system

$$\vec{p}_f = \vec{p}_i$$

- Specifically, the total momentum before the collision will equal the total momentum after the collision

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

The Archer

An archer stands at rest on frictionless ice and fires a 0.5-kg arrow horizontally at 50.0 m/s. The combined mass of the archer and bow is 60.0 kg. With what velocity does the archer move across the ice after firing the arrow?

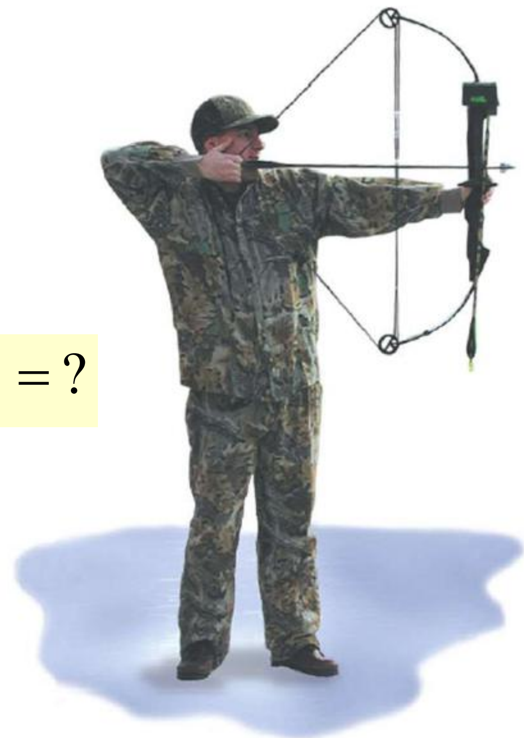
$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 = 60.0kg, m_2 = 0.5kg, v_{1i} = v_{2i} = 0, v_{2f} = 50m/s, v_{1f} = ?$$

$$0 = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{1f} = -\frac{m_2}{m_1} v_{2f} = -\frac{0.5kg}{60.0kg}(50.0m/s) = -0.417m/s$$



Conservation of Momentum

- A 100 kg man and 50 kg woman on ice skates stand facing each other. If the woman pushes the man backwards so that his final speed is 1 m/s, at what speed does she recoil?

  (A) 0

  (B) 0.5 m/s

  (C) 1 m/s

  (D) 1.414 m/s

  (E) 2 m/s

Types of Collisions

- Momentum is conserved in any collision
- **Inelastic collisions:** *rubber ball and hard ball*
  - Kinetic energy is not conserved
  - **Perfectly inelastic** collisions occur when the objects stick together
- **Elastic collisions:** *billiard ball*
  - both momentum and kinetic energy are conserved
- **Actual collisions**
  - Most collisions fall between elastic and perfectly inelastic collisions

Collisions Summary

- In an elastic collision, both momentum and kinetic energy are conserved
- In a non-perfect inelastic collision, momentum is conserved but kinetic energy is not. Moreover, the objects do not stick together
- In a perfectly inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same
- Elastic and perfectly inelastic collisions are limiting cases, most actual collisions fall in between these two types
- Momentum is conserved in all collisions

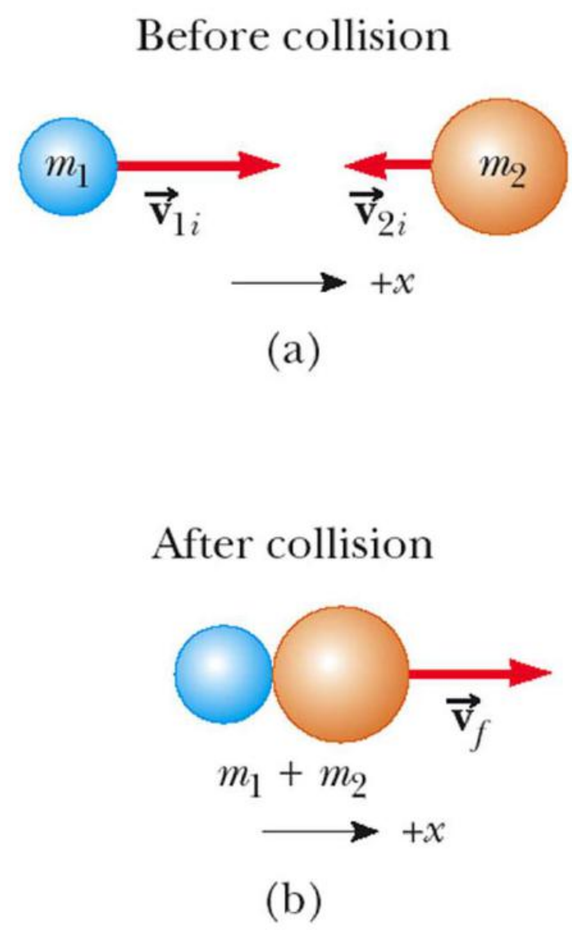
More about Perfectly Inelastic Collisions

- When two objects stick together after the collision, they have undergone a perfectly inelastic collision
- Conservation of momentum

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f}$$

$$\boxed{v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}}$$

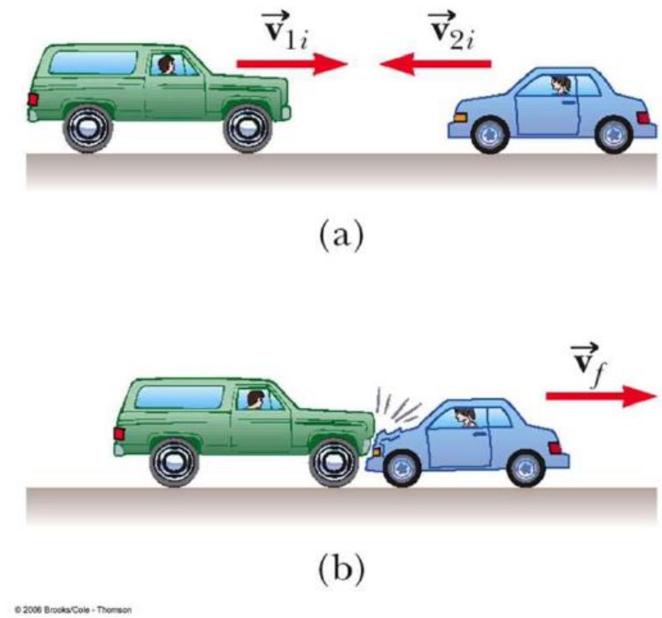
- Kinetic energy is NOT conserved



An SUV Versus a Compact

An SUV with mass 1.80×10^3 kg is travelling eastbound at +15.0 m/s, while a compact car with mass 9.00×10^2 kg is travelling westbound at -15.0 m/s. The cars collide head-on, becoming entangled.

Find the speed of the entangled cars after the collision.
Find the change in the velocity of each car.
Find the change in the kinetic energy of the system consisting of both cars.



An SUV Versus a Compact

Find the speed of the entangled cars after the collision.

$$m_1 = 1.80 \times 10^3 kg, v_{1i} = +15 m/s$$

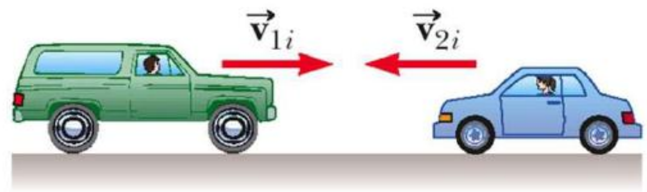
$$m_2 = 9.00 \times 10^2 kg, v_{2i} = -15 m/s$$

$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

$$v_f = +5.00 m/s$$



(a)

(b)

An SUV Versus a Compact

Find the change in the velocity of each car.

$$m_1 = 1.80 \times 10^3 kg, v_{1i} = +15m/s$$

$$m_2 = 9.00 \times 10^2 kg, v_{2i} = -15m/s$$

$$v_f = +5.00m/s$$

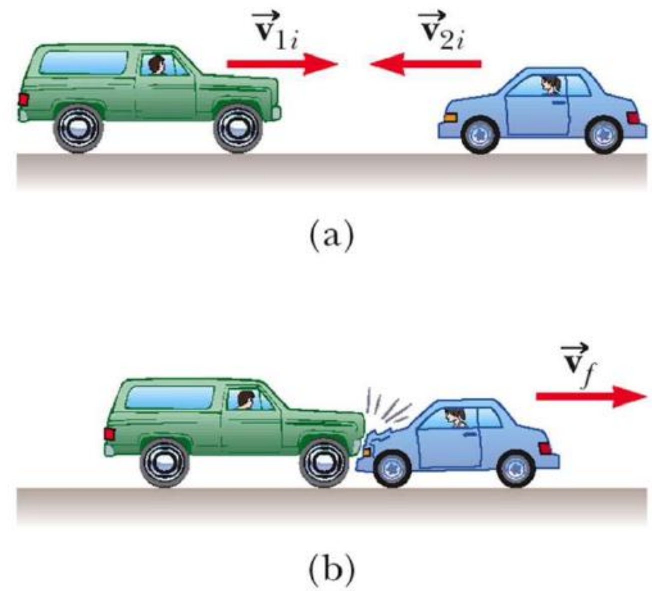
$$\Delta v_1 = v_f - v_{1i} = -10.0m/s$$

$$\Delta v_2 = v_f - v_{2i} = +20.0m/s$$

$$m_1 \Delta v_1 = m_1 (v_f - v_{1i}) = -1.8 \times 10^4 kg \cdot m/s$$

$$m_2 \Delta v_2 = m_2 (v_f - v_{2i}) = +1.8 \times 10^4 kg \cdot m/s$$

$$m_1 \Delta v_1 + m_2 \Delta v_2 = 0$$



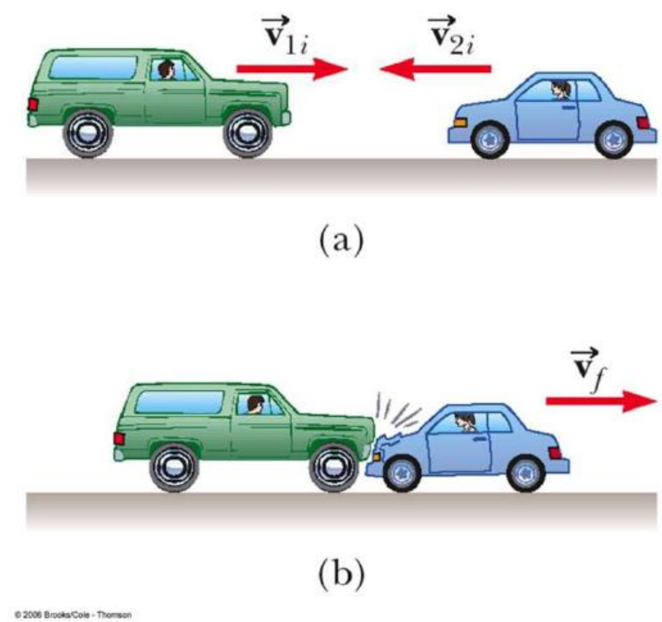
An SUV Versus a Compact

Find the change in the kinetic energy of the system consisting of both cars.

$$m_1 = 1.80 \times 10^3 kg, v_{1i} = +15 m/s$$

$$m_2 = 9.00 \times 10^2 kg, v_{2i} = -15 m/s$$

$$v_f = +5.00 m/s$$



$$KE_i = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = 3.04 \times 10^5 J$$

$$KE_f = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 = 3.38 \times 10^4 J$$

$$\Delta KE = KE_f - KE_i = -2.70 \times 10^5 J$$

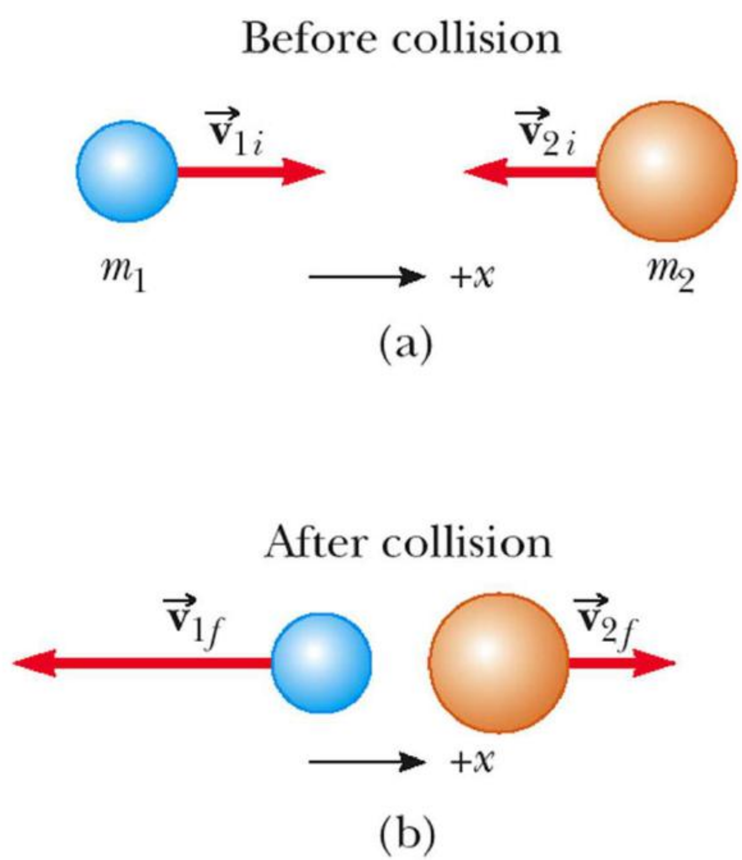
More About Elastic Collisions

- Both momentum and kinetic energy are conserved

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

- Typically have two unknowns
- Momentum is a vector quantity
  - Direction is important
  - Be sure to have the correct signs
- Solve the equations simultaneously



Elastic Collisions

- A simpler equation can be used in place of the KE equation

$$\frac{1}{2}m_1v_{1i}^2+\frac{1}{2}m_2v_{2i}^2=\frac{1}{2}m_1v_{1f}^2+\frac{1}{2}m_2v_{2f}^2$$

$$m_1(v_{1i}^2-v_{1f}^2)=m_2(v_{2f}^2-v_{2i}^2)$$

$$m_1(v_{1i}-v_{1f})(v_{1i}+v_{1f})=m_2(v_{2f}-v_{2i})(v_{2f}+v_{2i})$$

$$m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f} \qquad m_1(v_{1i}-v_{1f})=m_2(v_{2f}-v_{2i})$$

$$\boxed{v_{1i}+v_{1f}=v_{2f}+v_{2i}}$$

$$\boxed{m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2f}}$$

Summary of Types of Collisions

- In an elastic collision, both momentum and kinetic energy are conserved

$$\boxed{v_{1i} + v_{1f} = v_{2f} + v_{2i}} \qquad \boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$

- In an inelastic collision, momentum is conserved but kinetic energy is not

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$

- In a *perfectly* inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f}$$

Conservation of Momentum

- An object of mass m moves to the right with a speed v. It collides head-on with an object of mass $3m$ moving with speed $v/3$ in the opposite direction. If the two objects stick together, what is the speed of the combined object, of mass $4m$, after the collision?

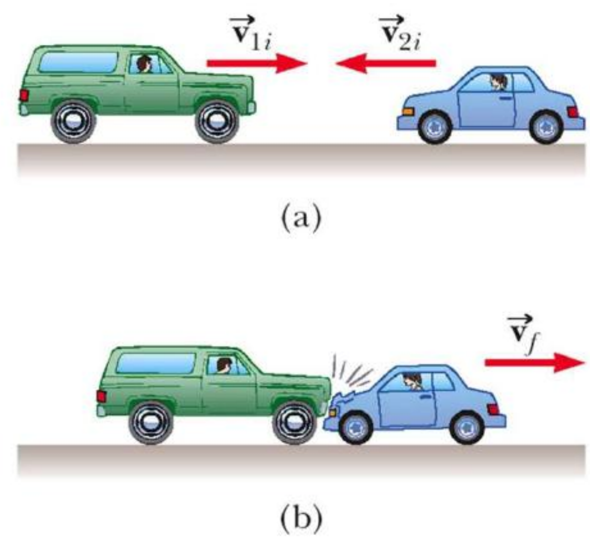
  (A) 0

  (B) v/2

  (C) v

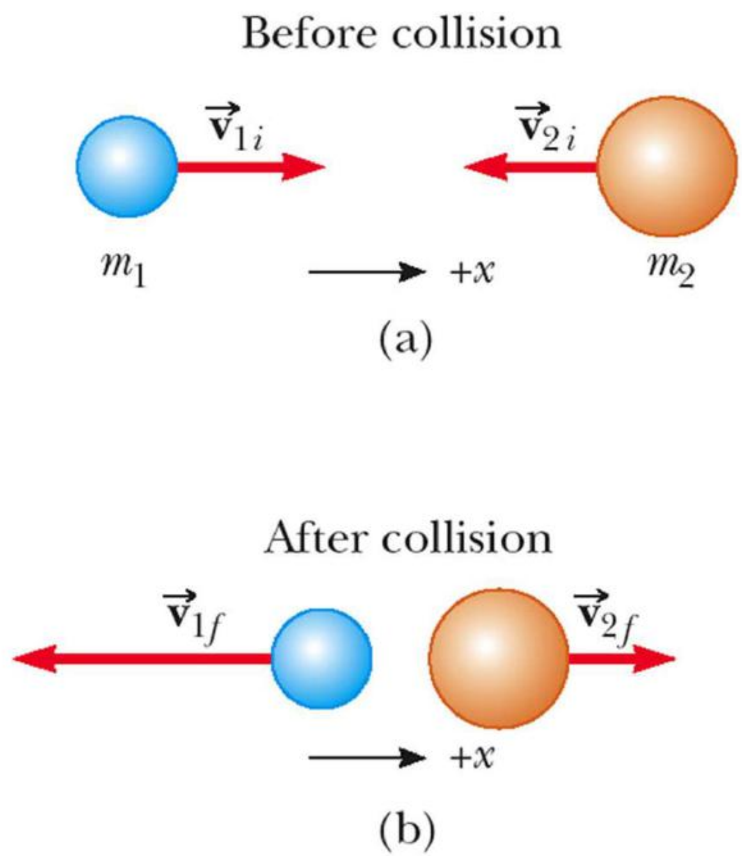
  (D) 2v

  (E) 4v



Problem Solving for 1D Collisions, 1

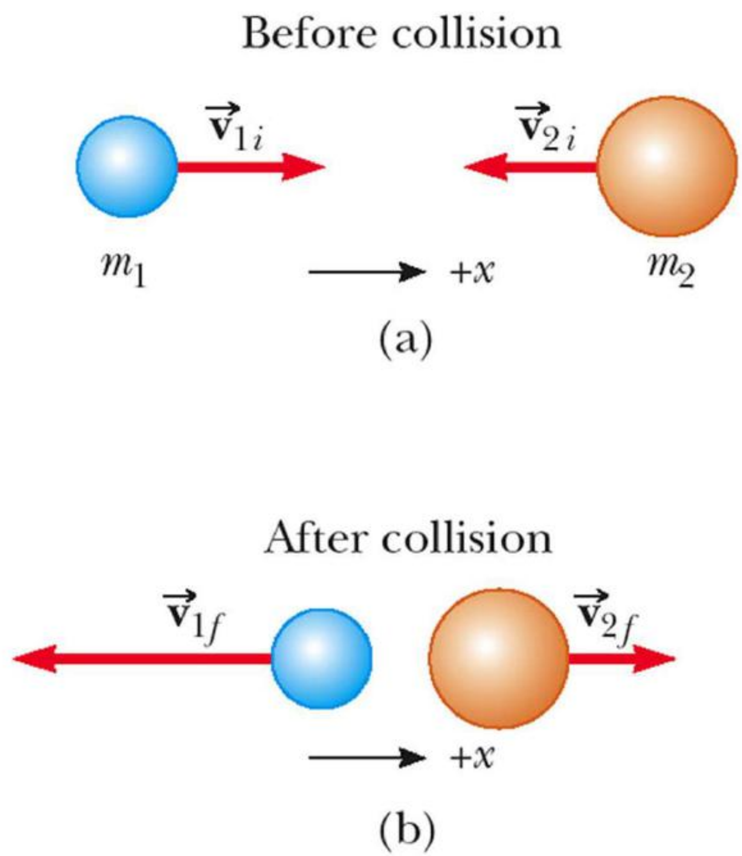
- **Coordinates:** Set up a coordinate axis and define the velocities with respect to this axis
  - It is convenient to make your axis coincide with one of the initial velocities
- **Diagram:** In your sketch, draw all the velocity vectors and label the velocities and the masses



Problem Solving for 1D Collisions, 2

- **Conservation of Momentum:** Write a general expression for the total momentum of the system *before* and *after* the collision
  - Equate the two total momentum expressions
  - Fill in the known values

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}}$$

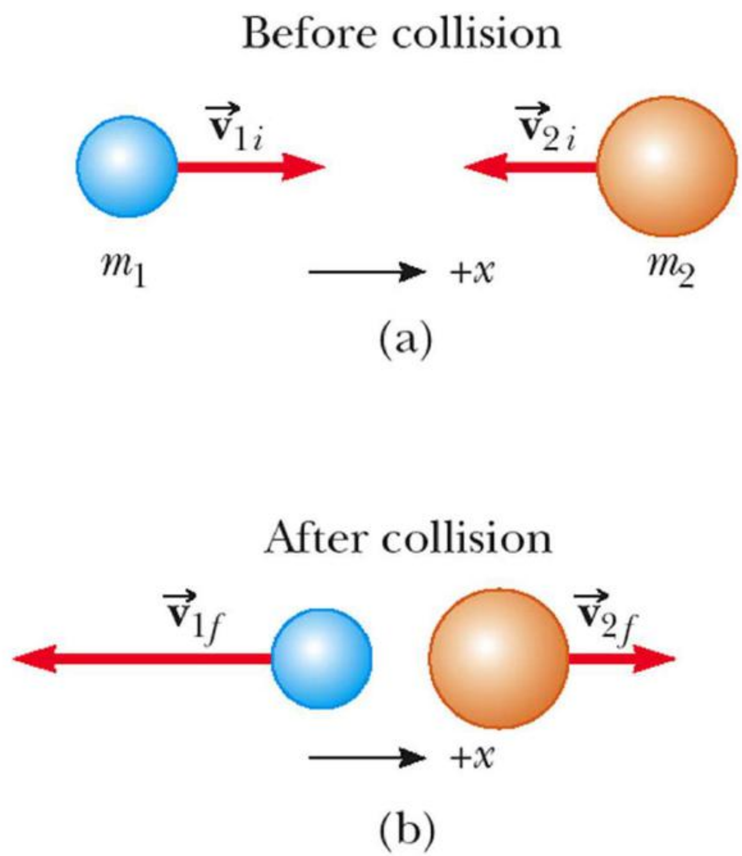


Problem Solving for 1D Collisions, 3

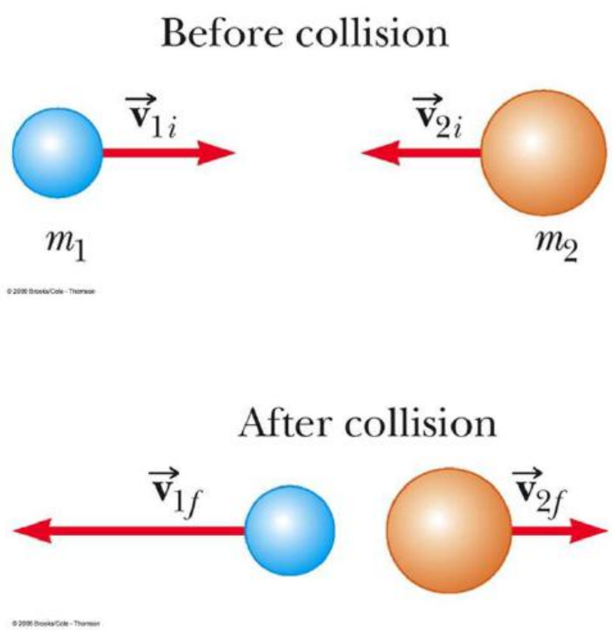
- **Conservation of Energy:** If the collision is elastic, write a second equation for conservation of KE, or the alternative equation
  - This only applies to perfectly elastic collisions

$$\boxed{v_{1i} + v_{1f} = v_{2f} + v_{2i}}$$

- **Solve:** the resulting equations simultaneously



One-Dimension vs Two-Dimension

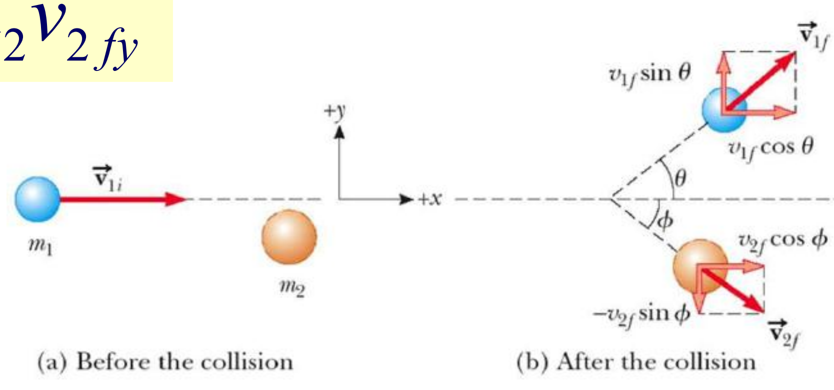


Two-Dimensional Collisions

- For a general collision of two objects in two-dimensional space, the conservation of momentum principle implies that the *total momentum of the system in each direction is conserved*

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$



Two-Dimensional Collisions

- The momentum is conserved in all directions
- Use subscripts for
  - Identifying the object
  - Indicating initial or final values
  - The velocity components

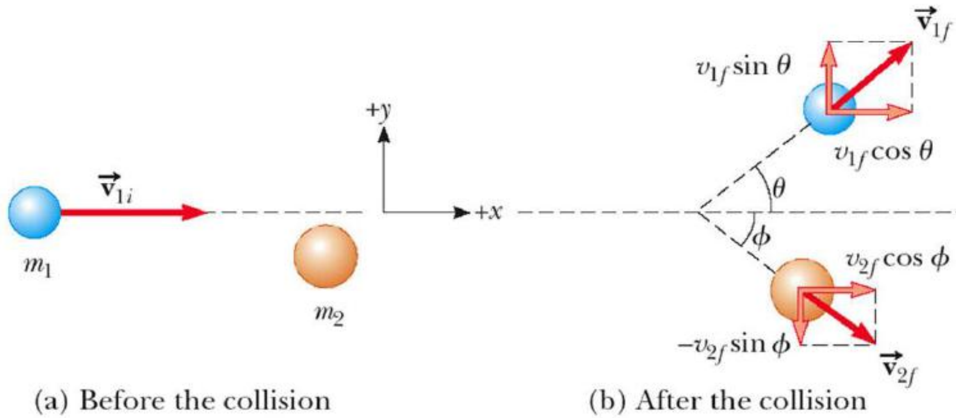
$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

- If the collision is elastic, use conservation of kinetic energy as a second equation
  - Remember, the simpler equation can only be used for one-dimensional situations

$$\cancel{v_{1i} + v_{1f} = v_{2f} + v_{2i}}$$

Glancing Collisions



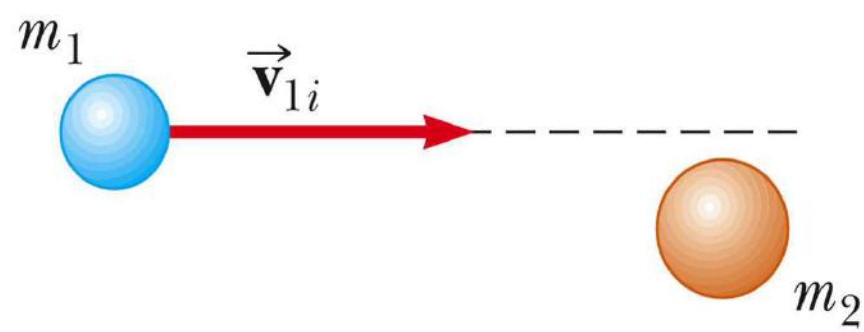
- The “after” velocities have x and y components
- Momentum is conserved in the x direction and in the y direction
- Apply conservation of momentum separately to each direction

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

2-D Collision, example

- Particle 1 is moving at velocity $\vec{\mathbf{v}}_{1i}$ and particle 2 is at rest
- In the *x*-direction, the initial momentum is $m_1 v_{1i}$
- In the *y*-direction, the initial momentum is 0



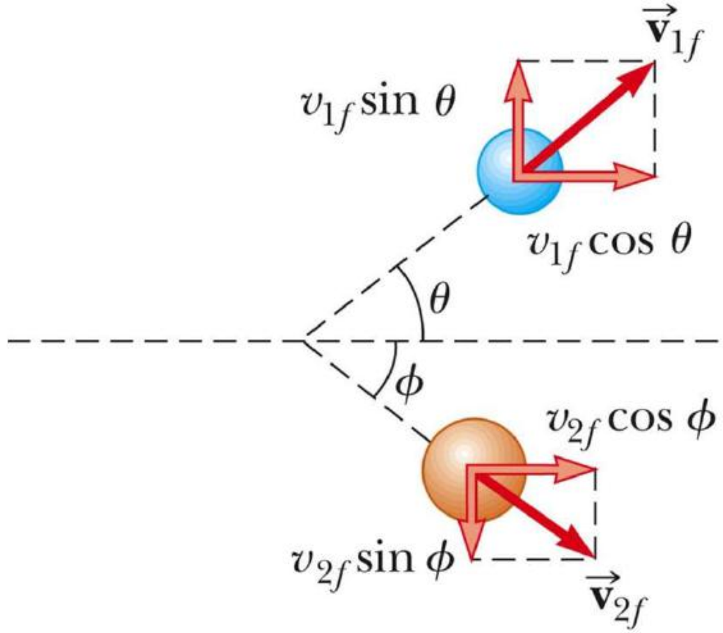
(a) Before the collision

2-D Collision, example cont

- After the collision, the momentum in the x-direction is $m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$
- After the collision, the momentum in the y-direction is $m_1 v_{1f} \sin\theta + m_2 v_{2f} \sin\phi$

$$m_1 v_{1i} + 0 = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$$

$$0 + 0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi$$



(b) After the collision

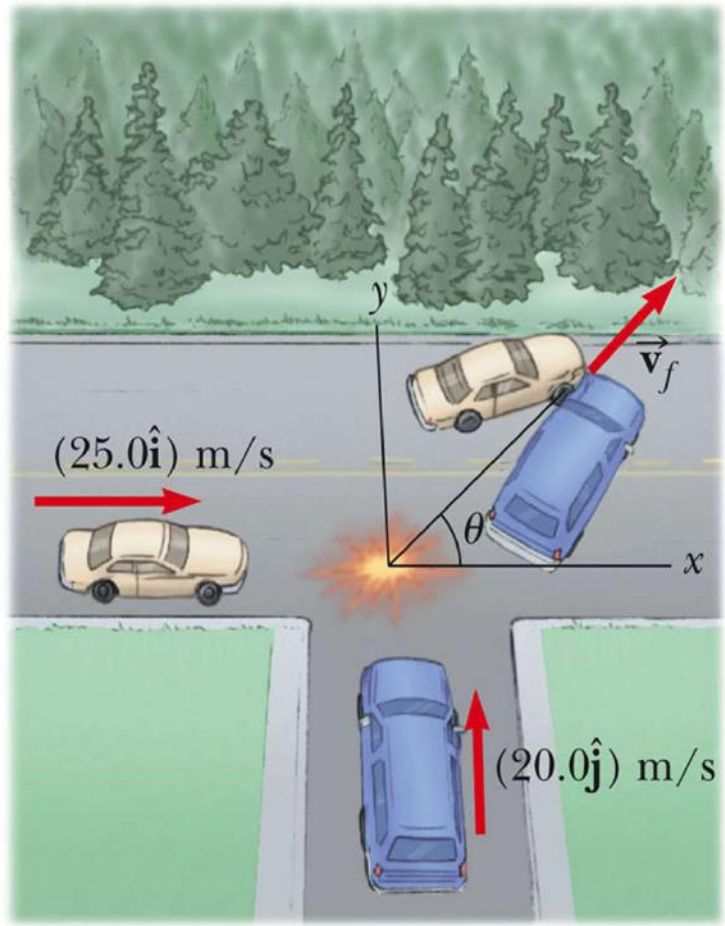
- If the collision is elastic, apply the kinetic energy equation

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

Collision at an Intersection

A car with mass 1.5×10^3 kg traveling east at a speed of 25 m/s collides at an intersection with a 2.5×10^3 kg van traveling north at a speed of 20 m/s. Find the magnitude and direction of the velocity of the wreckage after the collision, assuming that the vehicles undergo a perfectly inelastic collision and assuming that friction between the vehicles and the road can be neglected.

$$m_c = 1.5\times10^3 kg, m_v = 2.5\times10^3 kg$$
$$v_{cix} = 25m/s, v_{viy} = 20m/s, v_f = ? \theta = ?$$



Collision at an Intersection

$$m_c = 1.5\times10^3\,\text{kg}, m_v = 2.5\times10^3\,\text{kg}$$

$$v_{cix} = 25\,\text{m/s}, v_{viy} = 20\,\text{m/s}, v_f = ?\,\theta = ?$$

$$\sum p_{xi} = m_c v_{cix} + m_v v_{vix} = m_c v_{cix} = 3.75\times10^4\,\text{kg}\cdot\text{m/s}$$

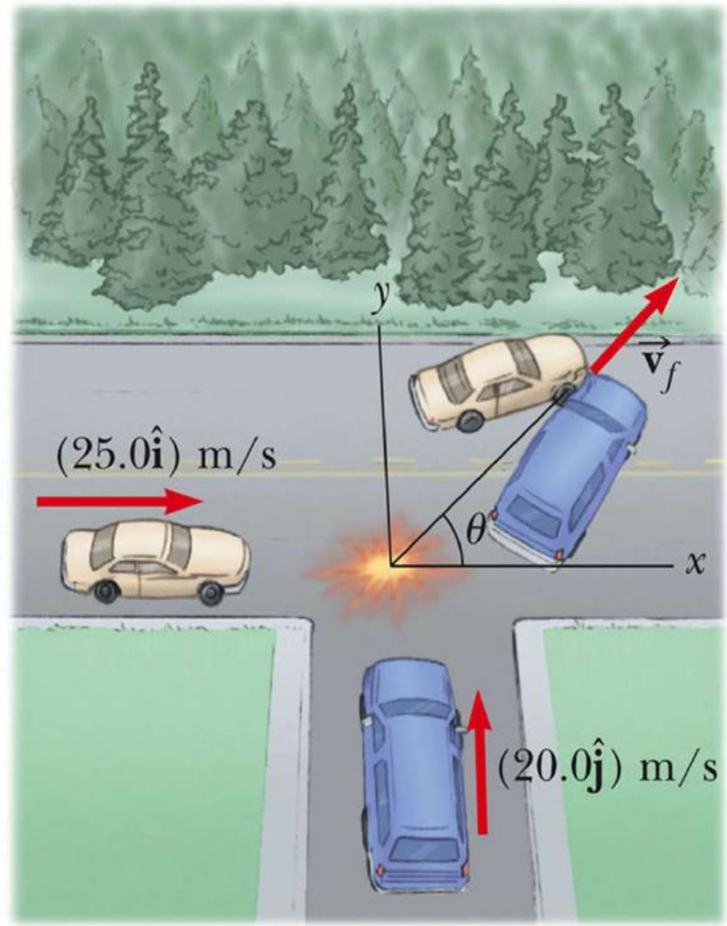
$$\sum p_{xf} = m_c v_{cfx} + m_v v_{vfx} = (m_c + m_v) v_f \cos\theta$$

$$3.75\times10^4\,\text{kg}\cdot\text{m/s} = (4.00\times10^3\,\text{kg}) v_f \cos\theta$$

$$\sum p_{yi} = m_c v_{ciy} + m_v v_{viy} = m_v v_{viy} = 5.00\times10^4\,\text{kg}\cdot\text{m/s}$$

$$\sum p_{yf} = m_c v_{cfy} + m_v v_{vfy} = (m_c + m_v) v_f \sin\theta$$

$$5.00\times10^4\,\text{kg}\cdot\text{m/s} = (4.00\times10^3\,\text{kg}) v_f \sin\theta$$



Collision at an Intersection

$$m_c = 1.5\times10^3 kg, m_v = 2.5\times10^3 kg$$

$$v_{cix} = 25m/s, v_{viy} = 20m/s, v_f = ?\theta = ?$$

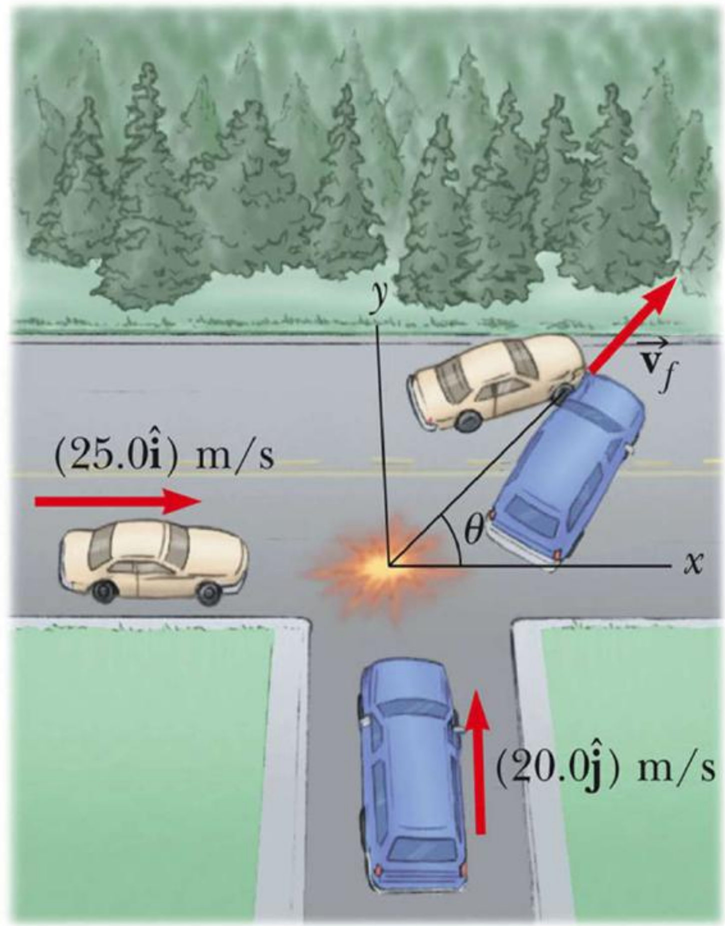
$$5.00\times10^4\,\text{kg}\cdot\text{m/s} = (4.00\times10^3\,\text{kg})v_f \sin\theta$$

$$3.75\times10^4\,\text{kg}\cdot\text{m/s} = (4.00\times10^3\,\text{kg})v_f \cos\theta$$

$$\tan\theta = \frac{5.00\times10^4\, kg\cdot m/s}{3.75\times10^4\, kg\cdot m/s} = 1.33$$

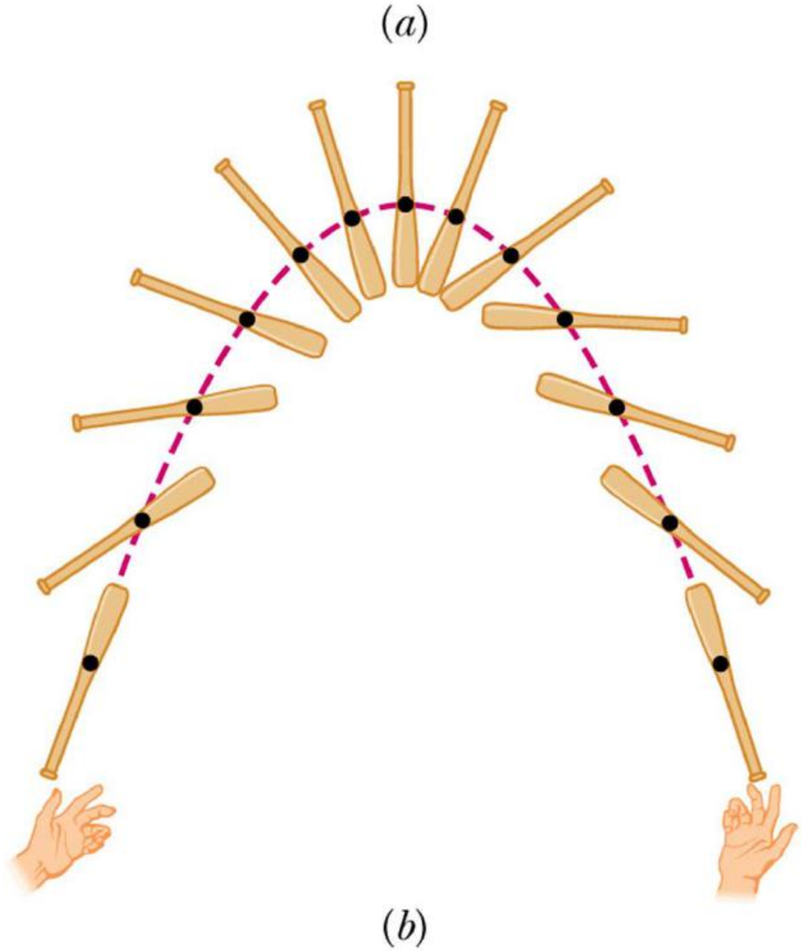
$$\theta = \tan^{-1}(1.33) = 53.1^\circ$$

$$v_f = \frac{5.00\times10^4\,\text{kg}\cdot\text{m/s}}{(4.00\times10^3\,\text{kg})\sin 53.1^\circ} = 15.6\,\text{m/s}$$



The Center of Mass

- How should we define the position of the moving body ?
- What is y for $U_g = mgy$?
- Take the average position of mass. Call “Center of Mass” (COM or CM)

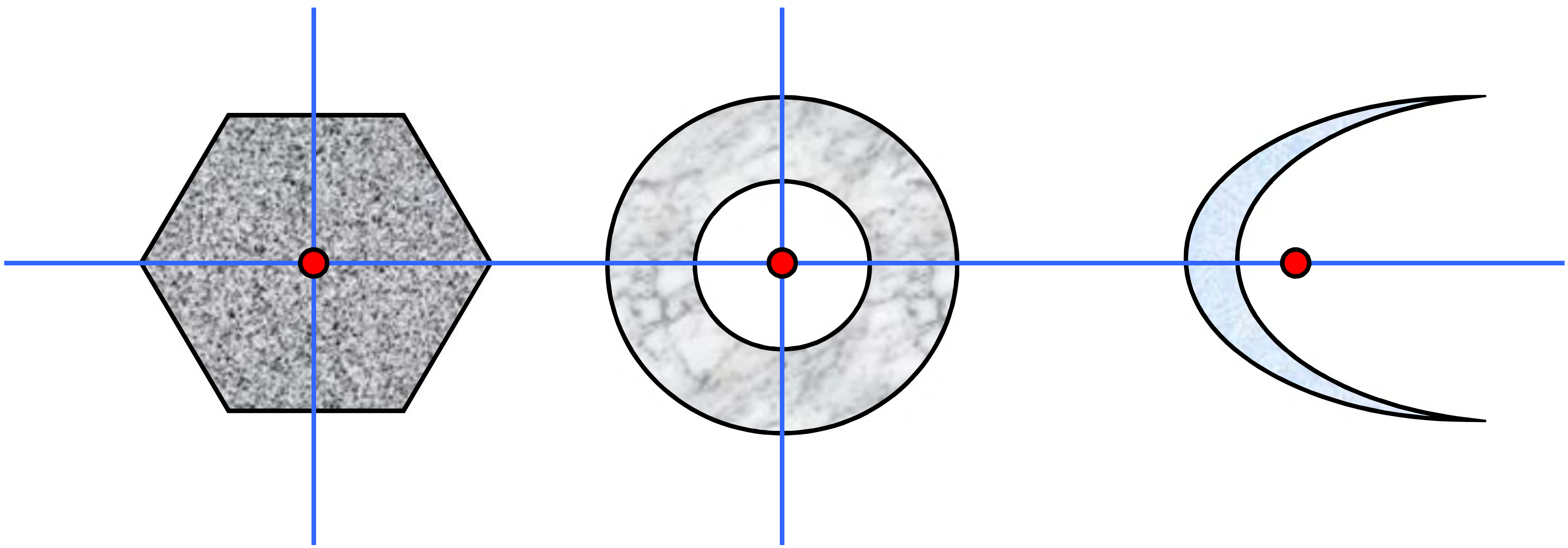


The Center of Mass

- There is a special point in a system or object, called the ***center of mass***, that moves as if all of the mass of the system is concentrated at that point
- The CM of an object or a system is the point, where the object or the system can be balanced in the uniform gravitational field

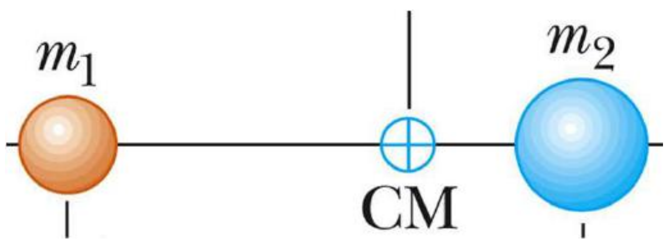
The Center of Mass

- The center of mass of any symmetric object lies on an axis of symmetry and on any plane of symmetry
  - If the object has uniform density
- The CM may reside inside the body, or outside the body

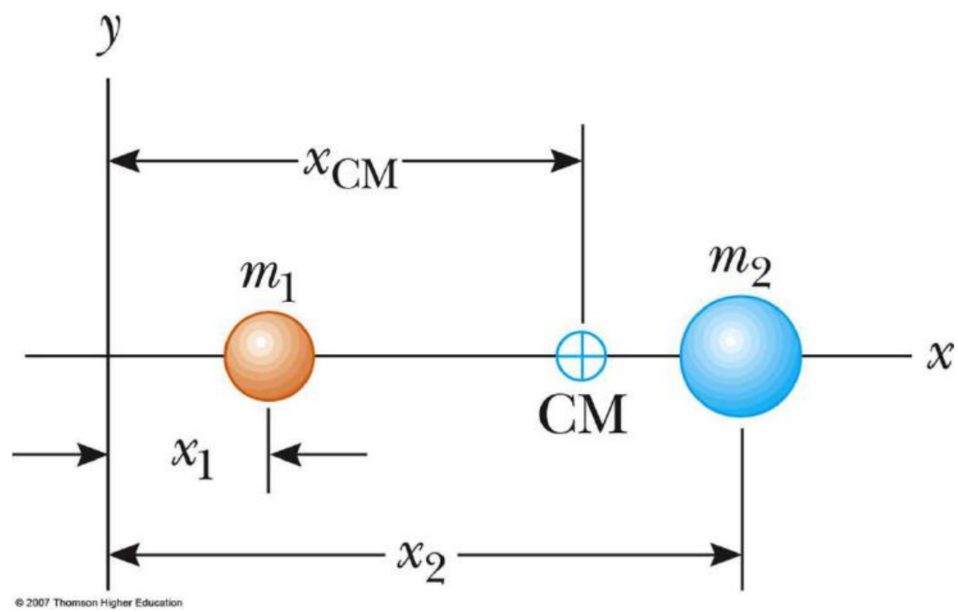


Where is the Center of Mass ?

- The center of mass of particles
- Two bodies in 1 dimension



$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$



Center of Mass for many particles in 3D?

Where is the Center of Mass ?

- Assume m_1 = 1 kg, m_2 = 3 kg, and x_1 = 1 m, x_2 = 5 m, where is the center of mass of these two objects?

  A) x_{CM} = 1 m

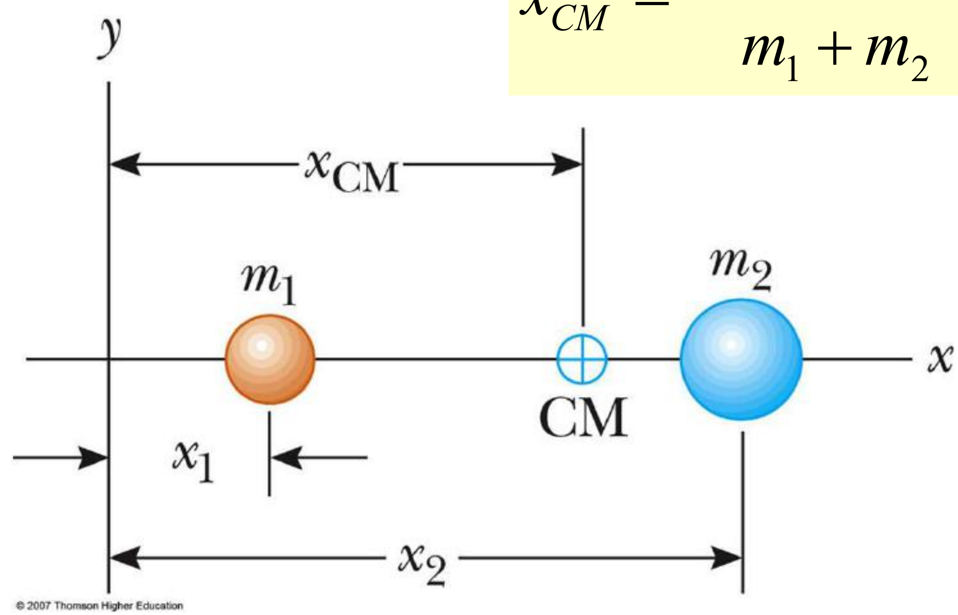
  B) x_{CM} = 2 m

  C) x_{CM} = 3 m

  D) x_{CM} = 4 m

  E) x_{CM} = 5 m

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$



Center of Mass for a System of Particles

- Two bodies and one dimension

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

- General case: n bodies and three dimension

$$x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i, \quad y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i, \quad z_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i$$

  – where $M = m_1 + m_2 + m_3 + \ldots$

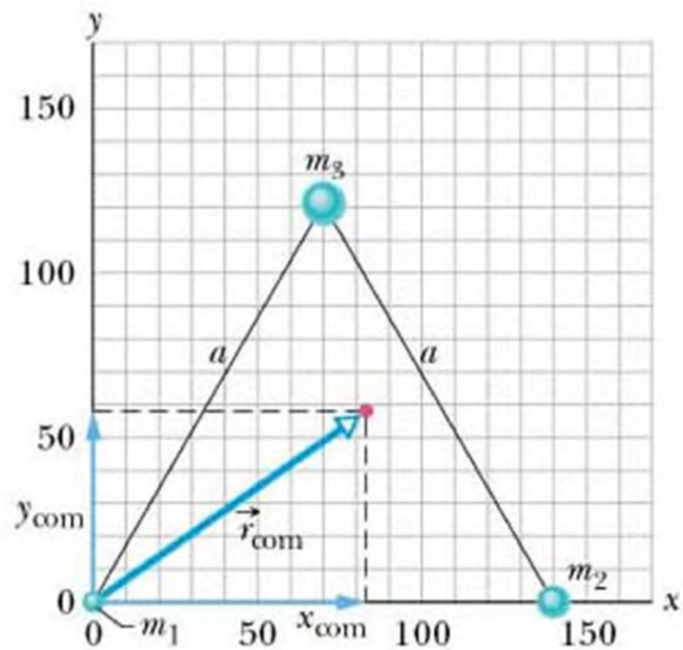
$$\vec{r}_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i \vec{r}_i$$

Sample Problem : Three particles of masses *m*1 = 1.2 kg, *m*2 = 2.5 kg, and *m*3 = 3.4 kg form an equilateral triangle of edge length *a* = 140 cm. Where is the center of mass of this system? (Hint: m1 is at (0,0), m2 is at (140 cm,0), and m3 is at (70 cm, 120 cm), as shown in the figure below.)

$$x_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$y_{CM} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$x_{CM} = 82.8\,\text{cm} \quad \text{and} \quad y_{CM} = 57.5\,\text{cm}$$



Motion of a System of Particles

- Assume the total mass, M, of the system remains constant
- We can describe the motion of the system in terms of the velocity and acceleration of the center of mass of the system
- We can also describe the momentum of the system and Newton's Second Law for the system

Velocity and Momentum of a System of Particles

- The velocity of the center of mass of a system of particles is

$$\vec{\mathbf{v}}_{\text{CM}} = \frac{d\vec{\mathbf{r}}_{\text{CM}}}{dt} = \frac{1}{M}\sum_i m_i \vec{\mathbf{v}}_i$$

- The momentum can be expressed as

$$M\vec{\mathbf{v}}_{\text{CM}} = \sum_i m_i \vec{\mathbf{v}}_i = \sum_i \vec{\mathbf{p}}_i = \vec{\mathbf{p}}_{\text{tot}}$$

- The total linear momentum of the system equals the total mass multiplied by the velocity of the center of mass

Acceleration and Force of the Center of Mass

- The acceleration of the center of mass can be found by differentiating the velocity with respect to time

$$\vec{\mathbf{a}}_{\mathrm{CM}} = \frac{d\vec{\mathbf{v}}_{\mathrm{CM}}}{dt} = \frac{1}{M}\sum_i m_i \vec{\mathbf{a}}_i$$

- The acceleration can be related to a force

$$M\vec{\mathbf{a}}_{\mathrm{CM}} = \sum_i \vec{\mathbf{F}}_i$$

- If we sum over all the internal forces, they cancel in pairs and the net force on the system is caused only by the external forces

Newton's Second Law for a System of Particles

- Since the only forces are external, the net external force equals the total mass of the system multiplied by the acceleration of the center of mass:

$$\sum \vec{\mathbf{F}}_{ext} = M\vec{\mathbf{a}}_{CM}$$

- The center of mass of a system of particles of combined mass *M* moves like an equivalent particle of mass *M* would move under the influence of the net external force on the system

Basic Physics 1
Lecture Module

Rahadian N, S.Si. M.Si.

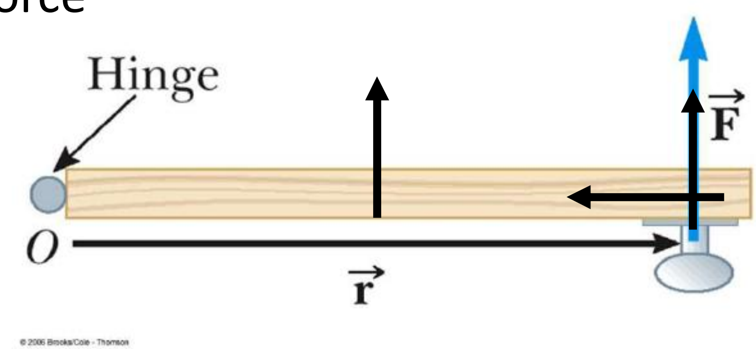
Rotational Dynamics

Rotational Dynamics

- Torque
- Moment of inertia
- Newton 2^{nd} law in rotation
- Rotational work
- Rotational kinetic energy
- Rotational energy conservation
- Rolling motion of a rigid object

Force vs. Torque

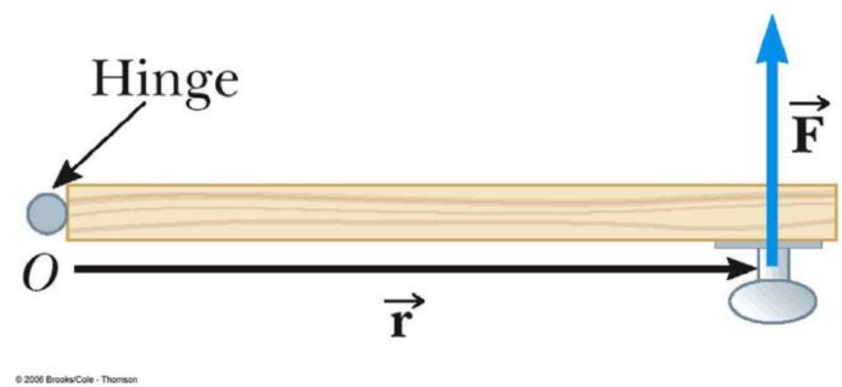
- Forces cause accelerations
- What cause angular accelerations ?
- A door is free to rotate about an axis through O
- There are three factors that determine the effectiveness of the force in opening the door:
  - The *magnitude* of the force
  - The *position* of the application of the force
  - The *angle* at which the force is applied



Torque Definition

- Torque, τ, is the tendency of a force to rotate an object about some axis
- Let **F** be a force acting on an object, and let **r** be a position vector from a rotational center to the point of application of the force, with **F** perpendicular to **r**. The magnitude of the torque is given by

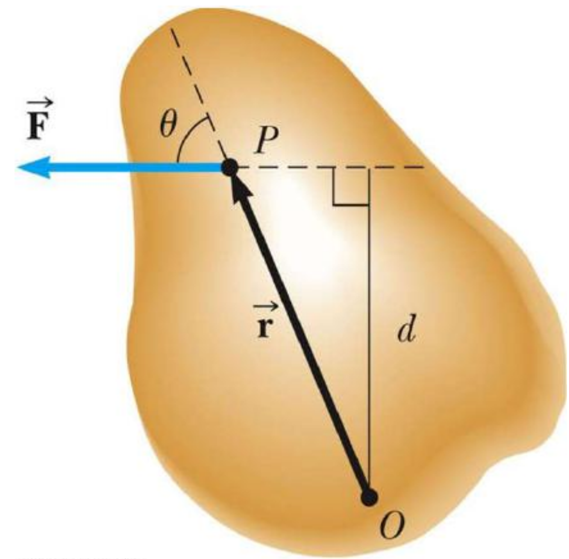
$$\tau = rF$$



Torque Units and Direction

- The SI units of torque are N·m
- Torque is a vector quantity
- Torque magnitude is given by

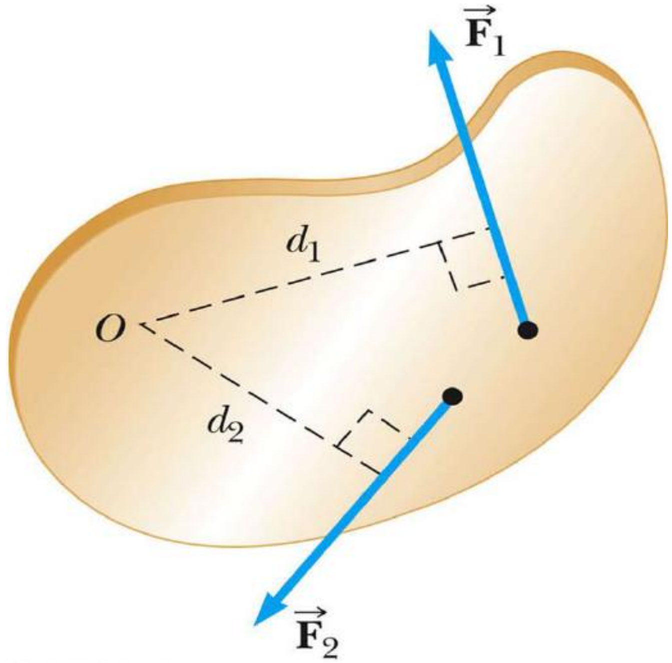
$$\tau = rF\sin\theta = Fd$$



- Torque will have direction
  - If the turning tendency of the force is counterclockwise, the torque will be positive
  - If the turning tendency is clockwise, the torque will be negative

Net Torque

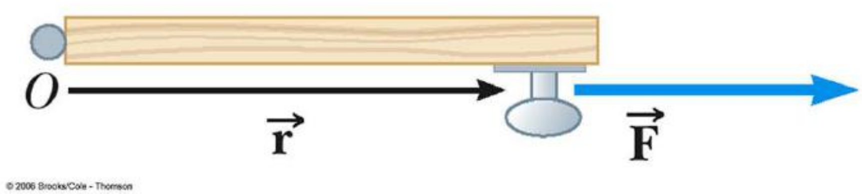
- The force $\vec{\mathbf{F}}_1$ will tend to cause a counterclockwise rotation about *O*
- The force $\vec{\mathbf{F}}_2$ will tend to cause a clockwise rotation about *O*
- $\Sigma\tau = \tau_1 + \tau_2 = F_1 d_1 - F_2 d_2$
- If $\Sigma\tau \neq 0$, starts rotating
- If $\Sigma\tau = 0$, rotation rate does not change

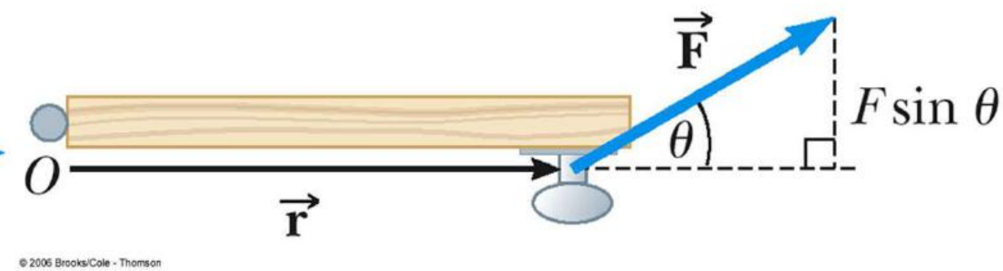


- *Rate of rotation of an object does not change, unless the object is acted on by a net torque*

General Definition of Torque

- The applied force is not always perpendicular to the position vector
- The component of the force *perpendicular* to the object will cause it to rotate
- When the force is parallel to the position vector, no rotation occurs
- When the force is at some angle, the perpendicular component causes the rotation

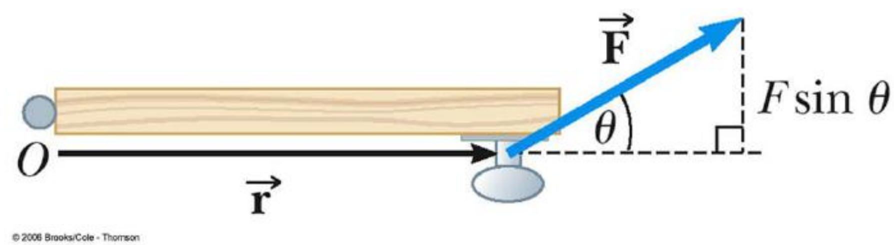




General Definition of Torque

- Let **F** be a force acting on an object, and let **r** be a position vector from a rotational center to the point of application of the force. The magnitude of the torque is given by

$$\tau = rF\sin\theta$$



- $\theta = 0^\circ$ or $\theta = 180^\circ$:

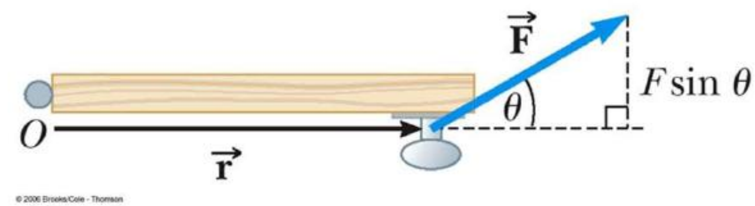
  torque are equal to zero
- $\theta = 90^\circ$ or $\theta = 270^\circ$: magnitude of torque attain to the maximum

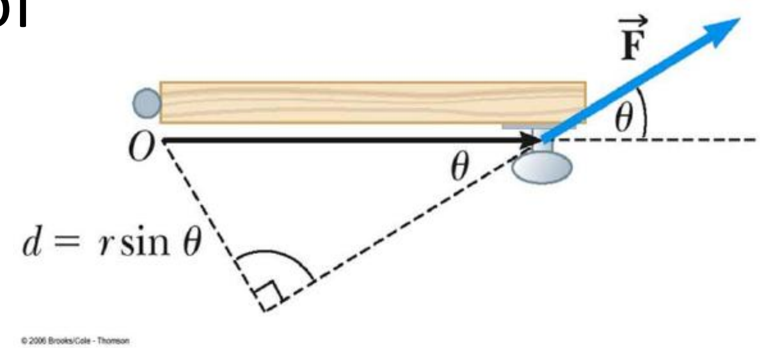
Understand sinθ

- The component of the force ($F \cos\theta$) has no tendency to produce a rotation
- The moment arm, *d*, is the *perpendicular* distance from the axis of rotation to a line drawn along the direction of the force

$$d = r \sin\theta$$

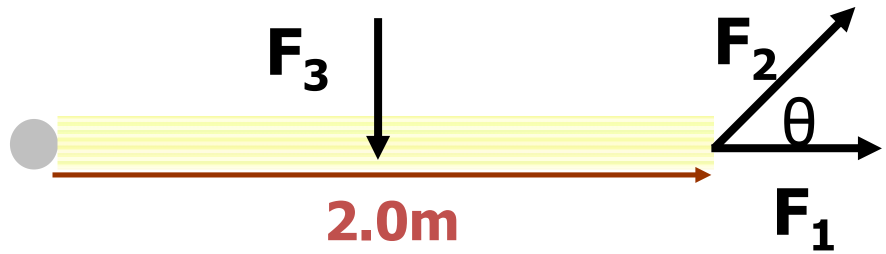
$$\tau = rF\sin\theta = Fd$$





The Swinging Door

- Three forces are applied to a door, as shown in figure. Suppose a wedge is placed 1.5 m from the hinges on the other side of the door. What minimum force must the wedge exert so that the force applied won't open the door? Assume $F_1 = 150$ N, $F_2 = 300$ N, $F_3 = 300$ N, $\theta = 30^\circ$

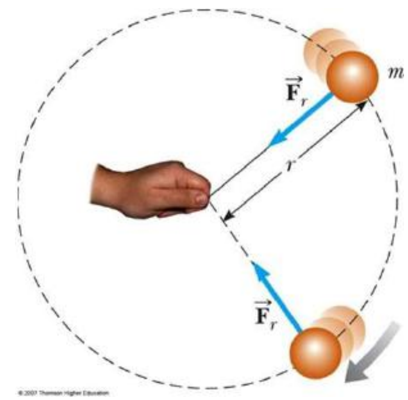


Moment of Inertia

- For a single particle, the definition of moment of inertia is

$$I = mr^2$$

  - m is the mass of the single particle
  - r is the rotational radius



- SI units of moment of inertia are kg·m^2

- Moment of inertia and mass of an object are different quantities

- It depends on both the quantity of matter and its distribution (through the r^2 term)

Moment of Inertia of Point Mass

- For a composite particle, the definition of moment of inertia is

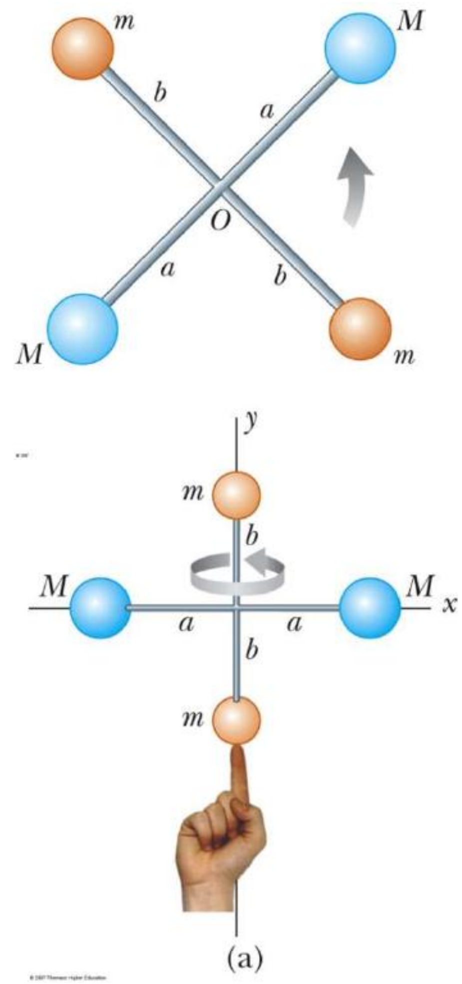
$$I = \sum m_i r_i^{\ 2} = m_1 r_1^{\ 2} + m_2 r_2^{\ 2} + m_3 r_3^{\ 2} + m_4 r_4^{\ 2} + \ldots$$

  - m_i is the mass of the ith single particle
  - r_i is the rotational radius of ith particle
- SI units of moment of inertia are kg·m^2
- Consider an unusual baton made up of four sphere fastened to the ends of very light rods
- Find *I* about an axis perpendicular to the page and passing through the point *O* where the rods cross

$$I = \sum m_i r_i^{\ 2} = mb^2 + Ma^2 + mb^2 + Ma^2 = 2Ma^2 + 2mb^2$$

The Baton Twirler

- Consider an unusual baton made up of four sphere fastened to the ends of very light rods. Each rod is 1.0m long ($a = b = 1.0$ m). M = 0.3 kg and m = 0.2 kg.
- (a) Find I about an axis perpendicular to the page and passing through the point where the rods cross. Find K_R if angular speed is ω
- (b) The majorette tries spinning her strange baton about the axis y, calculate I of the baton about this axis and K_R if angular speed is ω



Moment of Inertia of Extended Objects

- Divided the extended objects into many small volume elements, each of mass Δm_i
- We can rewrite the expression for I in terms of Δm

$$I = \lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i = \int r^2 dm$$

- Consider a small volume such that $dm = \rho\, dV$. Then

$$I = \int \rho r^2 dV$$

- If ρ is constant, the integral can be evaluated with known geometry, otherwise its variation with position must be known

Densities

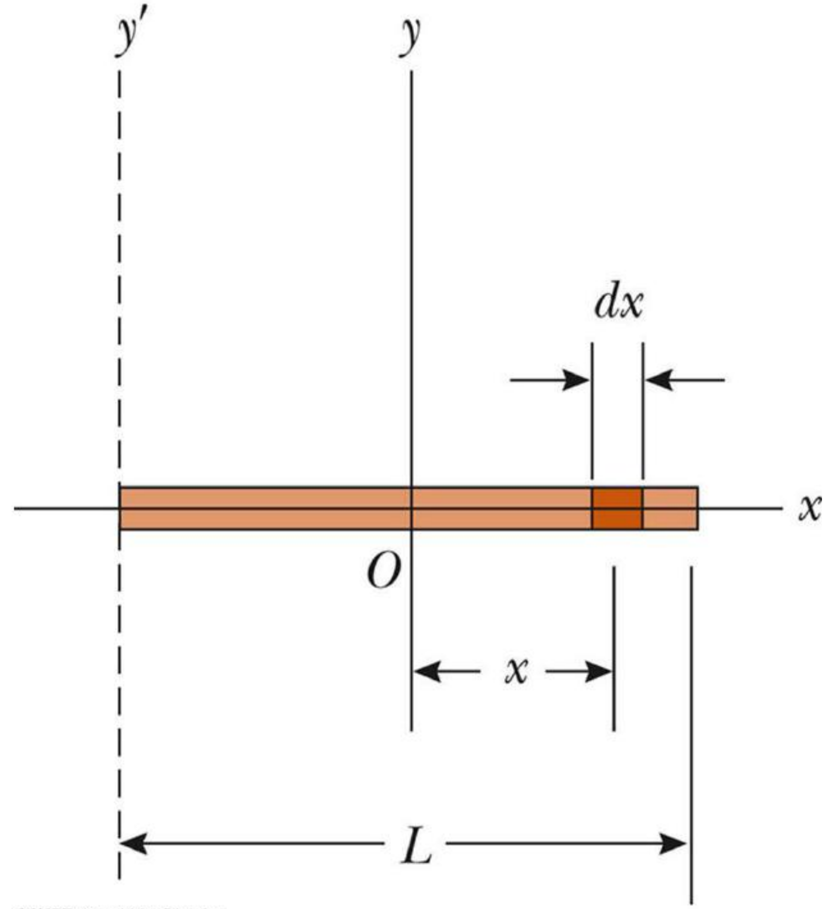
- You know the density (volume density) as mass/unit volume
  - $\rho = M/V = dm/dV$ $\qquad$ $\Rightarrow dm = \rho dV$
- We can define other densities such as surface density (mass/unit area)
  - $\sigma = M/A = dm/dA$ $\qquad$ $\Rightarrow dm = \sigma dV$
- Or linear density (mass/unit length)
  - $\lambda = M/L = dm/dx$ $\qquad$ $\Rightarrow dm = \lambda dV$

Moment of Inertia of a Uniform Rigid Rod

- The shaded area has a mass
  - $dm = \lambda\, dx$
- Then the moment of inertia is

$$I_y = \int r^2 dm = \int_{-L/2}^{L/2} x^2 \frac{M}{L}\, dx$$

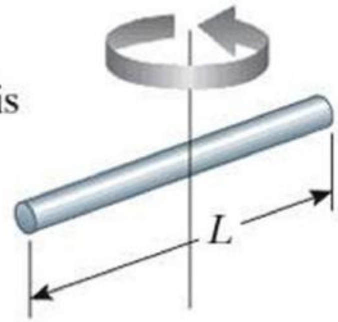
$$I = \frac{1}{12} ML^2$$



Moment of Inertia for some other common shapes

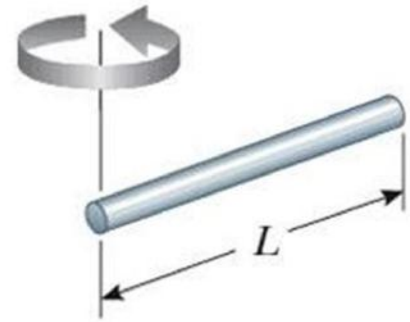
Long, thin rod with rotation axis through center

$$I_{CM} = \frac{1}{12} ML^2$$



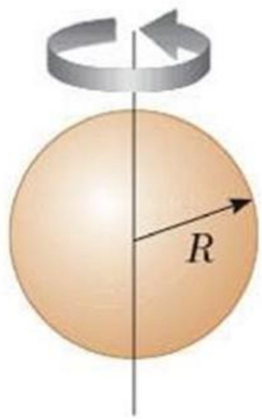
Long, thin rod with rotation axis through end

$$I = \frac{1}{3} ML^2$$



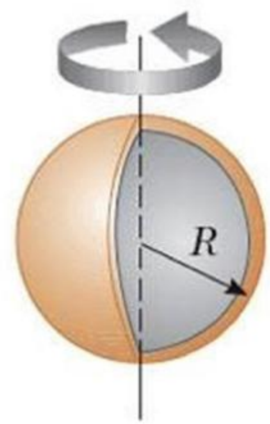
Solid sphere

$$I_{CM} = \frac{2}{5} MR^2$$

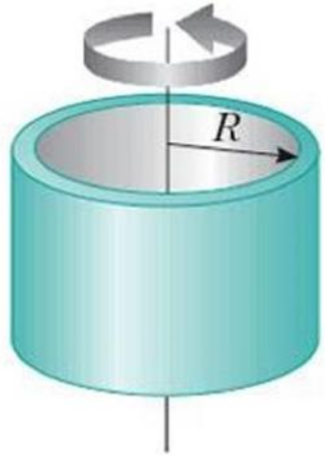


Thin spherical shell

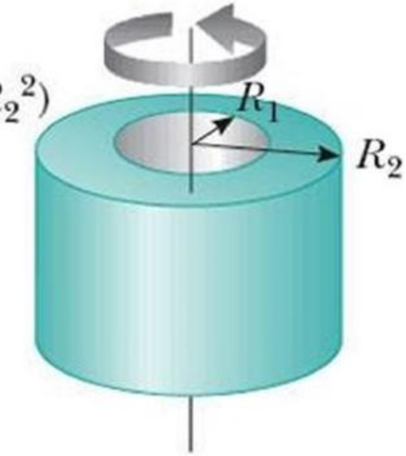
$$I_{CM} = \frac{2}{3} MR^2$$



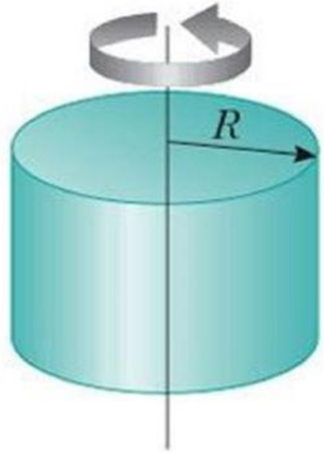
Hoop or thin cylindrical shell
$I_{CM} = MR^2$



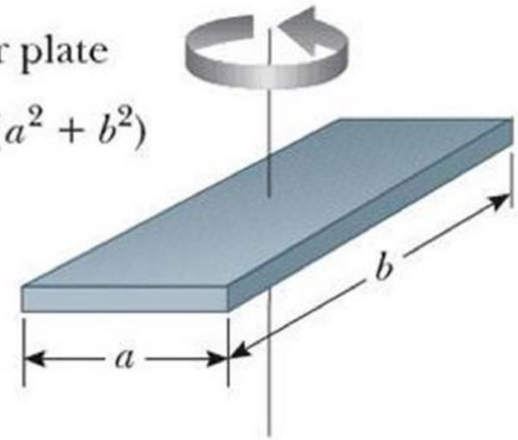
Hollow cylinder
$I_{CM} = \frac{1}{2}M(R_1{}^2 + R_2{}^2)$



Solid cylinder or disk
$I_{CM} = \frac{1}{2}MR^2$



Rectangular plate
$I_{CM} = \frac{1}{12}M(a^2 + b^2)$

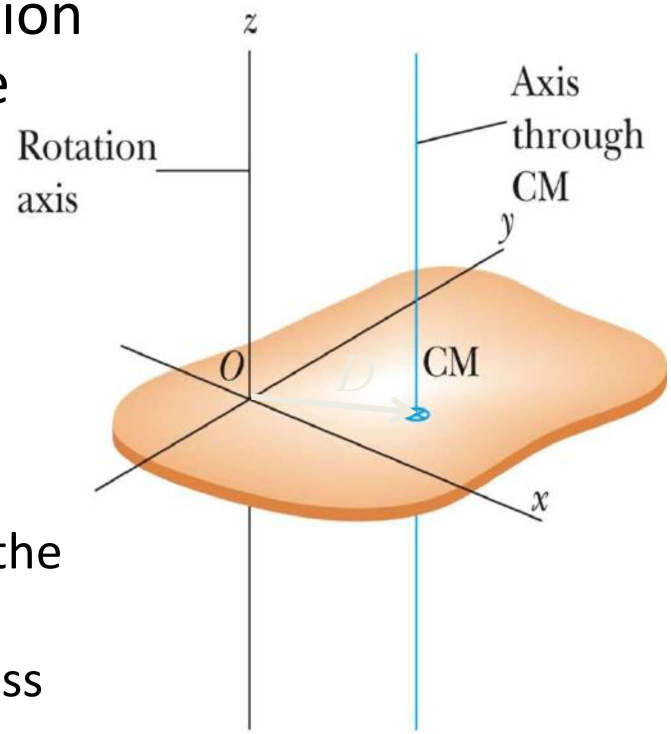


Parallel-Axis Theorem

- In the previous examples, the axis of rotation coincides with the axis of symmetry of the object
- For an arbitrary axis, the parallel-axis theorem often simplifies calculations
- The theorem states

$$I = I_{CM} + MD^2$$

  - I is about any axis parallel to the axis through the center of mass of the object
  - I_{CM} is about the axis through the center of mass
  - D is the distance from the center of mass axis to the arbitrary axis

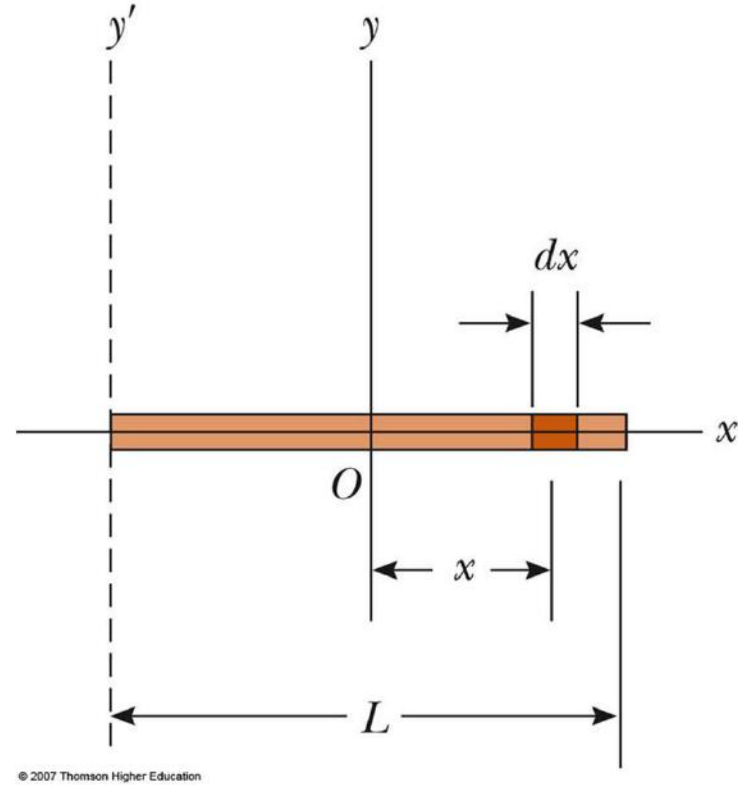


Moment of Inertia of a Uniform Rigid Rod

- The moment of inertia about y is

$$I_y = \int r^2 dm = \int_{-L/2}^{L/2} x^2 \frac{M}{L} dx$$

$$I = \frac{1}{12} ML^2$$



- The moment of inertia about y' is

$$I_{y'} = I_{CM} + MD^2 = \frac{1}{12} ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{3} ML^2$$

Newton's Second Law for a Rotating Object

- When a rigid object is subject to a net torque (≠0), it undergoes an angular acceleration

$$\Sigma\tau = I\alpha$$

- The angular acceleration is directly proportional to the net torque
- The angular acceleration is inversely proportional to the moment of inertia of the object
- The relationship is analogous to

$$\sum F = ma$$

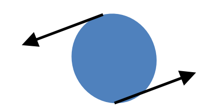
Strategy to use the Newton 2nd Law

- Draw or sketch system. Adopt coordinates, indicate rotation axes, list the known and unknown quantities, …
- Draw free body diagrams of key parts. Show forces at their points of application. Find torques about a (common) axis

- May need to apply Second Law twice, once to each part

  - Translation: $\mathbf{F}_{\mathbf{net}} = \sum \vec{\mathbf{F}}_{\mathbf{i}} = \mathbf{m}\vec{\mathbf{a}}$
  - Rotation: $\vec{\tau}_{\mathbf{net}} = \sum \vec{\tau}_{\mathbf{i}} = \mathbf{I}\vec{\alpha}$

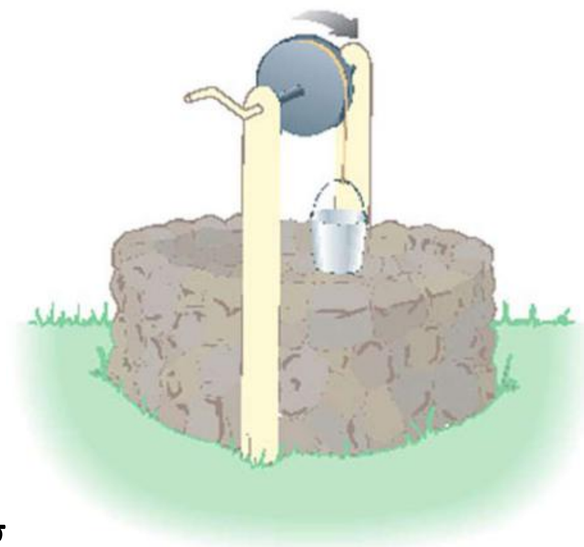
Note: can have $\mathbf{F}_{\mathbf{net}} = 0$ but $\tau_{\mathbf{net}} \neq 0$



- Make sure there are enough (N) equations; there may be constraint equations (extra conditions connecting unknowns)
- Simplify and solve the set of (simultaneous) equations.
- Find unknown quantities and check answers

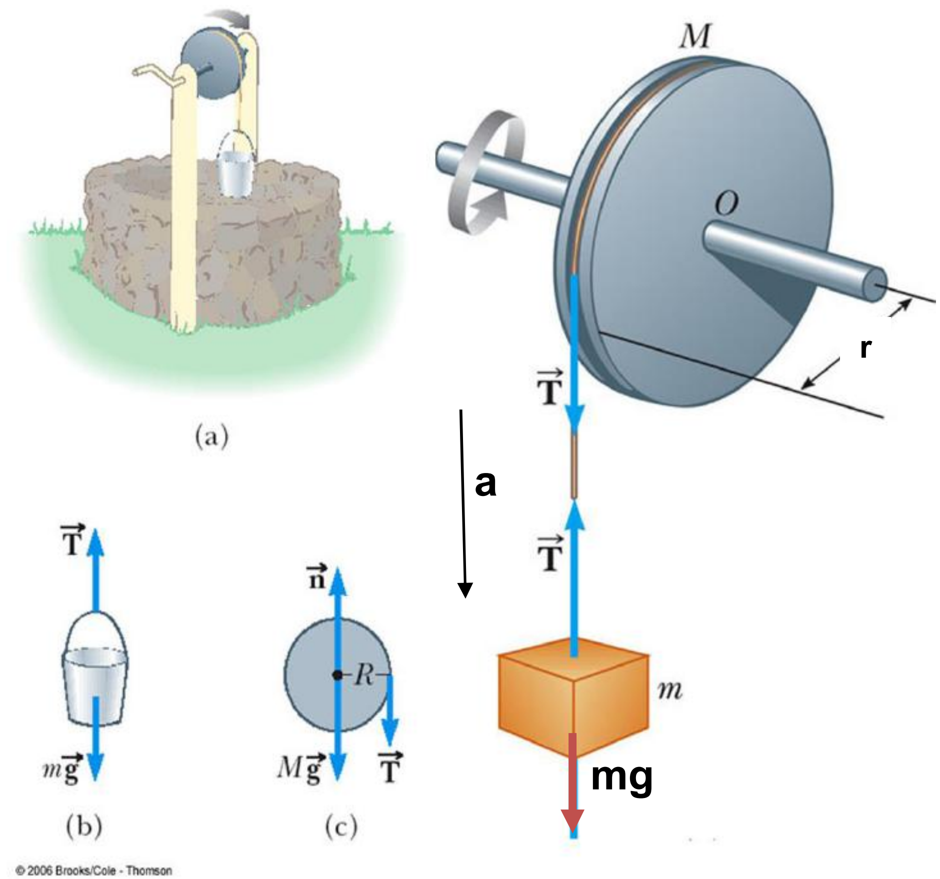
The Falling Object

- A solid, frictionless cylindrical reel of mass M = 2.5 kg and radius R = 0.2 m is used to draw water from a well. A bucket of mass m = 1.2 kg is attached to a cord that is wrapped around the cylinder.
- (a) Find the tension T in the cord and acceleration a of the object.
- (b) If the object starts from rest at the top of the well and falls for 3.0 s before hitting the water, how far does it fall ?

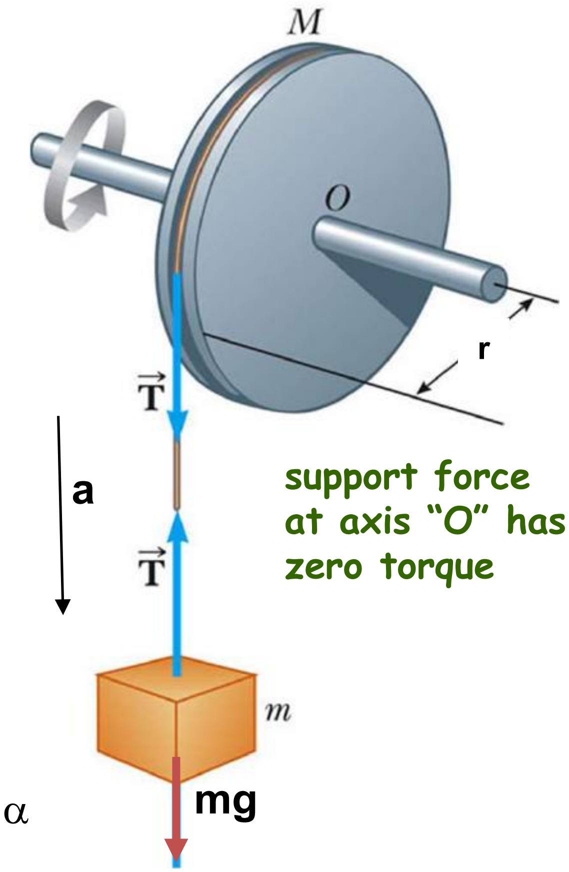


Newton 2nd Law for Rotation

- Draw free body diagrams of each object
- Only the cylinder is rotating, so apply
  $\Sigma \tau = I\alpha$
- The bucket is falling, but not rotating, so apply
  $\Sigma F = ma$
- Remember that $a = \alpha r$ and solve the resulting equations

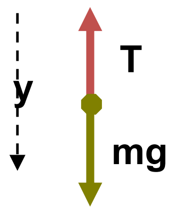


- Cord wrapped around disk, hanging weight
- Cord does not slip or stretch → constraint
- Disk's rotational inertia slows accelerations
- Let m = 1.2 kg, M = 2.5 kg, r =0.2 m



support force at axis "O" has zero torque

For mass m:

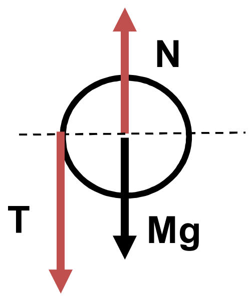


$$\sum F_y = ma = mg - T$$

$$\boxed{T = m(g-a)}$$

Unknowns: T, a

FBD for disk, with axis at "o":



$$\sum \tau_0 = \ +Tr \ = \ I\alpha \qquad I = \frac{1}{2}Mr^2$$

$$\boxed{\alpha = \frac{Tr}{I} = \frac{m(g-a)r}{\frac{1}{2}Mr^2}}$$

Unknowns: a, α

So far: 2 Equations, 3 unknowns →Need a constraint: $\boxed{a = \alpha r}$ **from "no slipping" assumption**

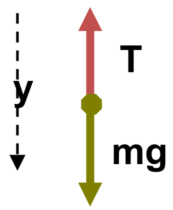
Substitute and solve:

$$\alpha = \frac{2mgr}{Mr^2} - \frac{2m\alpha r^2}{Mr^2} \qquad \alpha\left(1 + 2\frac{m}{M}\right) = \frac{2mg}{Mr}$$

$$\boxed{\alpha = \frac{mg}{r(m + M/2)} \quad (= 24\ rad/s^2)}$$

- Cord wrapped around disk, hanging weight
- Cord does not slip or stretch → constraint
- Disk's rotational inertia slows accelerations
- Let m = 1.2 kg, M = 2.5 kg, r =0.2 m

For mass m:



$$\sum F_y = ma = mg - T$$

$$\boxed{T = m(g - a)}$$

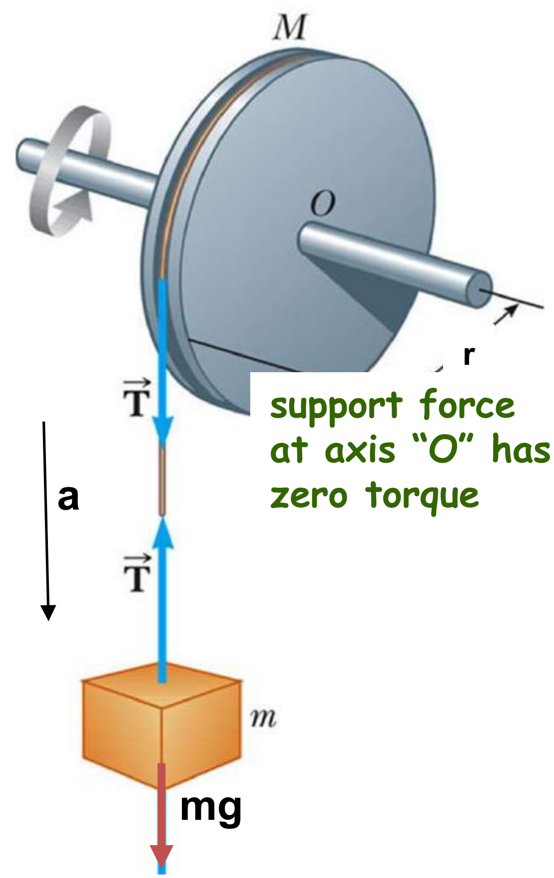
Unknowns: T, a

$$\alpha = \frac{mg}{r(m + M/2)} \quad (= 24 \text{ rad/s}^2)$$

$$a = \frac{mg}{(m + M/2)} \quad (= \ 4.8 \text{ m/s}^2)$$

$$\boxed{T = m(g - a) \ = 1.2(9.8 - 4.8) = 6 \text{ N}}$$

$$\boxed{x_f - x_i = \ v_i t + \frac{1}{2} a t^2 = 0 + \frac{1}{2}(4.8 \text{ m/s}^2)(3 \text{ s})^2 = 21.6 \text{ m}}$$



Rotational Kinetic Energy

- There is an analogy between the kinetic energies associated with linear motion ($K = ½\, mv^2$) and the kinetic energy associated with rotational motion ($K_R = ½\, I\omega^2$). Where *I* is the moment of inertia.

- Rotational kinetic energy is not a new type of energy, the form is different because it is applied to a rotating object

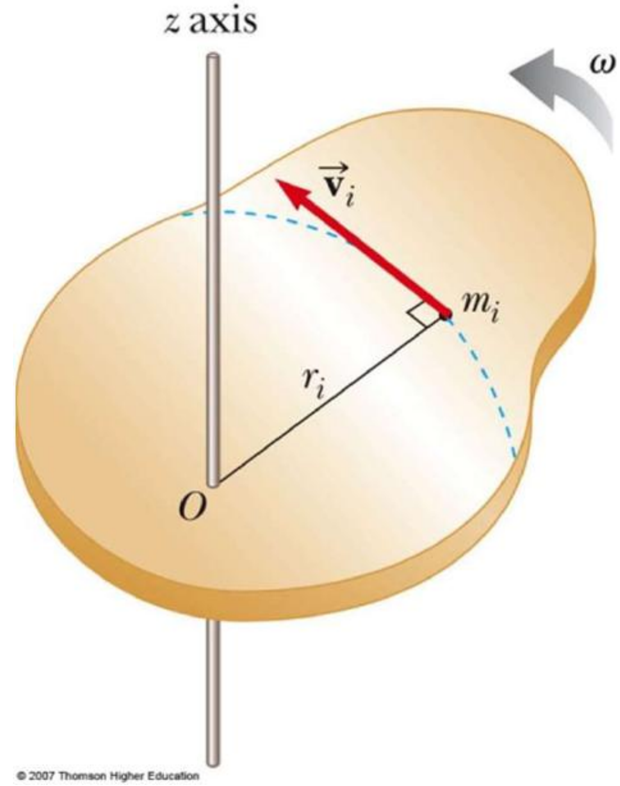
- Units of rotational kinetic energy are Joules (J)

- An object rotating about z axis with an angular speed, ω, has rotational kinetic energy

- The total rotational kinetic energy of the rigid object is the sum of the energies of all its particles

$$K_R = \sum_i K_i = \sum_i \frac{1}{2} m_i r_i^2 \omega^2$$

$$K_R = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2 = \frac{1}{2} I \omega^2$$



Work-Energy Theorem for pure Translational motion

- The work-energy theorem tells us

$$W_{net} = \Delta KE = KE_f - KE_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

- Kinetic energy is for point mass only, ignoring rotation.
- Work

$$W_{net} = \int dW = \int \vec{F} \cdot d\vec{s}$$

- Power

$$P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{s}}{dt} = \vec{F} \cdot \vec{v}$$

Mechanical Energy Conservation

- Energy conservation

$$W_{nc} = \Delta K + \Delta U$$

- When $W_{nc} = 0$,

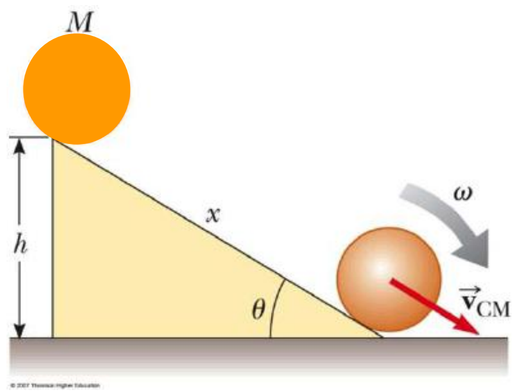
$$K_f + U_f = U_i + K_i$$

- The total mechanical energy is conserved and remains the same at all times

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

- Remember, this is for conservative forces, no dissipative forces such as friction can be present

Total Energy of a System



- A ball is rolling down a ramp
- Described by three types of energy
  - Gravitational potential energy

$$U = Mgh$$

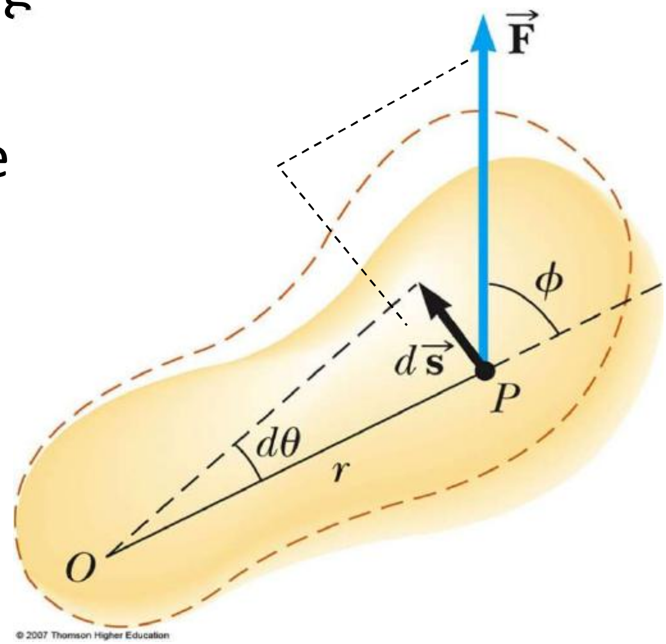
  - Translational kinetic energy $$K_t = \frac{1}{2}Mv_{CM}^2$$
  - Rotational kinetic energy $$K_r = \frac{1}{2}I\omega^2$$

- Total energy of a system $$E = \frac{1}{2}Mv_{CM}^2 + Mgh + \frac{1}{2}I\omega^2$$

Work done by a pure rotation

- Apply force F to mass at point r, causing rotation-only about axis
- Find the work done by F applied to the object at P as it rotates through an infinitesimal distance ds

$$dW = \vec{F} \cdot d\vec{s} = F\cos(90^\circ - \varphi)\,ds$$
$$= F\sin\varphi\,ds = Fr\sin\varphi\,d\theta$$



- Only transverse component of F does work – the same component that contributes to torque

$$dW = \tau d\theta$$

Work-Kinetic Theorem pure rotation

- As object rotates from θ_i to θ_f, work done by the torque

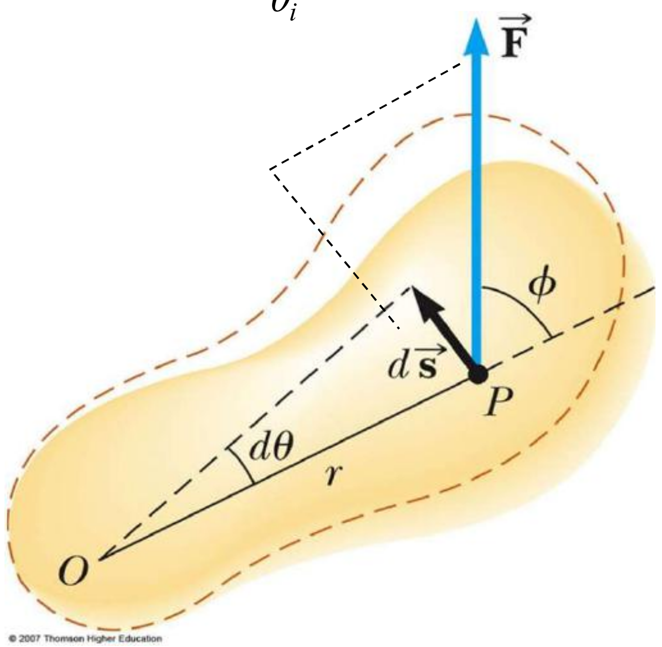
$$W = \int_{\theta_i}^{\theta_f} dW = \int_{\theta_i}^{\theta_f} \tau d\theta = \int_{\theta_i}^{\theta_f} I\alpha d\theta = \int_{\theta_i}^{\theta_f} I \frac{d\omega}{dt} d\theta = \int_{\theta_i}^{\theta_f} I\omega d\omega$$

- I is constant for rigid object

$$W = \int_{\theta_i}^{\theta_f} I\omega d\omega = I\int_{\theta_i}^{\theta_f} \omega d\omega = \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2$$

- Power

$$P = \frac{dW}{dt} = \tau \frac{d\theta}{dt} = \tau\omega$$



- An motor attached to a grindstone exerts a constant torque of 10 N-m. The moment of inertia of the grindstone is I = 2 kg-m^2. The system starts from rest.
  - Find the kinetic energy after 8 s

$$K_f = \frac{1}{2} I \omega_f^2 = 1600 J \Leftarrow \omega_f = \omega_i + \alpha t = 40 \text{ rad/s} \Leftarrow \alpha = \frac{\tau}{I} = 5 \text{ rad/s}^2$$

  - Find the work done by the motor during this time

$$W = \int_{\theta_i}^{\theta_f} \tau d\theta = \tau(\theta_f - \theta_i) = 10 \times 160 = 1600 J$$

$$(\theta_f - \theta_i) = \omega_i t + \frac{1}{2}\alpha t^2 = 160 \text{ rad} \qquad W = K_f - K_i = \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2 = 1600 \text{ J}$$

  - Find the average power delivered by the motor

$$P_{avg} = \frac{dW}{dt} = \frac{1600}{8} = 200 \text{ watts}$$

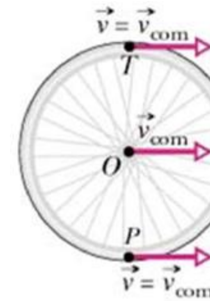
  - Find the instantaneous power at t = 8 s

$$P = \tau\omega = 10 \times 40 = 400 \text{ watts}$$

Work-Energy Theorem

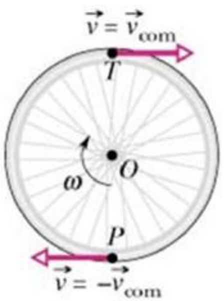
- For pure translation

$$W_{net} = \Delta K_{cm} = K_{cm,f} - K_{cm,i} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$



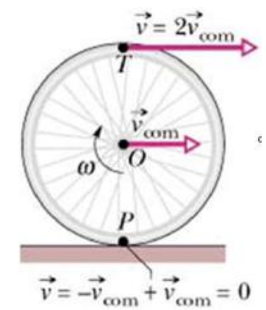
- For pure rotation

$$W_{net} = \Delta K_{rot} = K_{rot,f} - K_{rot,i} = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$



- Rolling: pure rotation + pure translation

$$W_{net} = \Delta K_{total} = (K_{rot,f} + K_{cm,f}) - (K_{rot,i} + K_{cm,i})$$
$$= \left(\frac{1}{2}I\omega_f^2 + \frac{1}{2}mv_f^2\right) - \left(\frac{1}{2}I\omega_i^2 + \frac{1}{2}mv_i^2\right)$$



Energy Conservation

- Energy conservation

$$W_{nc} = \Delta K_{total} + \Delta U$$

- When $W_{nc} = 0$,

$$K_{rot,f} + K_{cm,f} + U_f = K_{rot,i} + K_{cm,i} + U_i$$

- The total mechanical energy is conserved and remains the same at all times

$$\frac{1}{2}I\omega_i^2 + \frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}I\omega_f^2 + \frac{1}{2}mv_f^2 + mgy_f$$

- Remember, this is for conservative forces, no dissipative forces such as friction can be present

Total Energy of a Rolling System

- A ball is rolling down a ramp
- Described by three types of energy
  - Gravitational potential energy

$$U = Mgh$$

  - Translational kinetic energy

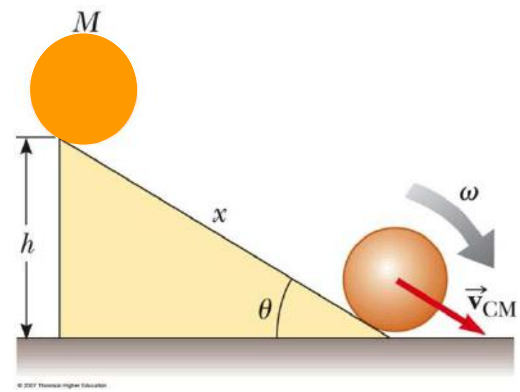
$$K_t = \frac{1}{2}Mv^2$$

  - Rotational kinetic energy

$$K_r = \frac{1}{2}I\omega^2$$

- Total energy of a system

$$E = \frac{1}{2}Mv^2 + Mgh + \frac{1}{2}I\omega^2$$



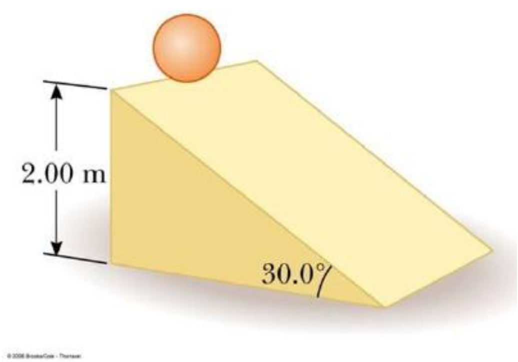
Problem Solving Hints

- Choose two points of interest
  - One where all the necessary information is given
  - The other where information is desired
- Identify the conservative and non-conservative forces
- Write the general equation for the Work-Energy theorem if there are non-conservative forces
  - Use Conservation of Energy if there are no non-conservative forces
- Use $v = r\omega$ to combine terms
- Solve for the unknown

A Ball Rolling Down an Incline

- A ball of mass M and radius R starts from rest at a height of h and rolls down a 30° slope, what is the linear speed of the ball when it leaves the incline? Assume that the ball rolls without slipping.

$$\frac{1}{2}mv_i^2 + mgy_i + \frac{1}{2}I\omega_i^2 = \frac{1}{2}mv_f^2 + mgy_f + \frac{1}{2}I\omega_f^2$$



$$0 + Mgh + 0 = \frac{1}{2}Mv_f^2 + 0 + \frac{1}{2}I\omega_f^2$$

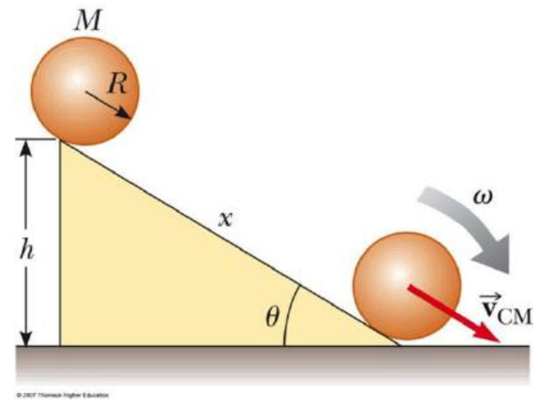
$$I = \frac{2}{5}MR^2 \qquad \omega_f = \frac{v_f}{R}$$

$$Mgh = \frac{1}{2}Mv_f^2 + \frac{1}{2}\frac{2}{5}MR^2\frac{v_f^2}{R^2} = \frac{1}{2}Mv_f^2 + \frac{1}{5}Mv_f^2$$

$$v_f = (\frac{10}{7}gh)^{1/2}$$

Rotational Work and Energy

- A ball rolls without slipping down incline A, starting from rest. At the same time, a box starts from rest and slides down incline B, which is identical to incline A except that it is frictionless. Which arrives at the bottom first?
- Ball rolling:

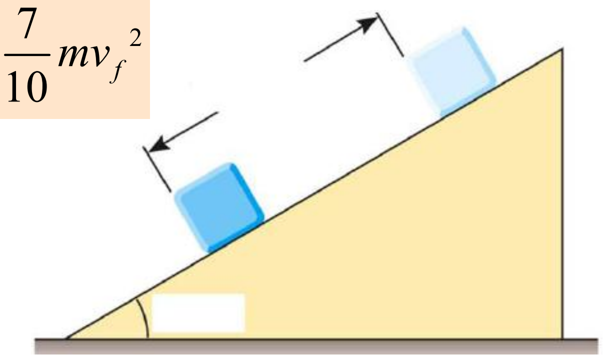


$$\frac{1}{2}mv_i^{\ 2} + mgy_i + \frac{1}{2}I\omega_i^{\ 2} = \frac{1}{2}mv_f^{\ 2} + mgy_f + \frac{1}{2}I\omega_f^{\ 2}$$

$$mgh = \frac{1}{2}mv_f^{\ 2} + \frac{1}{2}I\omega_f^{\ 2}$$

$$= \frac{1}{2}mv_f^{\ 2} + \frac{1}{2}\left(\frac{2}{5}mR^2\right)(v_f/R)^2 = \frac{7}{10}mv_f^{\ 2}$$

- Box sliding



$$\frac{1}{2}mv_i^{\ 2} + mgy_i = \frac{1}{2}mv_f^{\ 2} + mgy_f$$

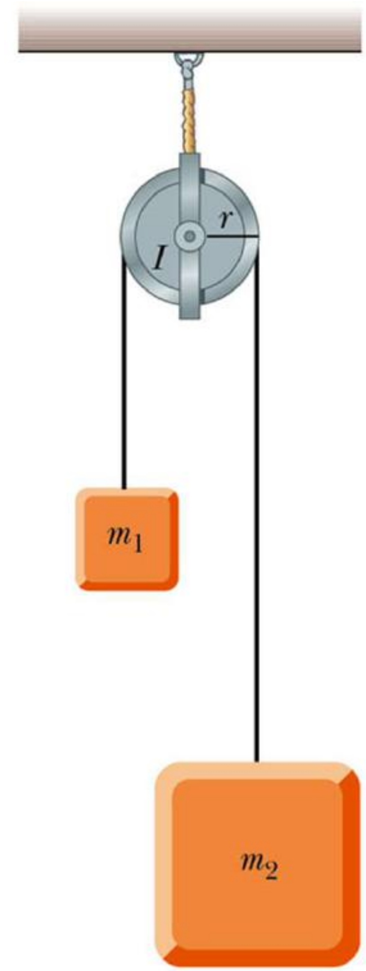
$$\text{sliding: } mgh = \frac{1}{2}mv_f^{\ 2}$$

$$\text{rolling: } mgh = \frac{7}{10}mv_f^{\ 2}$$

Blocks and Pulley

- Two blocks having different masses m_1 and m_2 are connected by a string passing over a pulley. The pulley has a radius R and moment of inertia I about its axis of rotation. The string does not slip on the pulley, and the system is released from rest.

- Find the translational speeds of the blocks after block 2 descends through a distance h.
- Find the angular speed of the pulley at that time.



- Find the translational speeds of the blocks after block 2 descends through a distance h.

$$K_{rot,f} + K_{cm,f} + U_f = K_{rot,i} + K_{cm,i} + U_i$$

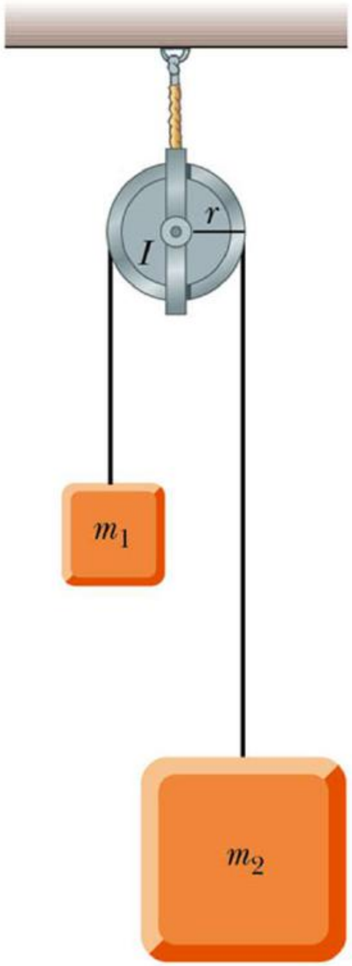
$$(\frac{1}{2}m_1 v_f^{\;2} + \frac{1}{2}m_2 v_f^{\;2} + \frac{1}{2}I\omega_f^{\;2}) + (m_1 gh - m_2 gh) = 0 + 0 + 0$$

$$\frac{1}{2}(m_1 + m_2 + \frac{I}{R^2})v_f^2 = m_2 gh - m_1 gh$$

$$v_f = \left[\frac{2(m_2 - m_1)gh}{m_1 + m_2 + I/R^2}\right]^{1/2}$$

- Find the angular speed of the pulley at that time.

$$\omega_f = \frac{v_f}{R} = \frac{1}{R}\left[\frac{2(m_2 - m_1)gh}{m_1 + m_2 + I/R^2}\right]^{1/2}$$



Basic Physics 1
Lecture Module

Rahadian N, S.Si. M.Si.

Statics Structure

Statics Structure

- Equilibrium

- Static equilibrium conditions
  - Net external force must equal zero
  - Net external torque must equal zero

- Center of mass

- Center of gravity

- Solving static equilibrium problems

Static and Dynamic Equilibrium

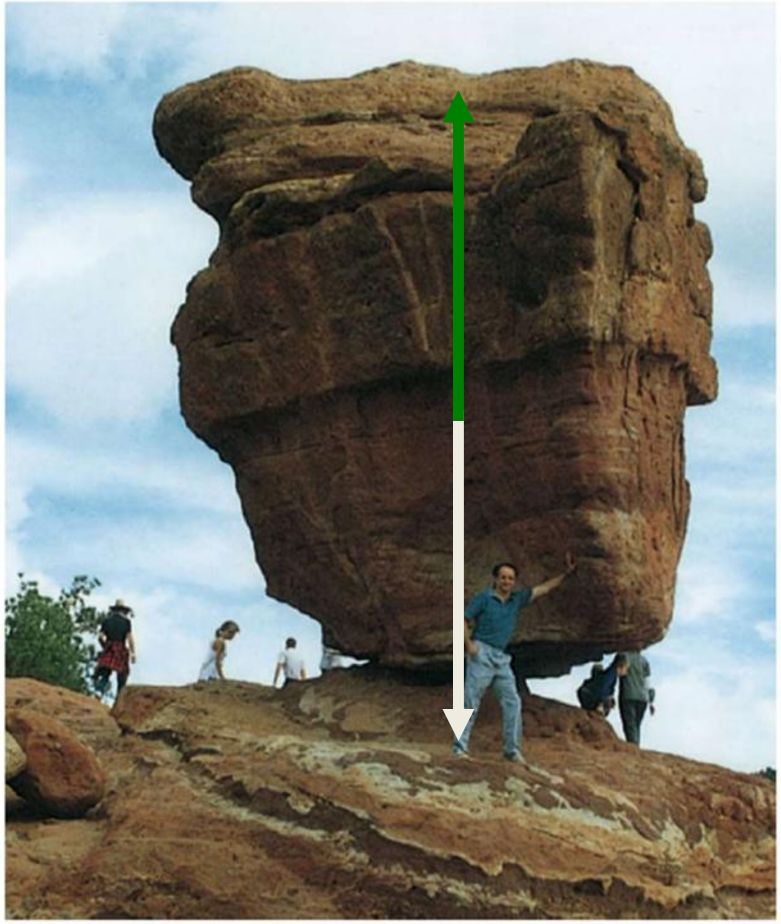
- Equilibrium implies the object is at rest (static) or its center of mass moves with a constant velocity (dynamic)
- We will consider only with the case in which linear and angular velocities are equal to zero, called **"static equilibrium"** : $v_{CM} = 0$ and $\omega = 0$
- Examples
  - *Book on table*
  - *Hanging sign*
  - *Ceiling fan – off*
  - *Ceiling fan – on*
  - *Ladder leaning against wall*

Conditions for Equilibrium

- The first condition of equilibrium is a statement of translational equilibrium
- The net external force on the object must equal zero

$$\vec{F}_{net} = \sum \vec{F}_{ext} = m\vec{a} = 0$$

- It states that the translational acceleration of the object's center of mass must be zero

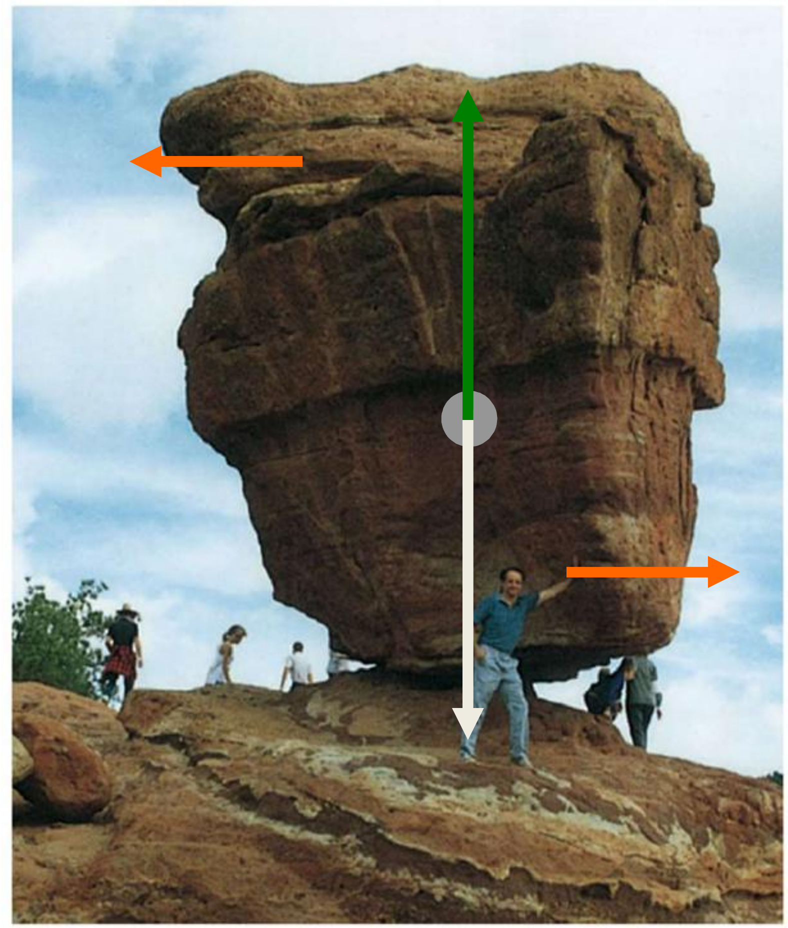


Conditions for Equilibrium

- If the object is modeled as a particle, then this is the only condition that must be satisfied

$$\vec{F}_{net} = \sum \vec{F}_{ext} = 0$$

- For an extended object to be in equilibrium, a second condition must be satisfied
- This second condition involves the rotational motion of the extended object

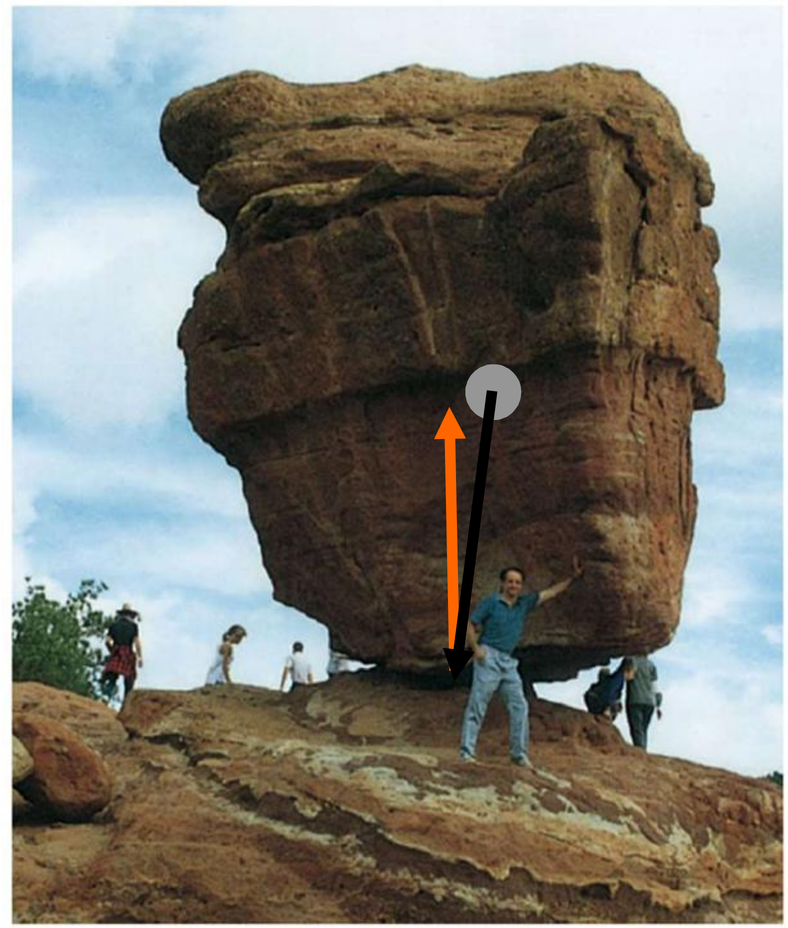


Conditions for Equilibrium

- The second condition of equilibrium is a statement of rotational equilibrium
- The net external torque on the object must equal zero

$$\vec{\tau}_{net} = \sum \vec{\tau}_{ext} = I\vec{\alpha} = 0$$

- It states the angular acceleration of the object to be zero
- This must be true for any axis of rotation



Conditions for Equilibrium

- The net force equals zero $\sum \vec{\mathbf{F}} = 0$
  - If the object is modeled as a particle, then this is the only condition that must be satisfied
- The net torque equals zero $\sum \vec{\tau} = 0$
  - This is needed if the object cannot be modeled as a particle
- These conditions describe the rigid objects in the equilibrium analysis model

Static Equilibrium

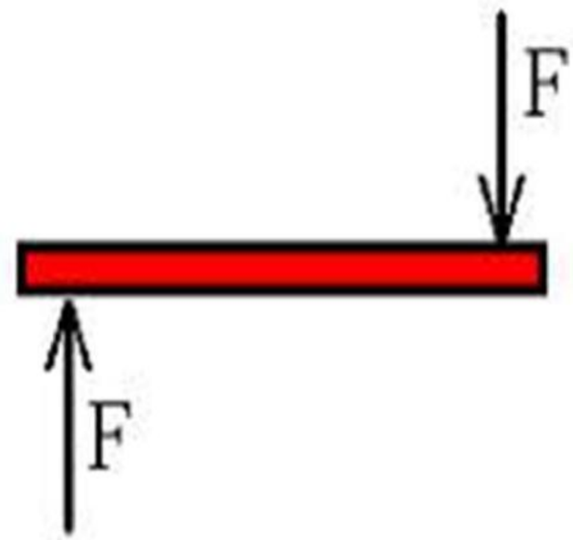
Consider a light rod subject to the two forces of equal magnitude as shown in figure. Choose the correct statement with regard to this situation:

(A) The object is in force equilibrium but not torque equilibrium.

(B) The object is in torque equilibrium but not force equilibrium

(C) The object is in both force equilibrium and torque equilibrium

(D) The object is in neither force equilibrium nor torque equilibrium



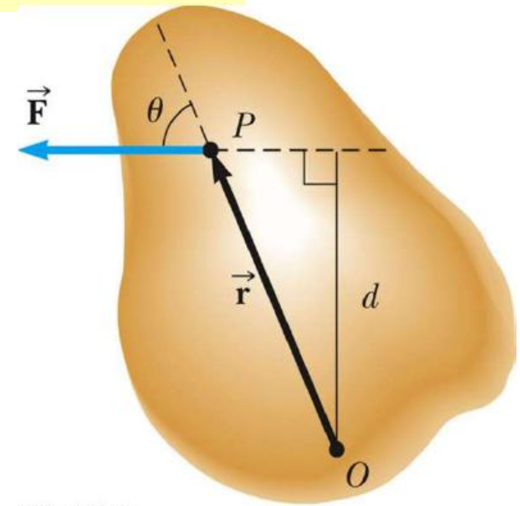
Equilibrium Equations

- For simplicity, We will restrict the applications to situations in which all the forces lie in the xy plane.
- Equation 1: $\vec{F}_{net} = \sum \vec{F}_{ext} = 0: \; F_{net,x} = 0 \; F_{net,y} = 0 \; \cancel{F_{net,z} = 0}$
- Equation 2: $\vec{\tau}_{net} = \sum \vec{\tau}_{ext} = 0: \; \cancel{\tau_{net,x} = 0} \; \cancel{\tau_{net,y} = 0} \; \tau_{net,z} = 0$
- There are three resulting equations

$$F_{net,x} = \sum F_{ext,x} = 0$$

$$F_{net,y} = \sum F_{ext,y} = 0$$

$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$



Alternative Equations of Equilibrium

If all the forces acting on the rigid body are planar and all the couples are perpendicular to the plane of the body, equations of equilibrium become two dimensional. In two dimensional problems, in alternative to the above set of equations, two more sets of equations can be employed in the solution of problems.

$$\Sigma F_x = 0 \qquad \Sigma M_A = 0 \qquad \Sigma M_B = 0$$

$$\Sigma M_A = 0 \qquad \Sigma M_B = 0 \qquad \Sigma M_C = 0$$

Points A, B and C in the latter set cannot lie along the same line, if they do, trivial equations will be obtained.

Free Body Diagram

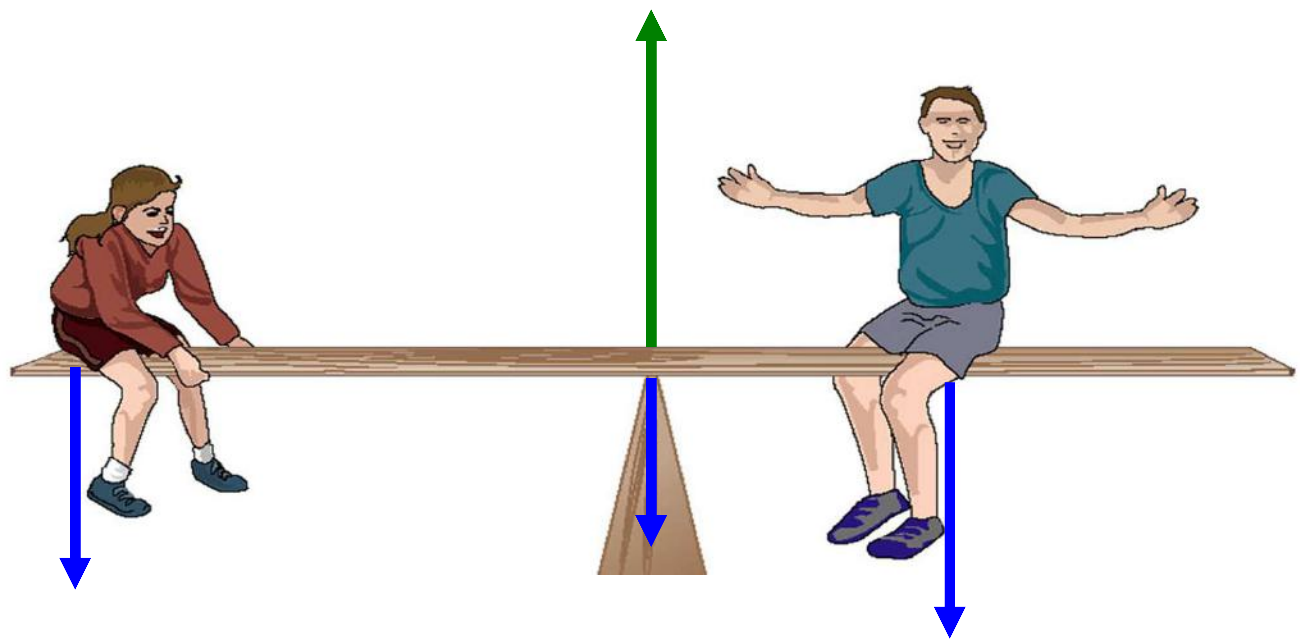
The first step in the analysis of the equilibrium of rigid bodies must be to draw the "***free body diagram***" of the body in question.

1) If there exists, identify the two force members in the problem.
2) Decide which system to isolate.
3) Isolate the chosen system by drawing a diagram which represents its ***complete external boundary***.
4) If not given with the problem, select a coordinate system which appropriately suits with the given forces and/or dimensions.
5) Identify ***all*** forces which act ***on*** the isolated system applied by removing the contacting or attracting bodies, and represent them in their proper positions on the diagram.
6) Write the equations of equilibrium and solve for the unknowns.

Case Study

A seesaw consisting of a uniform board of mass m_{pl} and length L supports at rest a father and daughter with masses M and m, respectively. The support is under the center of gravity of the board, the father is a distance d from the center, and the daughter is a distance 2.00 m from the center.

- A) Find the magnitude of the upward force **n** exerted by the support on the board.
- B) Find where the father should sit to balance the system at rest.



A) Find the magnitude of the upward force **n** exerted by the support on the board.

B) Find where the father should sit to balance the system at rest.

$$F_{net,y} = n - mg - Mg - m_{pl}g = 0$$

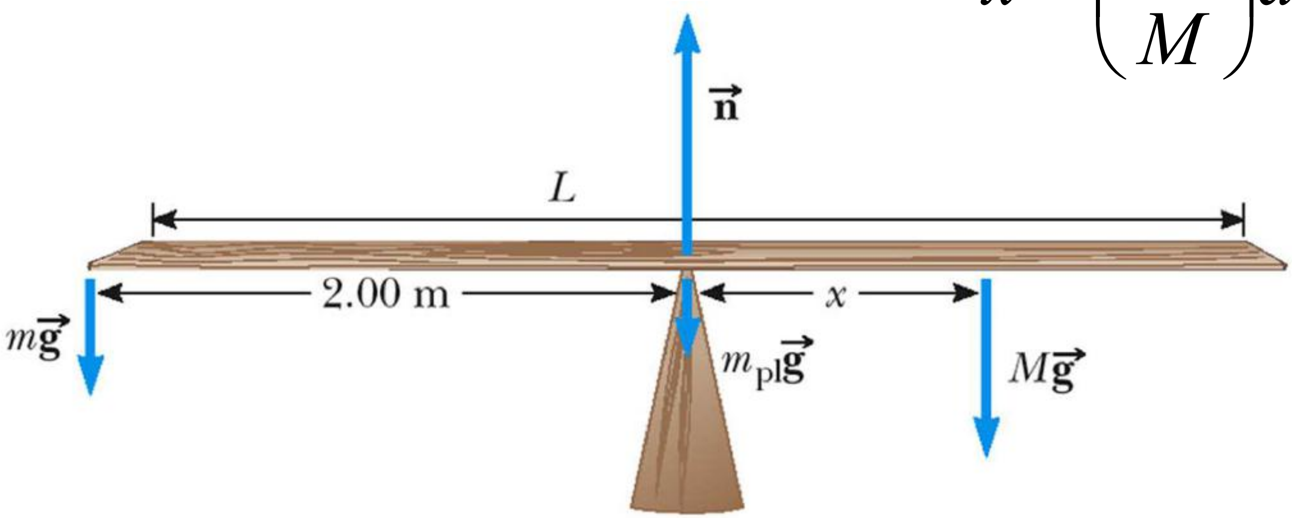
$$n = mg + Mg + m_{pl}g$$

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$

$$= mgd - Mgx + 0 + 0 = 0$$

$$mgd = Mgx$$

$$x = \left(\frac{m}{M}\right)d = \frac{2m}{M} < 2.00\ \text{m}$$



$$F_{net,x} = \sum F_{ext,x} = 0$$

$$F_{net,y} = \sum F_{ext,y} = 0$$

$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$

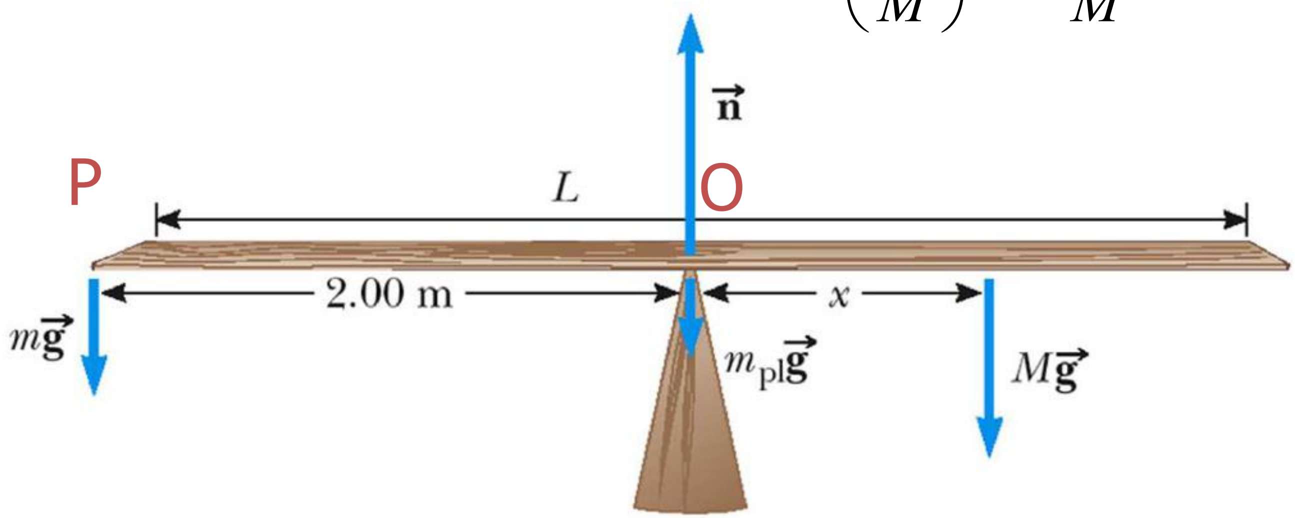
B) Find where the father should sit to balance the system at rest.

Rotation axis O

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$
$$= mgd - Mgx + 0 + 0 = 0$$
$$mgd = Mgx$$
$$x = \left(\frac{m}{M}\right)d = \frac{2m}{M}$$

Rotation axis P

$$\tau_{net,z} = \tau_d + \tau_f + \tau_{pl} + \tau_n$$
$$= 0 - Mg(d + x) - m_{pl}gd + nd = 0$$
$$-Mgd - Mgx - m_{pl}gd + (Mg + mg + m_{pl}g)d = 0$$
$$mgd = Mgx$$
$$x = \left(\frac{m}{M}\right)d = \frac{2m}{M}$$



$$F_{net,x} = \sum F_{ext,x} = 0$$
$$F_{net,y} = \sum F_{ext,y} = 0$$
$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$

Axis of Rotation

- The net torque is about an axis through any point in the xy plane
- Does it matter which axis you choose for calculating torques?
- NO. The choice of an axis is arbitrary
- If an object is in translational equilibrium and the net torque is zero about one axis, then the net torque must be zero about any other axis
- We should be smart to choose a rotation axis to simplify problems

Where is the Center of Mass ?

- Assume m_1 = 1 kg, m_2 = 3 kg, and x_1 = 1 m, x_2 = 5 m, where is the center of mass of these two objects?

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

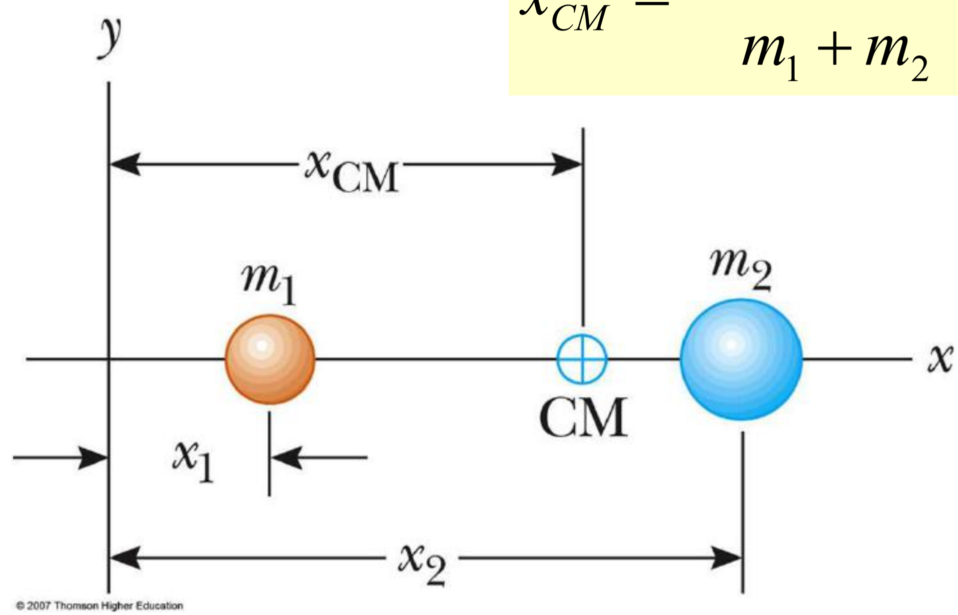
A) x_{CM} = 1 m

B) x_{CM} = 2 m

C) x_{CM} = 3 m

D) x_{CM} = 4 m

E) x_{CM} = 5 m

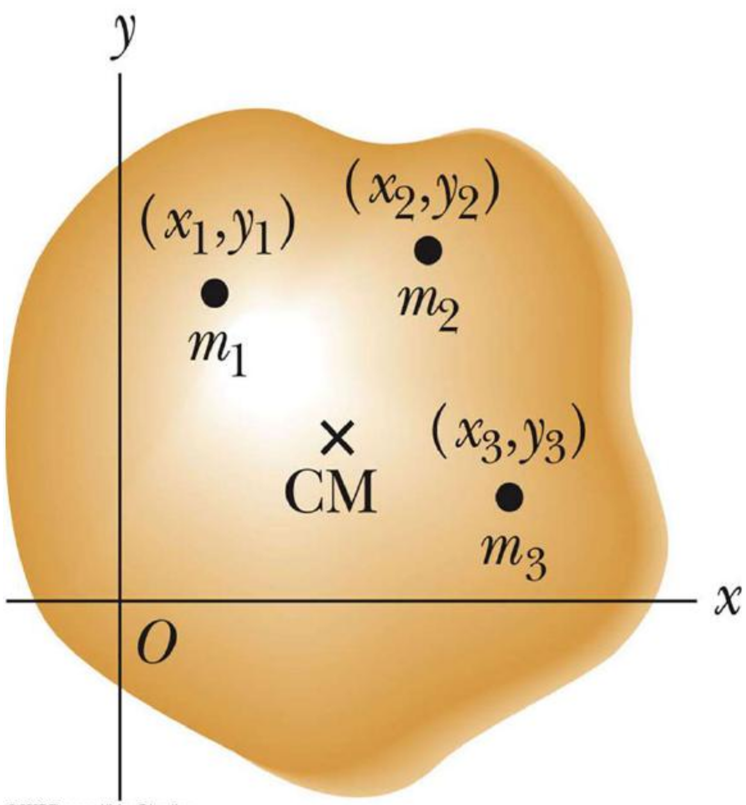


Center of Mass (CM)

- An object can be divided into many small particles
  - Each particle will have a specific mass and specific coordinates
- The x coordinate of the center of mass will be

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

- Similar expressions can be found for the y coordinates



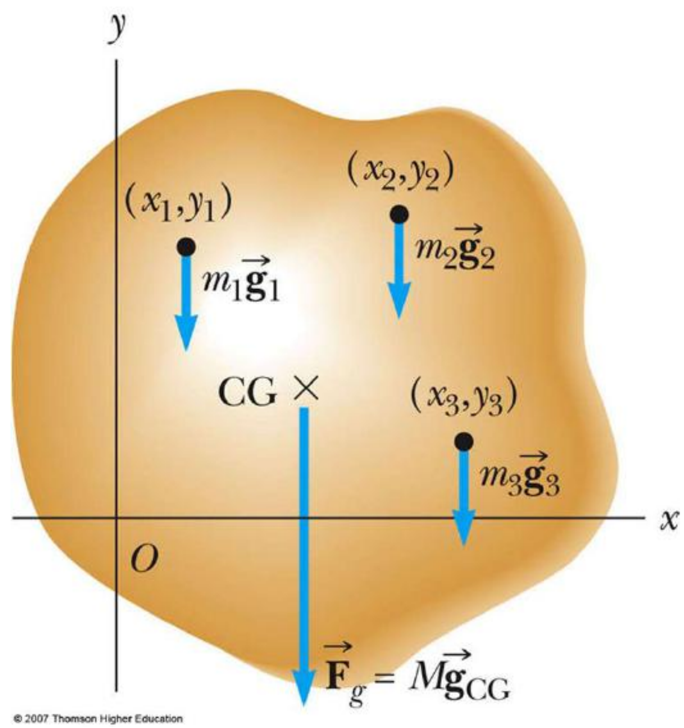
Center of Gravity

- The torque due to the gravitational force on an object of mass M is the force Mg acting at the center of gravity of the object
- If g is uniform over the object, then the center of gravity of the object coincides with its center of mass
- If the object is homogeneous and symmetrical, the center of gravity coincides with its geometric center

Center of Gravity (CG)

- All the various gravitational forces acting on all the various mass elements are equivalent to a single gravitational force acting through a single point called the center of gravity (CG)

$$Mg_{CG}x_{CG} = (m_1 + m_2 + m_3 + \cdots)g_{CG}x_{CG}$$
$$= m_1 g_1 x_1 + m_2 g_2 x_2 + m_3 g_3 x_3 + \cdots$$



- If

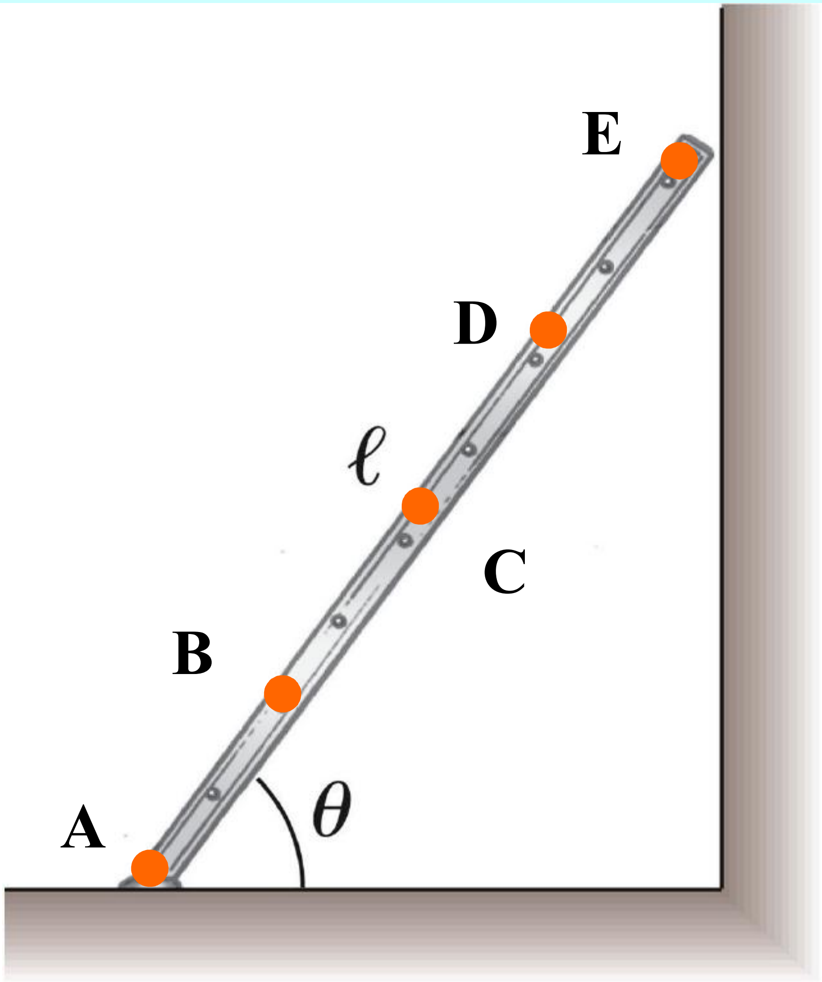
$$g_1 = g_2 = g_3 = \cdots$$

- then

$$x_{CG} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum m_i x_i}{\sum m_i}$$

CG of a Ladder

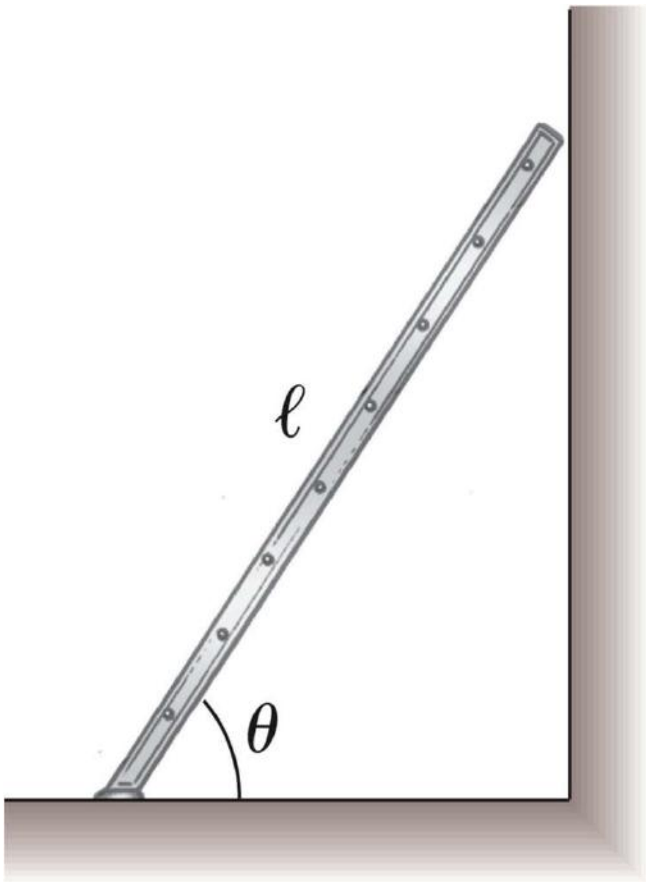
- A uniform ladder of length l rests against a smooth, vertical wall. When you calculate the torque due to the gravitational force, you have to find center of gravity of the ladder. The center of gravity should be located at



(a)

Ladder Example

- A uniform ladder of length l rests against a smooth, vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. Find the minimum angle θ at which the ladder does not slip.



(a)

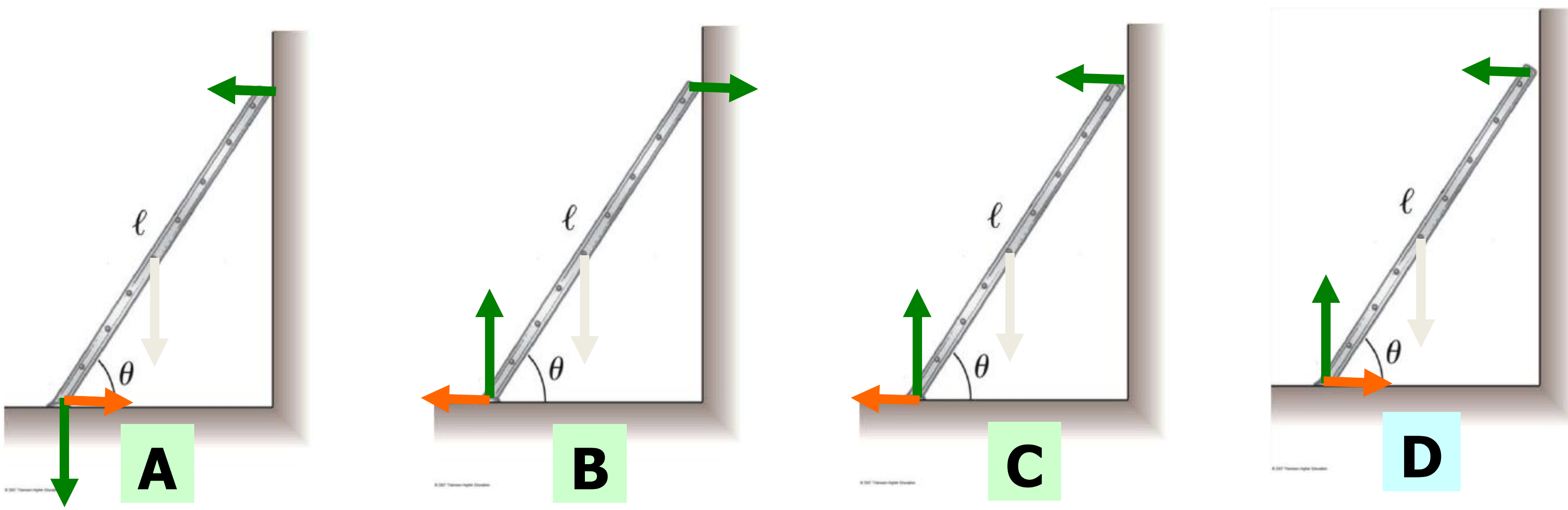
Problem-Solving Strategy 1

- Draw sketch, decide what is in or out the system
- Draw a free body diagram (FBD)
- Show and label all external forces acting on the object
- Indicate the locations of all the forces
- Establish a convenient coordinate system
- Find the components of the forces along the two axes
- Apply the first condition for equilibrium
- Be careful of signs

$$F_{net,x} = \sum F_{ext,x} = 0$$
$$F_{net,y} = \sum F_{ext,y} = 0$$

Which free-body diagram is correct?

- A uniform ladder of length l rests against a smooth, vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. gravity: blue, friction: orange, normal: green



- A uniform ladder of length l rests against a smooth, vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is μ_s = 0.40. Find the minimum angle θ at which the ladder does not slip.

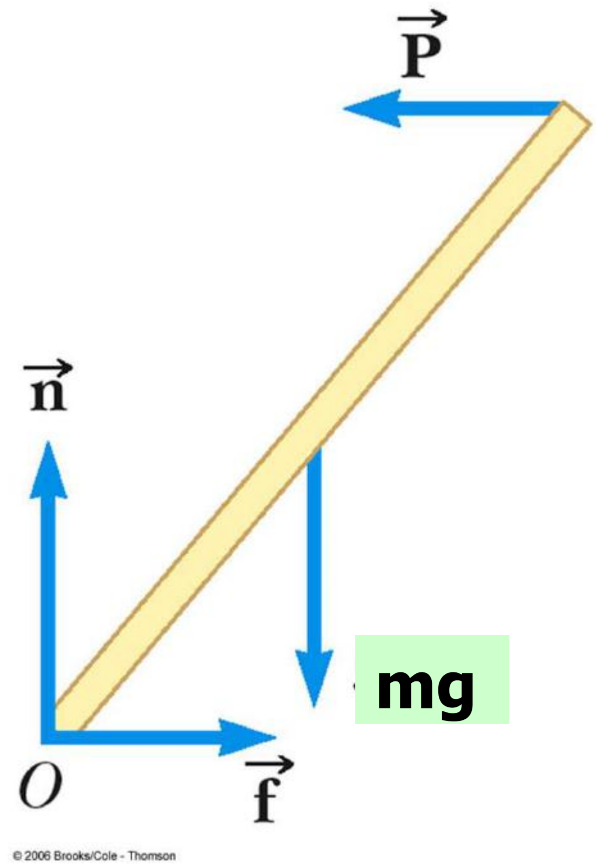
$$\sum F_x = f_x - P = 0$$

$$\sum F_y = n - mg = 0$$

$$P = f_x$$

$$n = mg$$

$$P = f_{x,\max} = \mu_s n = \mu_s mg$$



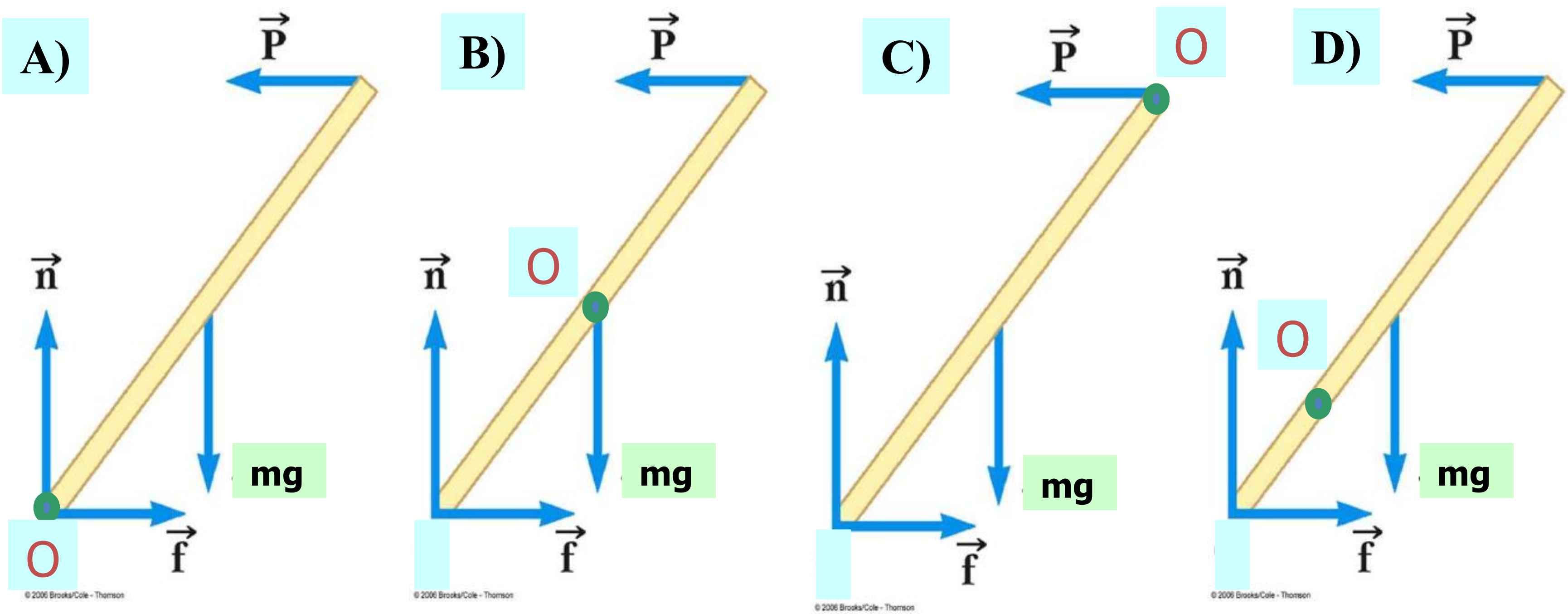
Problem-Solving Strategy 2

- Choose a convenient axis for calculating the net torque on the object
  - Remember the choice of the axis is arbitrary
- Choose an origin that simplifies the calculations as much as possible
  - A force that acts along a line passing through the origin produces a zero torque
- Be careful of sign with respect to rotational axis
  - positive if force tends to rotate object in CCW
  - negative if force tends to rotate object in CW
  - zero if force is on the rotational axis
- Apply the second condition for equilibrium

$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$

Choose an origin O that simplifies the calculations as much as possible ?

- A uniform ladder of length l rests against a smooth, vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is μ_s = 0.40. Find the minimum angle.



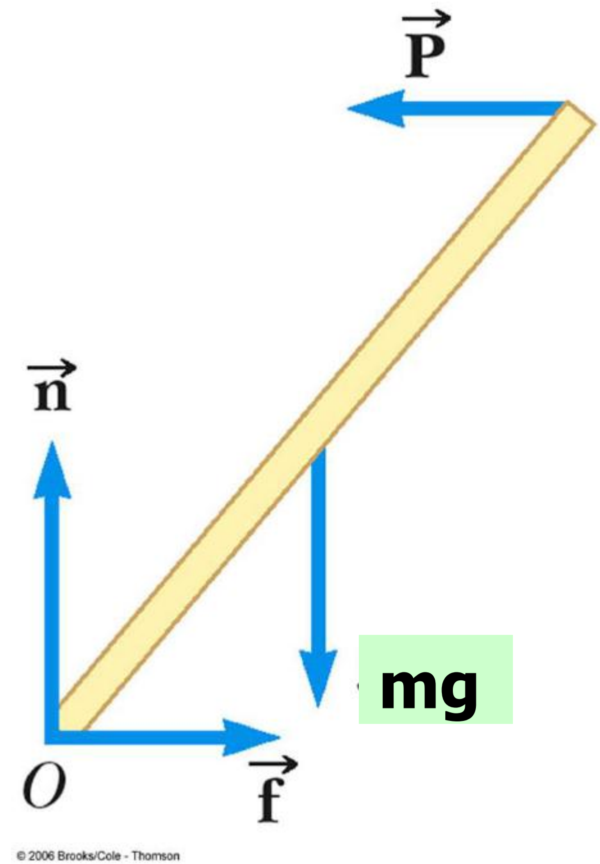
- A uniform ladder of length l rests against a smooth, vertical wall. The mass of the ladder is m, and the coefficient of static friction between the ladder and the ground is $\mu_s = 0.40$. Find the minimum angle θ at which the ladder does not slip.

$$\sum \tau_O = \tau_n + \tau_f + \tau_g + \tau_P$$

$$= 0 + 0 + Pl \sin\theta_{\min} - mg\frac{l}{2}\cos\theta_{\min} = 0$$

$$\frac{\sin\theta_{\min}}{\cos\theta_{\min}} = \tan\theta_{\min} = \frac{mg}{2P} = \frac{mg}{2\mu_s mg} = \frac{1}{2\mu_s}$$

$$\theta_{\min} = \tan^{-1}\left(\frac{1}{2\mu_s}\right) = \tan^{-1}\left[\frac{1}{2(0.4)}\right] = 51^\circ$$



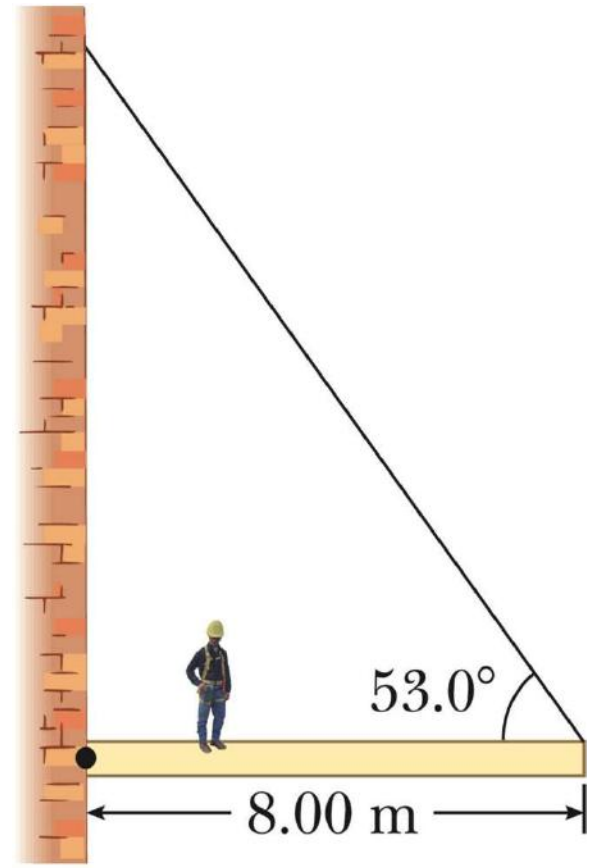
Problem-Solving Strategy 3

- The two conditions of equilibrium will give a system of equations
- Solve the equations simultaneously
- Make sure your results are consistent with your free body diagram
- If the solution gives a negative for a force, it is in the opposite direction to what you drew in the free body diagram
- Check your results to confirm

$$F_{net,x} = \sum F_{ext,x} = 0$$
$$F_{net,y} = \sum F_{ext,y} = 0$$
$$\tau_{net,z} = \sum \tau_{ext,z} = 0$$

Horizontal Beam Example

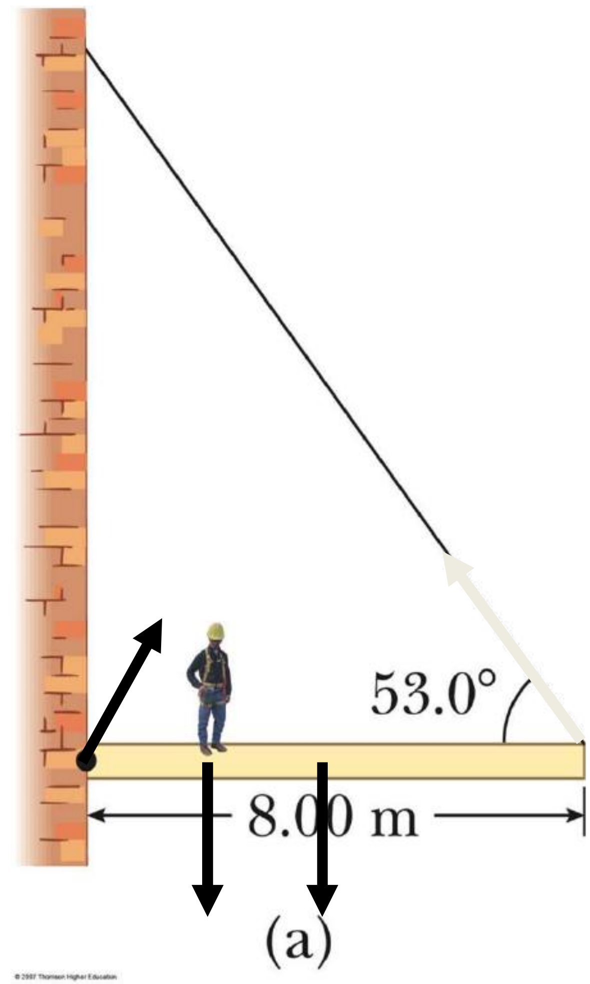
- A uniform horizontal beam with a length of l = 8.00 m and a weight of W_b = 200 N is attached to a wall by a pin connection. Its far end is supported by a cable that makes an angle of ϕ = 53° with the beam. A person of weight W_p = 600 N stands a distance d = 2.00 m from the wall. Find the tension in the cable as well as the magnitude and direction of the force exerted by the wall on the beam.



(a)

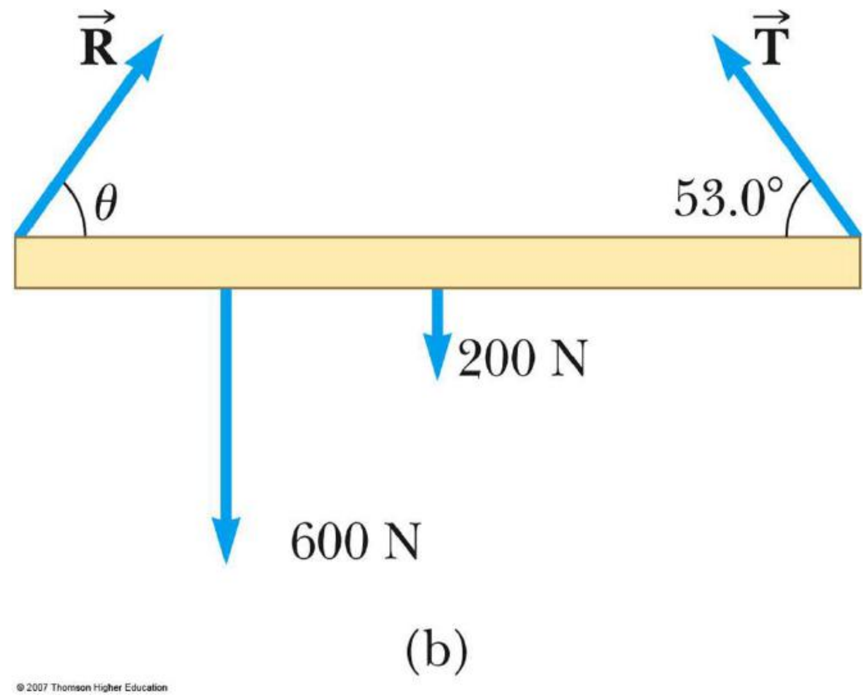
Horizontal Beam Example

- The beam is uniform
  - So the center of gravity is at the geometric center of the beam
- The person is standing on the beam
- What are the tension in the cable and the force exerted by the wall on the beam?



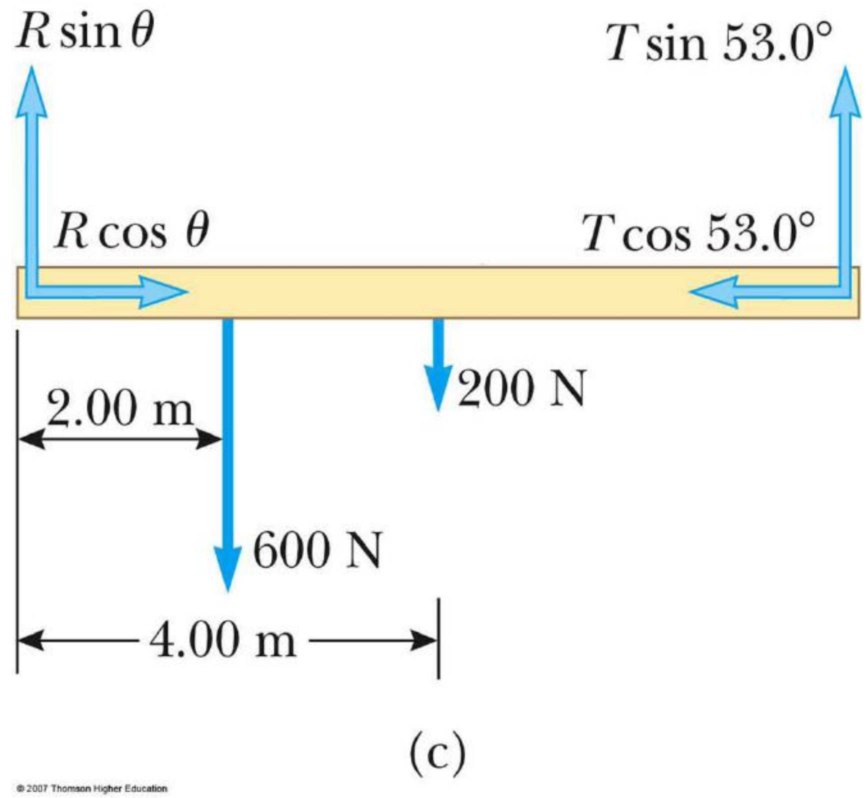
Horizontal Beam Example, 2

- Analyze
  - Draw a free body diagram
  - Use the pivot in the problem (at the wall) as the pivot
    - This will generally be easiest
  - Note there are three unknowns (T, R, θ)



Horizontal Beam Example, 3

- The forces can be resolved into components in the free body diagram
- Apply the two conditions of equilibrium to obtain three equations
- Solve for the unknowns



Horizontal Beam Example, 3

$$\sum \tau_z = (T\sin\phi)(l) - W_p d - W_b(\frac{l}{2}) = 0$$

$$T = \frac{W_p d + W_b(\frac{l}{2})}{l\sin\phi} = \frac{(600N)(2m)+(200N)(4m)}{(8m)\sin 53^\circ} = 313N$$

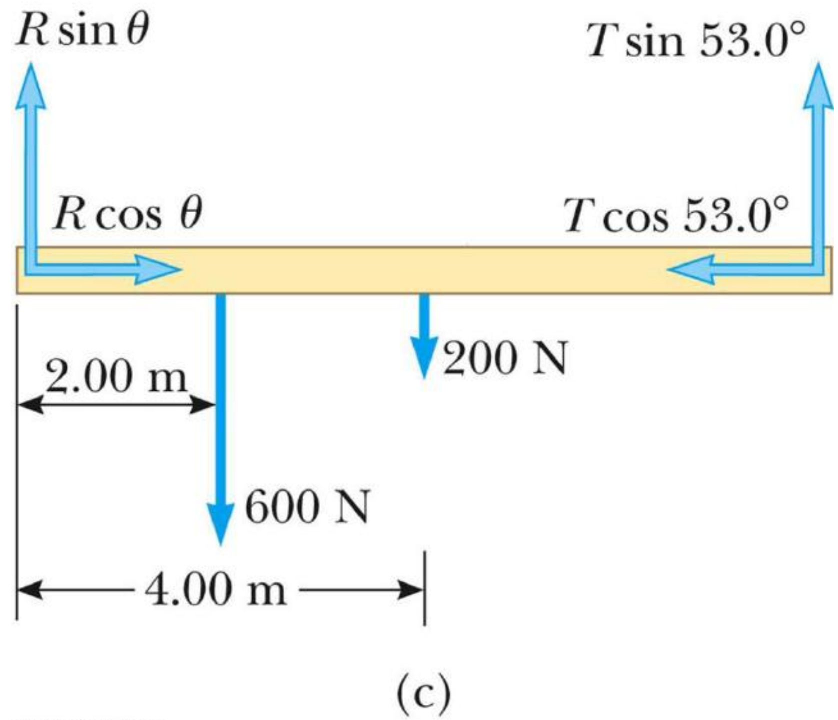
$$\sum F_x = R\cos\theta - T\cos\phi = 0$$

$$\sum F_y = R\sin\theta + T\sin\phi - W_p - W_b = 0$$

$$\frac{R\sin\theta}{R\cos\theta} = \tan\theta = \frac{W_p + W_b - T\sin\phi}{T\sin\phi}$$

$$\theta = \tan^{-1}\left(\frac{W_p + W_b - T\sin\phi}{T\sin\phi}\right) = 71.7^\circ$$

$$R = \frac{T\cos\phi}{\cos\theta} = \frac{(313N)\cos 53^\circ}{\cos 71.7^\circ} = 581N$$



Basic Physics 1
Lecture Module

Rahadian N, S.Si. M.Si.

Fluid Mechanics

Fluid Mechanics

- Density
- Pressure
- Pascal
- Archimedes and buoyancy
- Surface tension
- Fluid flow
- Continuity
- Bernoulli
- Viscosity and Viscous Drag
- Diffusion and osmosis

Preface

Why must the shark keep moving to stay afloat while the small fish can remain at the same level with little effort?

We begin with fluids at rest and then move on to the more complex field of fluid dynamics. Its called fluid mechanics, then go to fluids engineering.

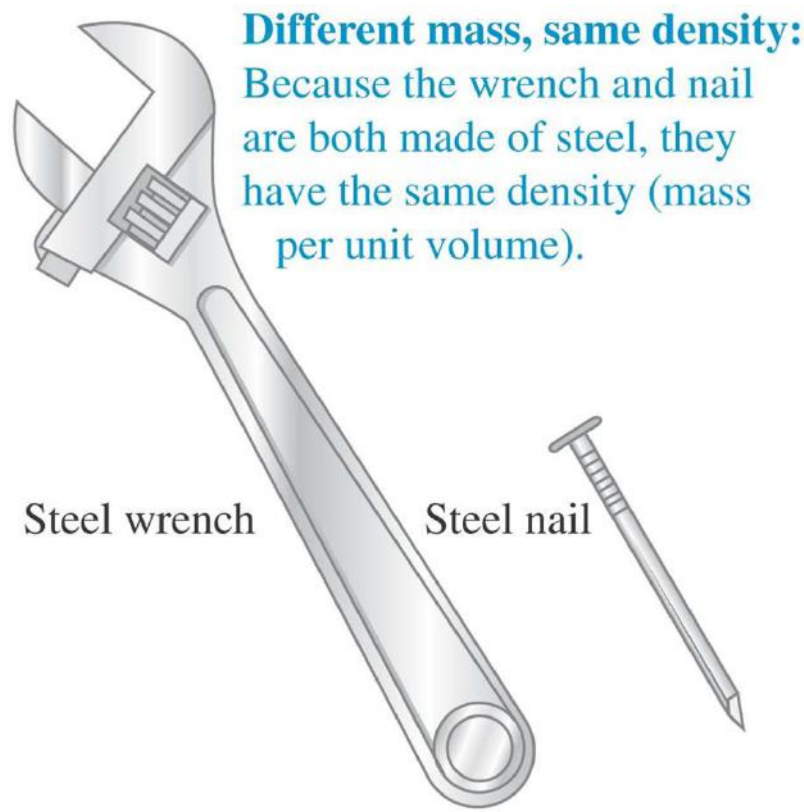
Density

- The *density* of a material is its mass per unit volume:

$$\rho = m/V$$

- The *specific gravity* of a material is its density compared to that of water at 4°C.

- How much does the air in a room weigh? Using data table on the next slide.



Densities of Some Common Substances

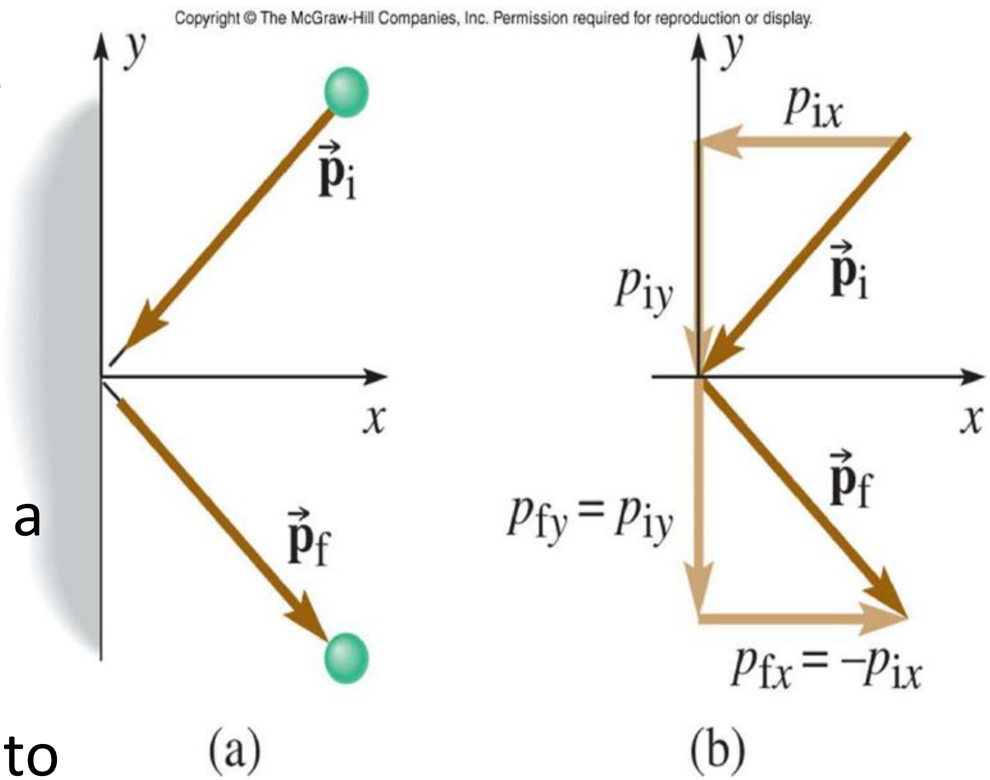
Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Air (1 atm, 20°C)	1.20	Iron, steel	7.8×10^3
Ethanol	0.81×10^3	Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ice	0.92×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Mercury	13.6×10^3
Blood	1.06×10^3	Gold	19.3×10^3
Glycerine	1.26×10^3	Platinum	21.4×10^3
Concrete	2×10^3	White dwarf star	10^{10}
Aluminum	2.7×10^3	Neutron star	10^{18}

*To obtain the densities in grams per cubic centimeter, simply divide by 10^3.

Pressure

Pressure arises from the collisions between the particles of a fluid with another object (container walls for example).

There is a momentum change (impulse) that is away from the container walls. There must be a force exerted on the particle by the wall. By Newton's 3rd Law, there is a force on the wall due to the particle.

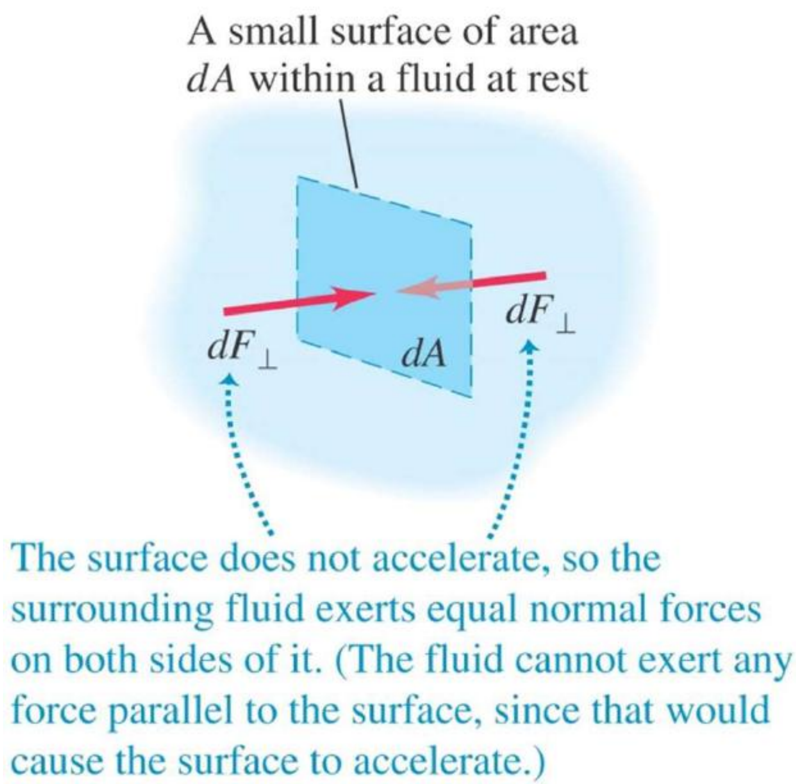


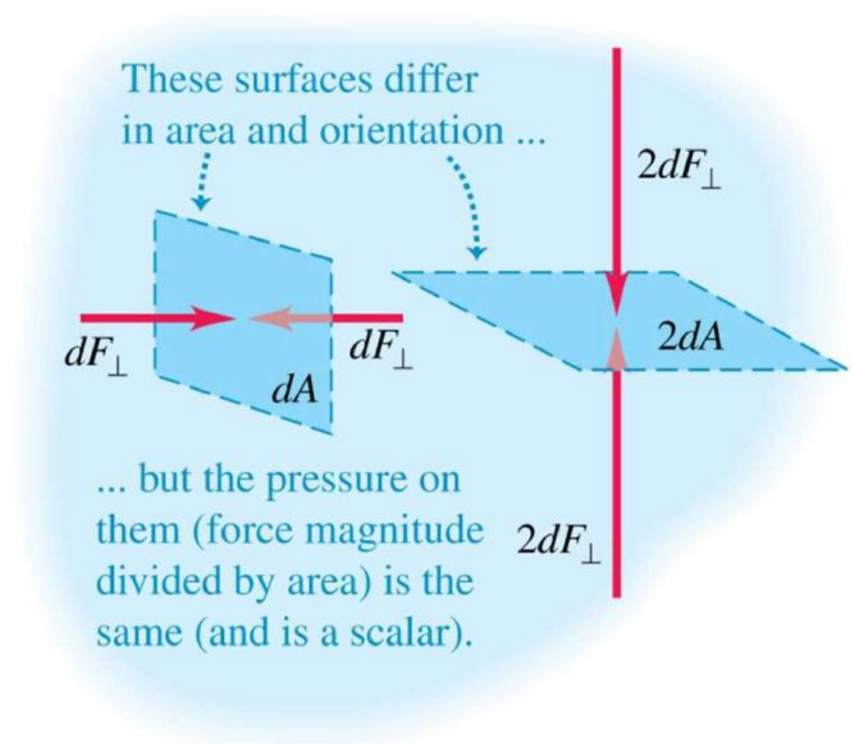
Pressure is defined as $P = \dfrac{F}{A}$.

The units of pressure are N/m^2 and are called Pascals (Pa).

Pressure in A Fluid

- The pressure in a fluid is the normal force per unit area: $p = dF_{\perp}/dA$. Refer to Figures below

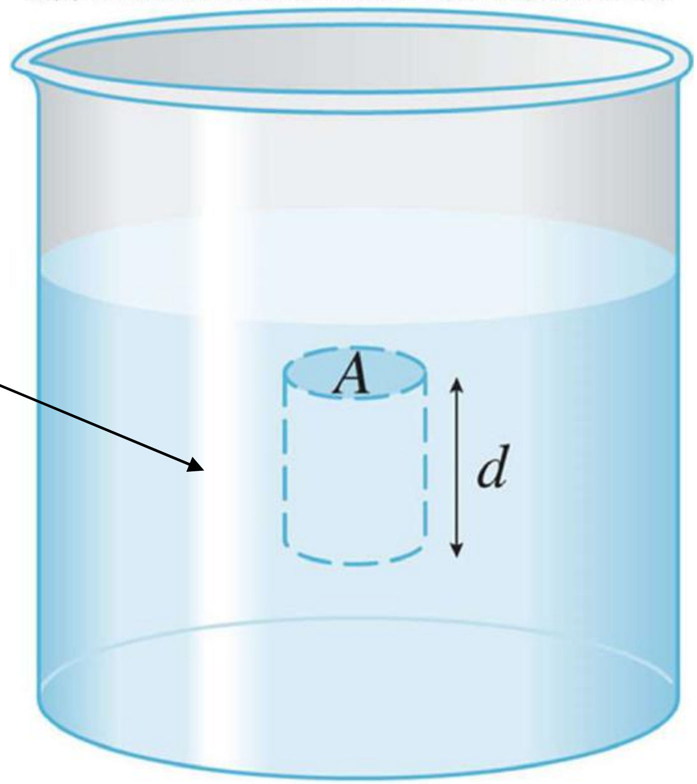




Gravity's Effect on Fluid Pressure

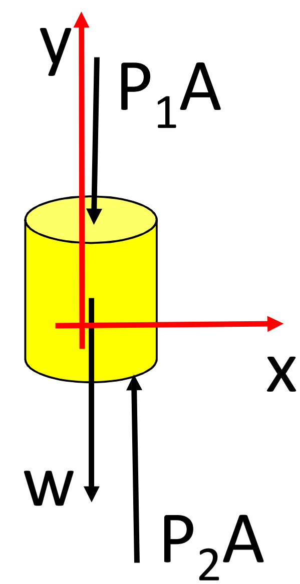
An imaginary cylinder of fluid

Imaginary cylinder can be any size



(a)

FBD for the fluid cylinder



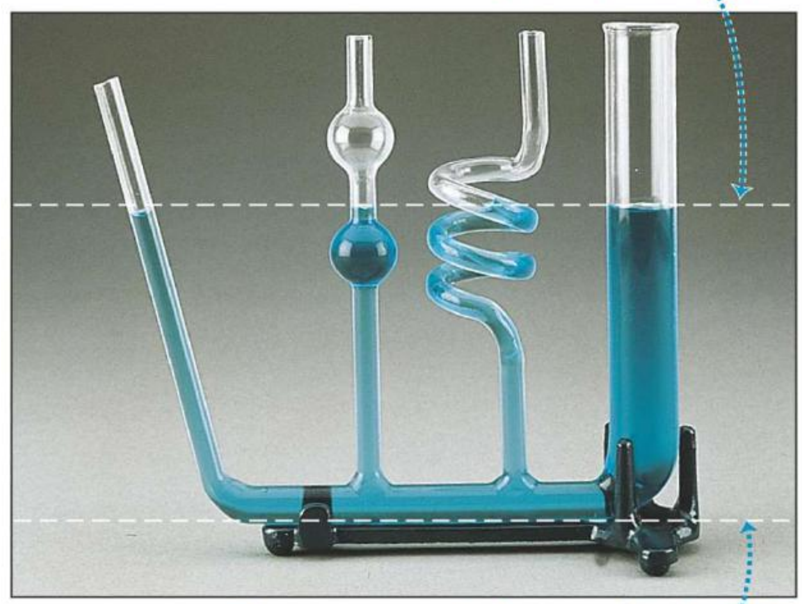
Pressure at Depth in a Fluid

- The pressure at a depth h in a fluid of uniform density is given by $P = P_0 + \rho g h$. As Figure at the right illustrates, the shape of the container does not matter.

- The *gauge pressure* is the pressure above atmospheric pressure. The *absolute pressure* is the total pressure.

- Follow the figure, which involves both gauge and absolute pressure.

The pressure at the top of each liquid column is atmospheric pressure, p_0.



The pressure at the bottom of each liquid column has the same value p.

The difference between p and p_0 is ρgh, where h is the distance from the top to the bottom of the liquid column. Hence all columns have the same height.

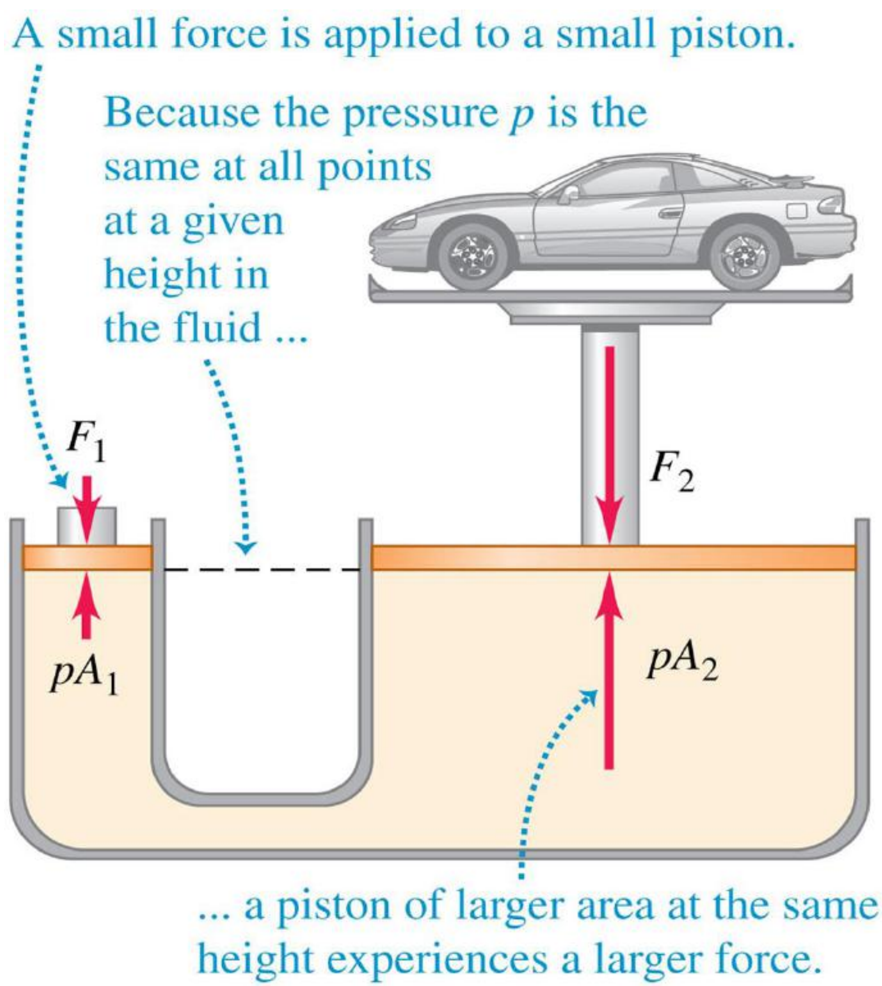
Pascal's Law

- *Pascal's law*: Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel.
- The hydraulic life shown in Figure is an application of Pascal's law.

$$\Delta P \text{ at point } 1 = \Delta P \text{ at point } 2$$

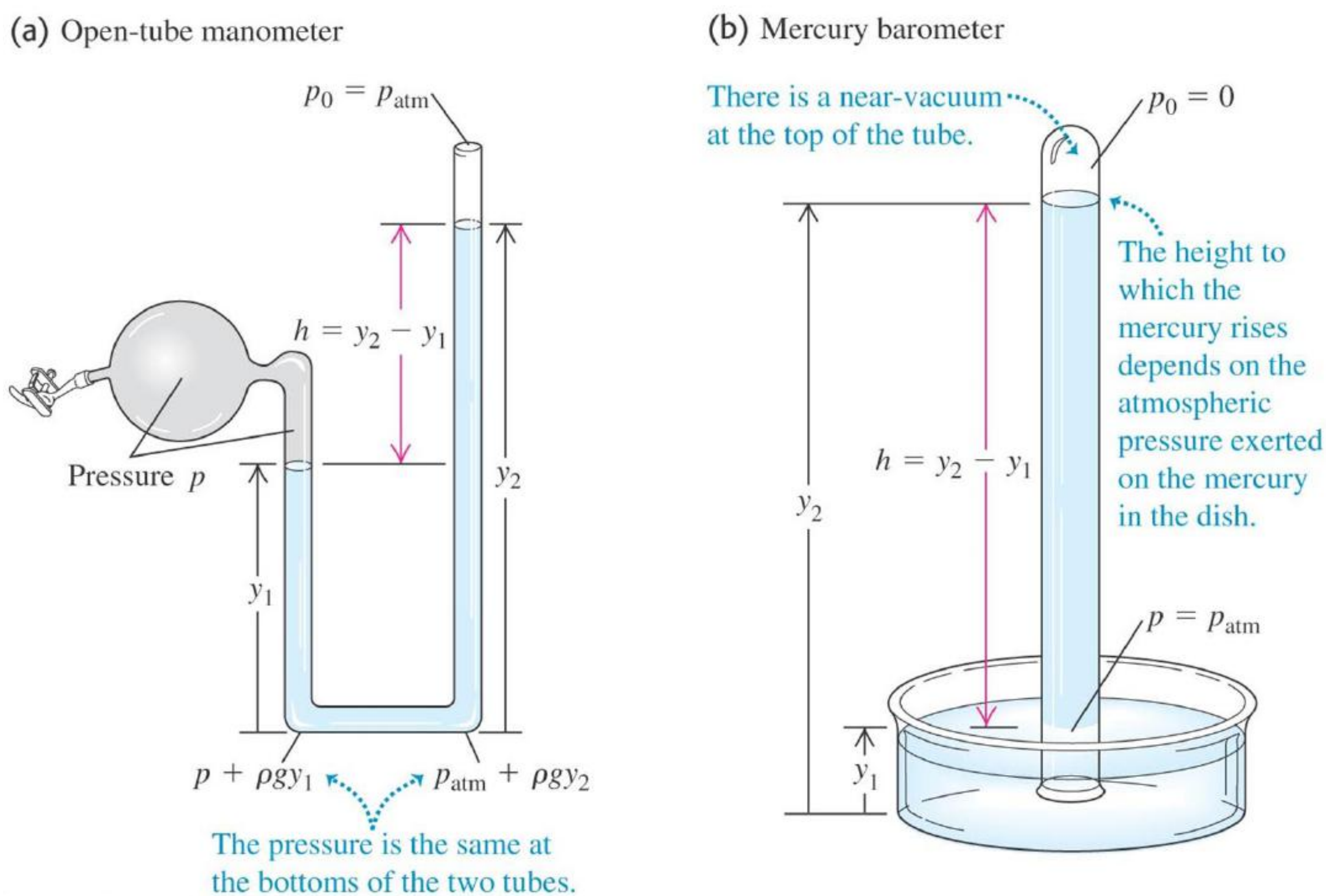
$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$F_2 = \left(\frac{A_2}{A_1}\right) F_1$$

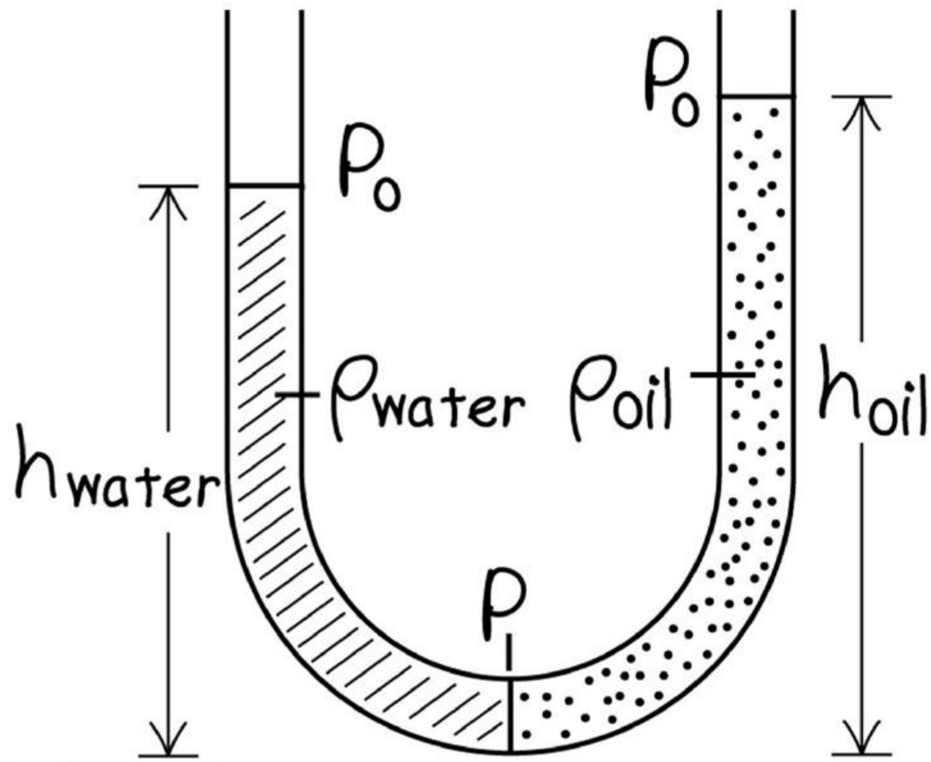


Two Types of Pressure Gauge

- Figure below shows two types of gauges for measuring pressure.

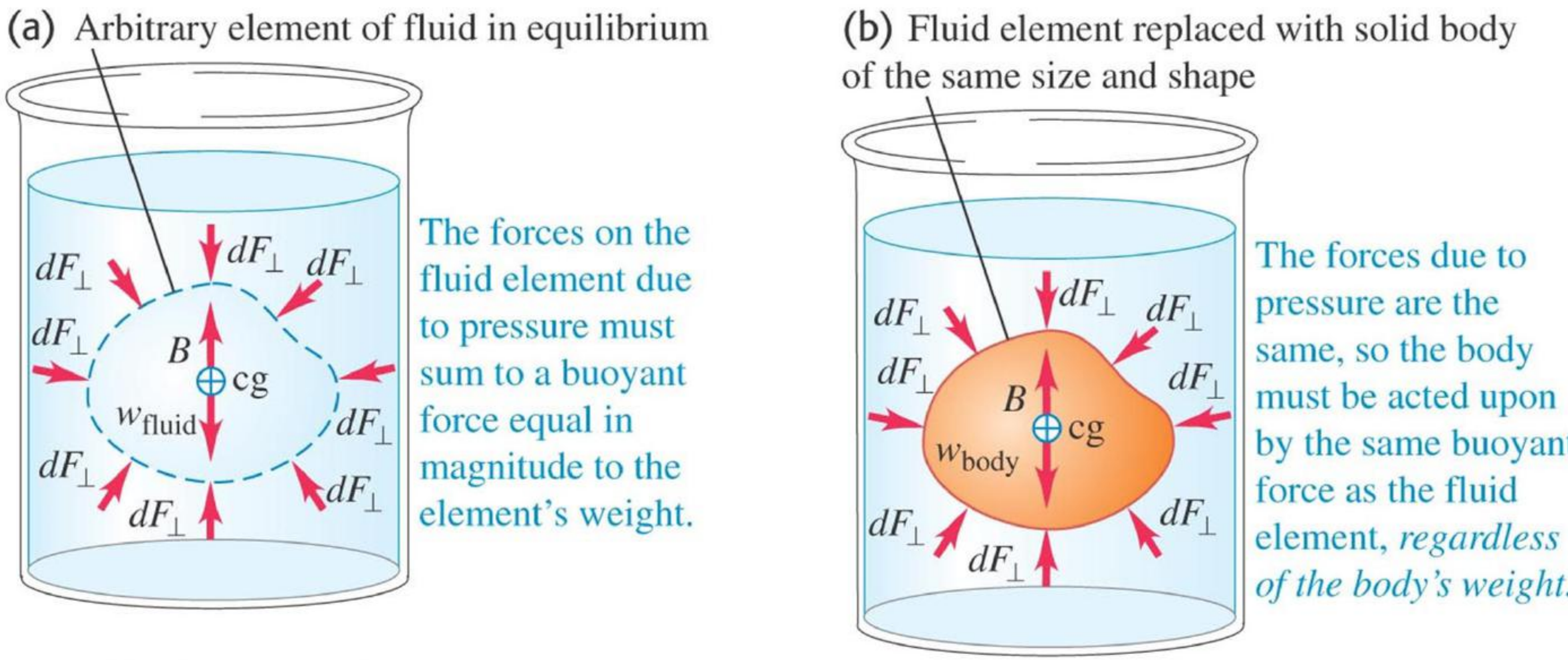


A tale of two fluids



Archimedes

- When a body is completely or partially immersed in a fluid, the fluid exerts an upward force (the "buoyant force") on the body equal to the weight of the fluid displaced by the body. See Figure below.



Buoyancy

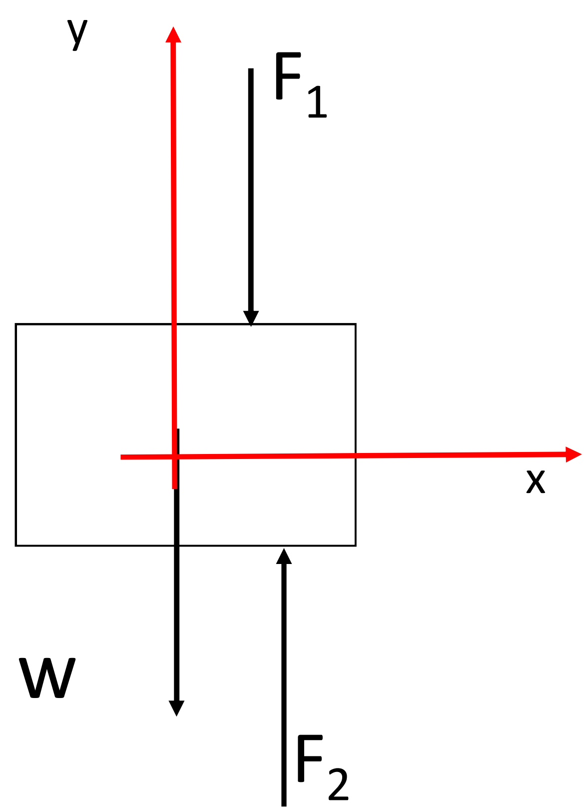
Buoyant force = the weight of the fluid displaced

The magnitude of the buoyant force is:

$$F_B = F_2 - F_1$$
$$= P_2 A - P_1 A$$
$$= (P_2 - P_1) A$$

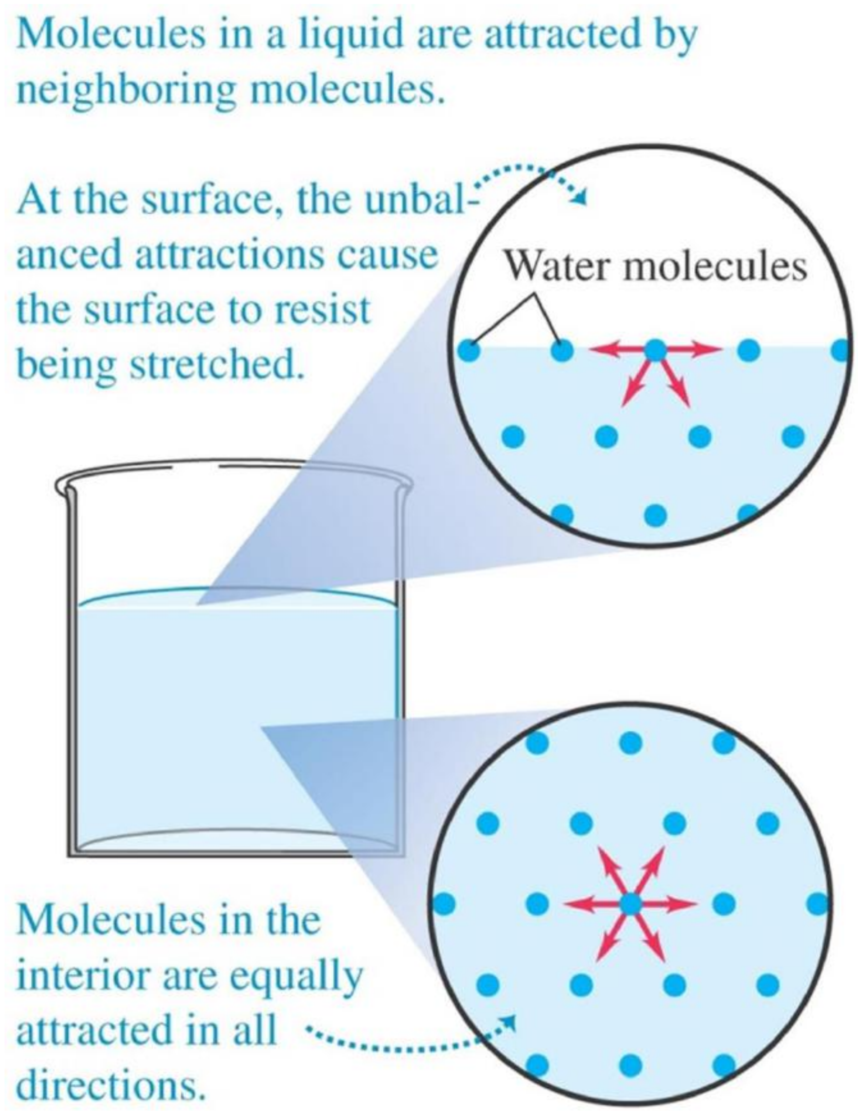
From before: $P_2 - P_1 = \rho g d$

The result is $F_B = \rho g d A = \rho g V$



Surface Tension

- The surface of a liquid behaves like a membrane under tension, so it resists being stretched. This force is the *surface tension*. See Figure at the right.
- The surface tension allows the insect shown at the right to walk on water.
- The surface tension is a force per unit length.



Fluid Flow

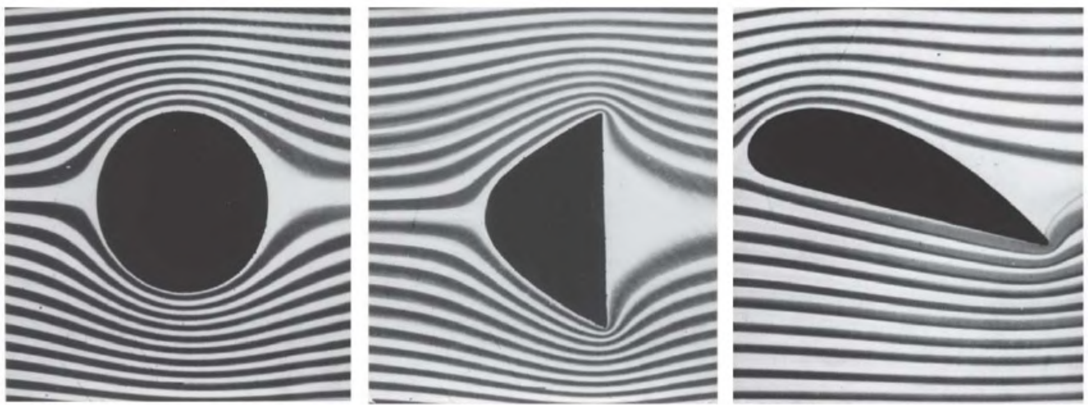
A moving fluid will exert forces parallel to the surface over which it moves, unlike a static fluid. This gives rise to a viscous force that impedes the forward motion of the fluid. A **steady flow** is one where the velocity at a given point in a fluid is constant.

$V_1 \neq V_2$

Steady flow is laminar; the fluid flows in layers. The path that the fluid in these layers takes is called a streamline. Streamlines do not cross. Crossing streamlines would indicate a volume of fluid with two different velocities at the same time. An **ideal fluid** is incompressible, undergoes laminar flow, and has no viscosity.

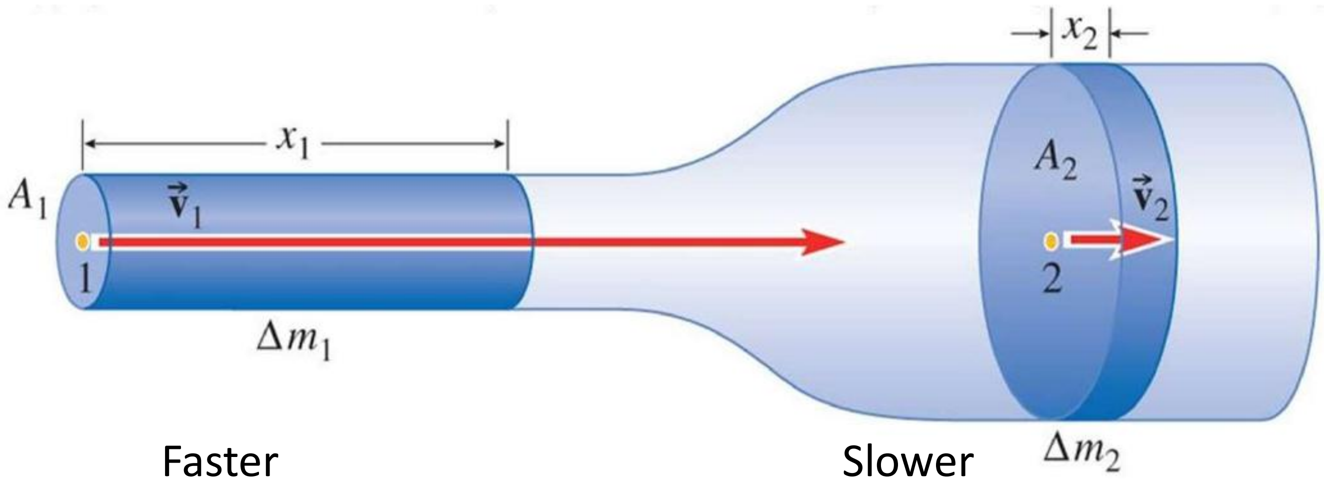
Streamline Flow

- The flow lines in the bottom figure are *laminar* because adjacent layers slide smoothly past each other.



- In the figure at the right, the upward flow is laminar at first but then becomes *turbulent flow*.

The Continuity



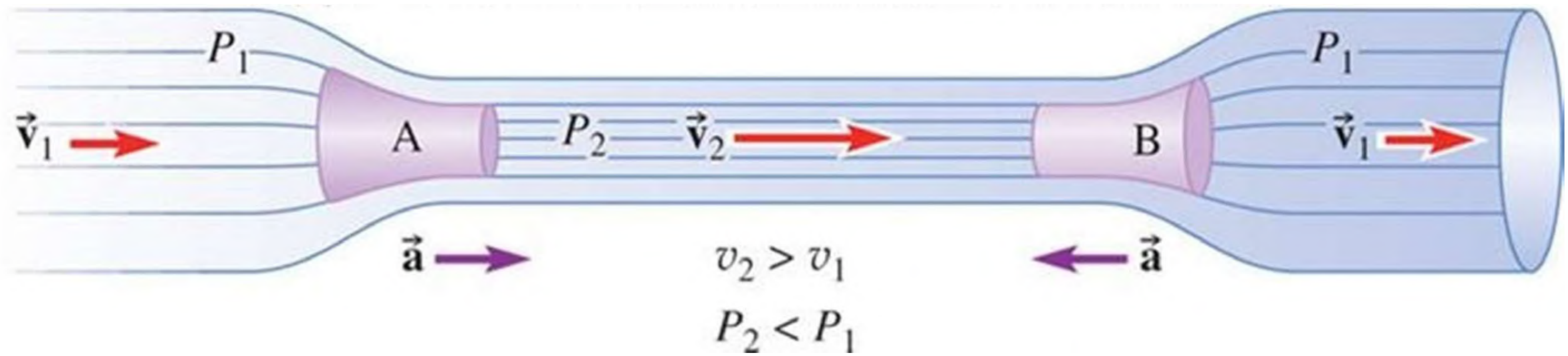
$$\frac{\Delta m}{\Delta t} = \rho A v$$ is the mass flow rate (units kg/s)

$$\frac{\Delta V}{\Delta t} = A v$$ is the volume flow rate (units m^3/s)

The continuity equation is $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

If the fluid is incompressible, then $\rho_1 = \rho_2$.

Bernoulli



$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

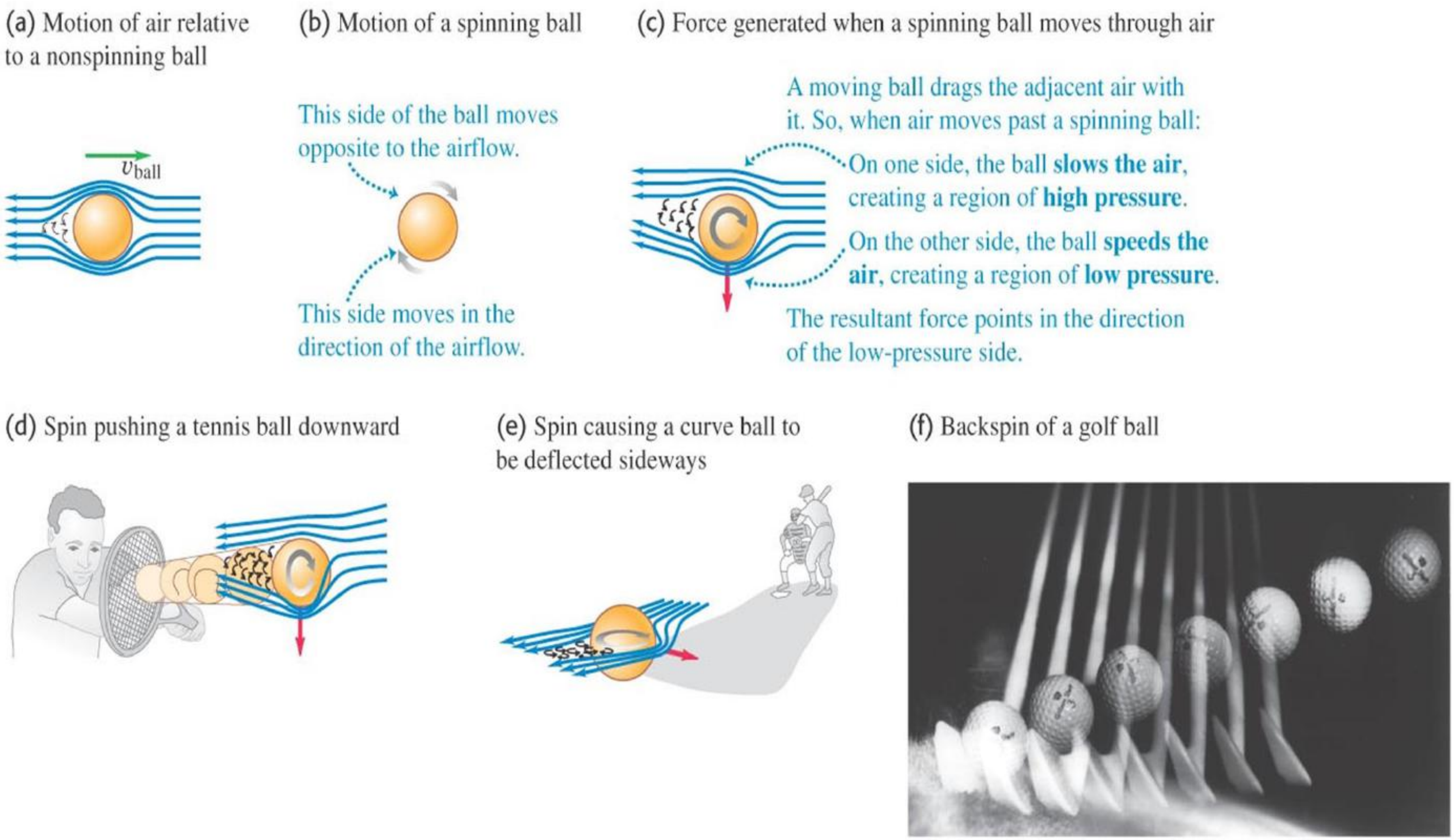
Work per unit volume done by the fluid

Potential energy per unit volume

Kinetic energy per unit volume

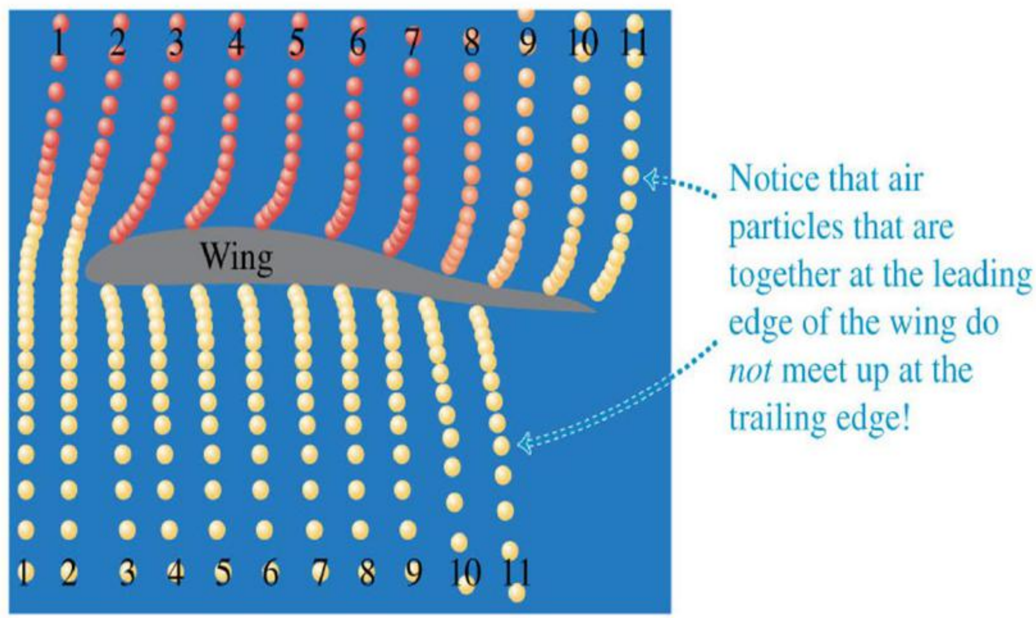
Points 1 and 2 must be on the same streamline

A Curve Ball

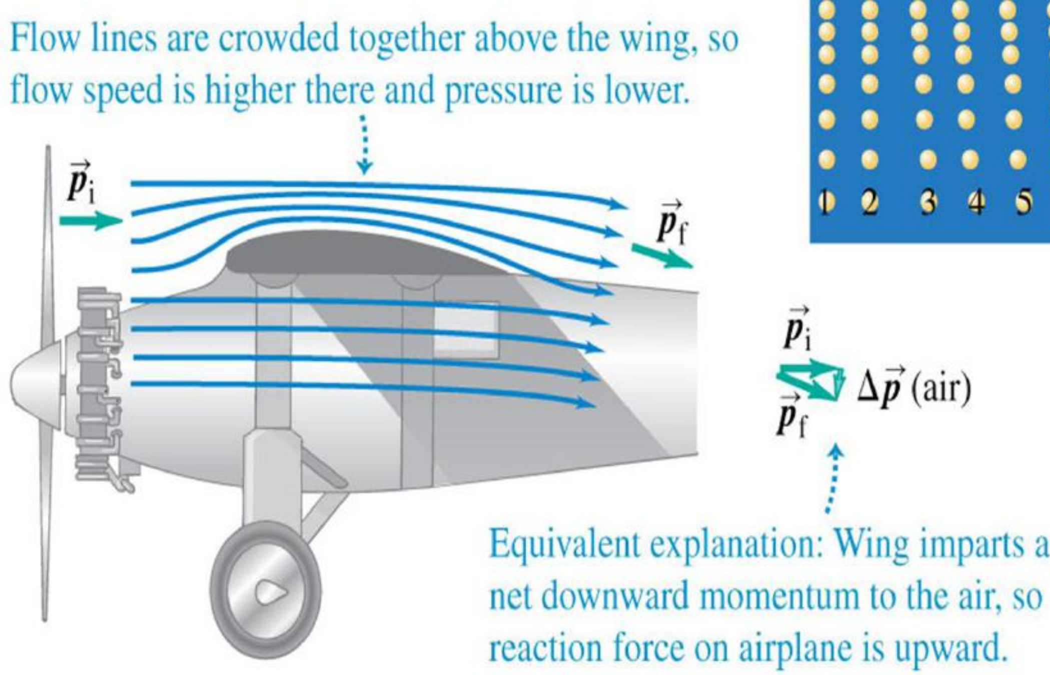

(a) Motion of air relative to a nonspinning ball
v_{ball}
(b) Motion of a spinning ball
This side of the ball moves opposite to the airflow.
This side moves in the direction of the airflow.
(c) Force generated when a spinning ball moves through air
A moving ball drags the adjacent air with it. So, when air moves past a spinning ball:
On one side, the ball **slows the air**, creating a region of **high pressure**.
On the other side, the ball **speeds the air**, creating a region of **low pressure**.
The resultant force points in the direction of the low-pressure side.
(d) Spin pushing a tennis ball downward
(e) Spin causing a curve ball to be deflected sideways
(f) Backspin of a golf ball

Lift on Airplane Wing

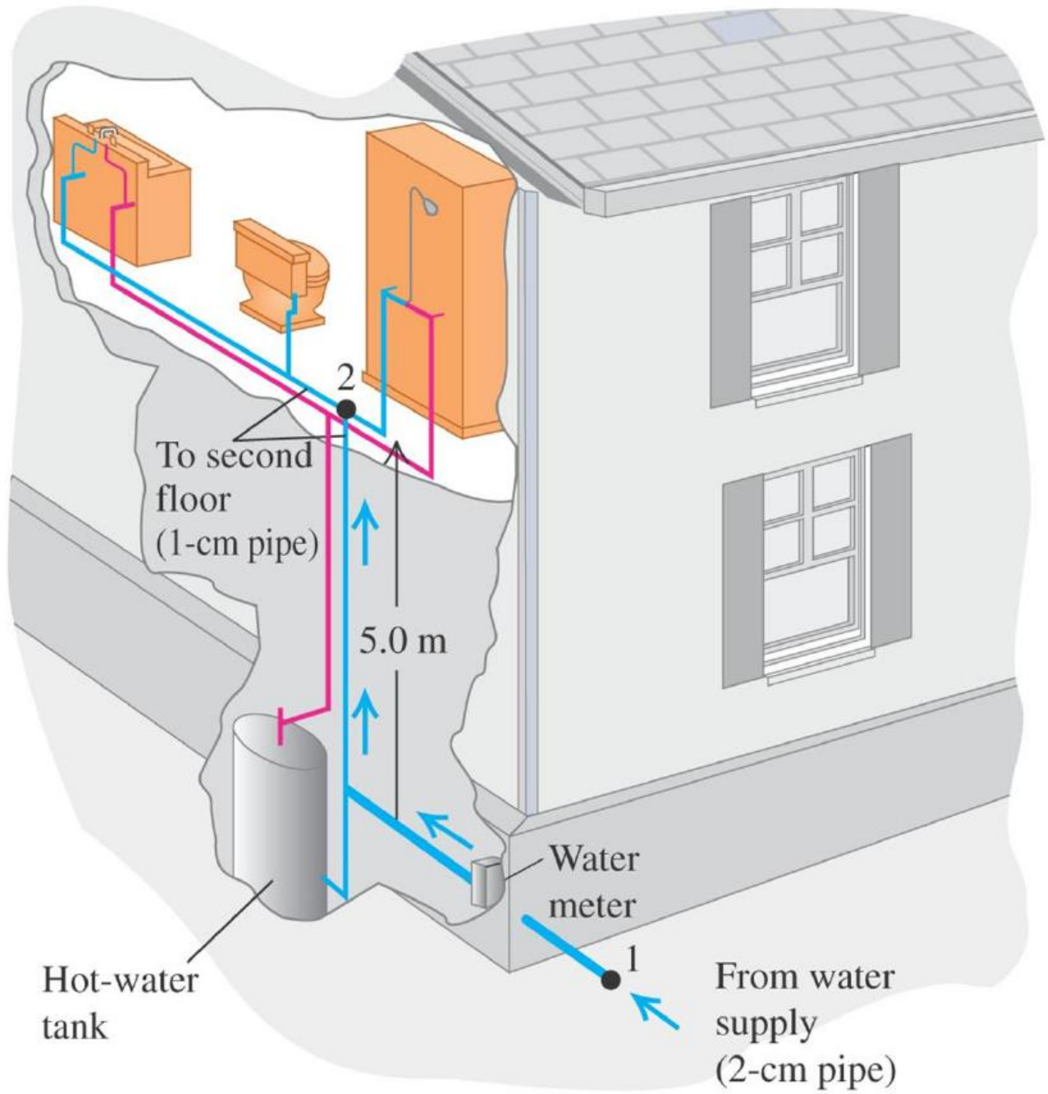
(b) Computer simulation of air parcels flowing around a wing, showing that air moves much faster over the top than over the bottom.



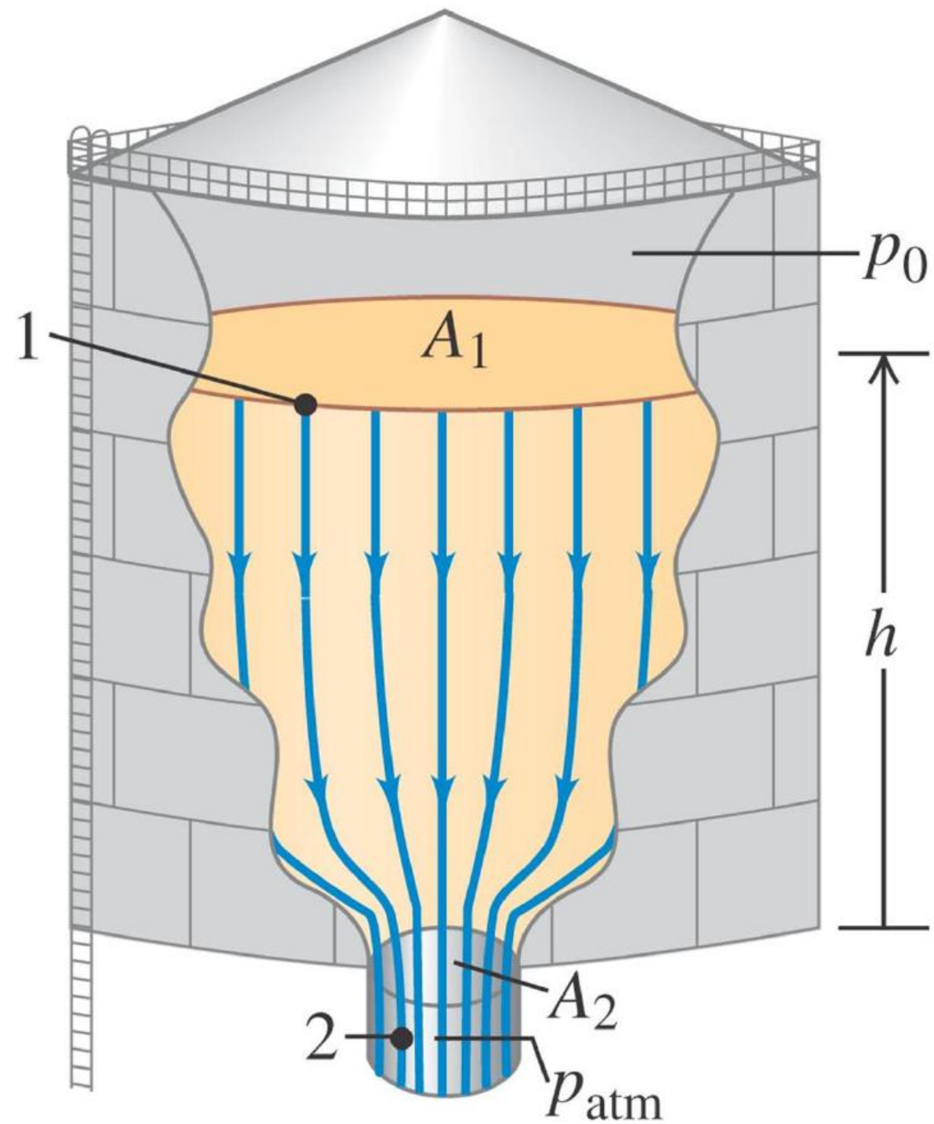
(a) Flow lines around an airplane wing



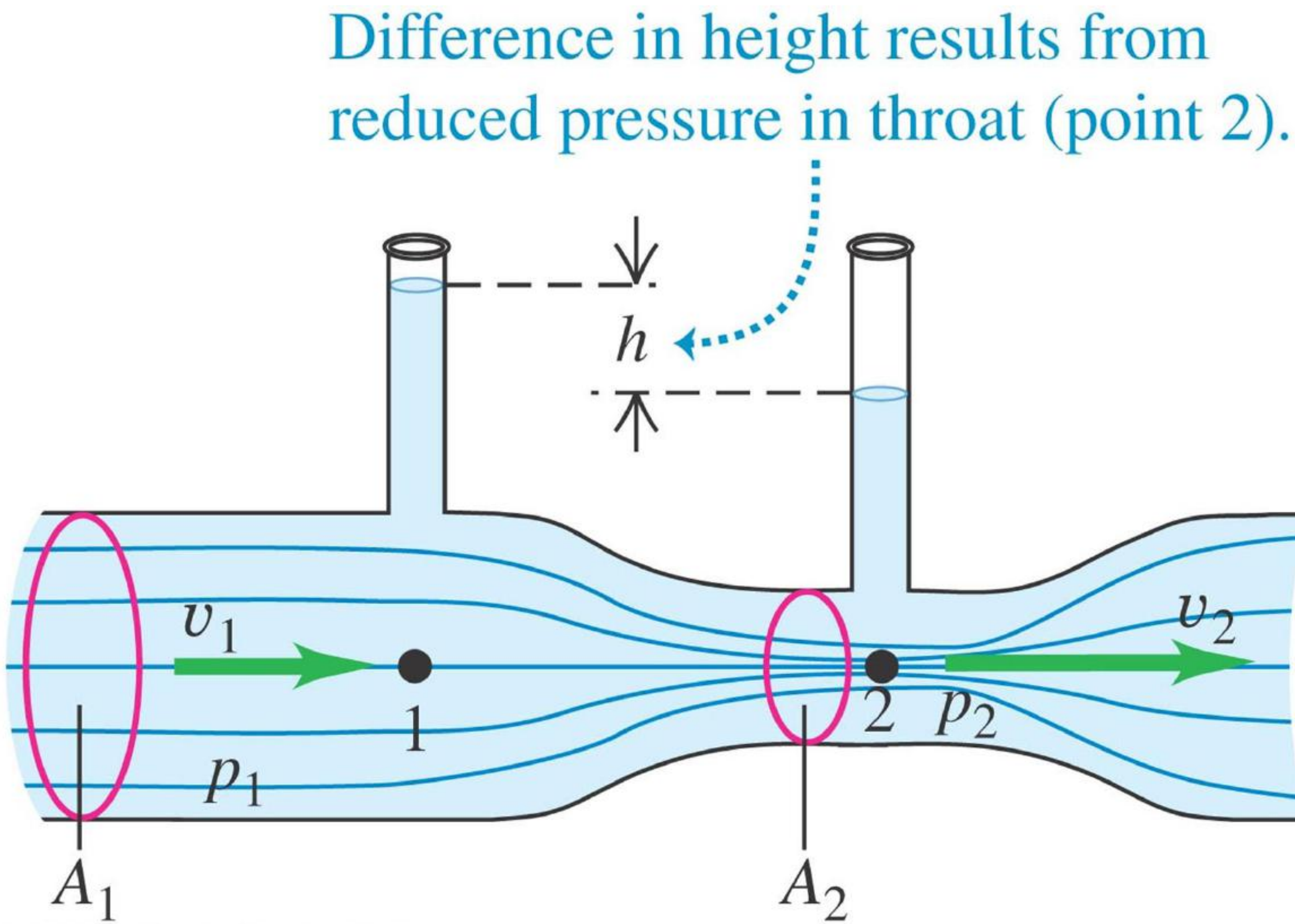
Water Pressure in The Home



Speed of Efflux



The Venturi meter



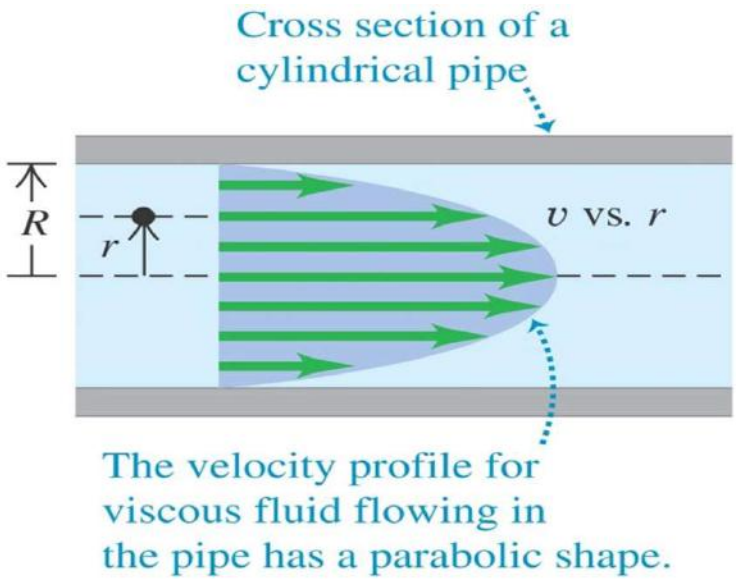
Viscosity

- A real fluid has viscosity (fluid friction). This implies a pressure difference needs to be maintained across the ends of a pipe for fluid to flow.
- **Viscosity (η)** also causes the existence of a velocity gradient across a pipe. A fluid flows more rapidly in the center of the pipe and more slowly closer to the walls of the pipe.
- The volume flow rate for laminar flow of a viscous fluid is given by Poiseuille's Law.

$$\frac{\Delta V}{\Delta t} = \frac{\pi}{8}\frac{\Delta P/L}{\eta} r^4 \qquad F = \eta A \frac{v}{d}$$

Viscosity and Turbulence

- *Viscosity* is internal friction in a fluid. *Turbulence* is irregular chaotic flow that is no longer laminar.



Viscous Drag

- The viscous drag force on a *sphere* is given by Stokes' law.

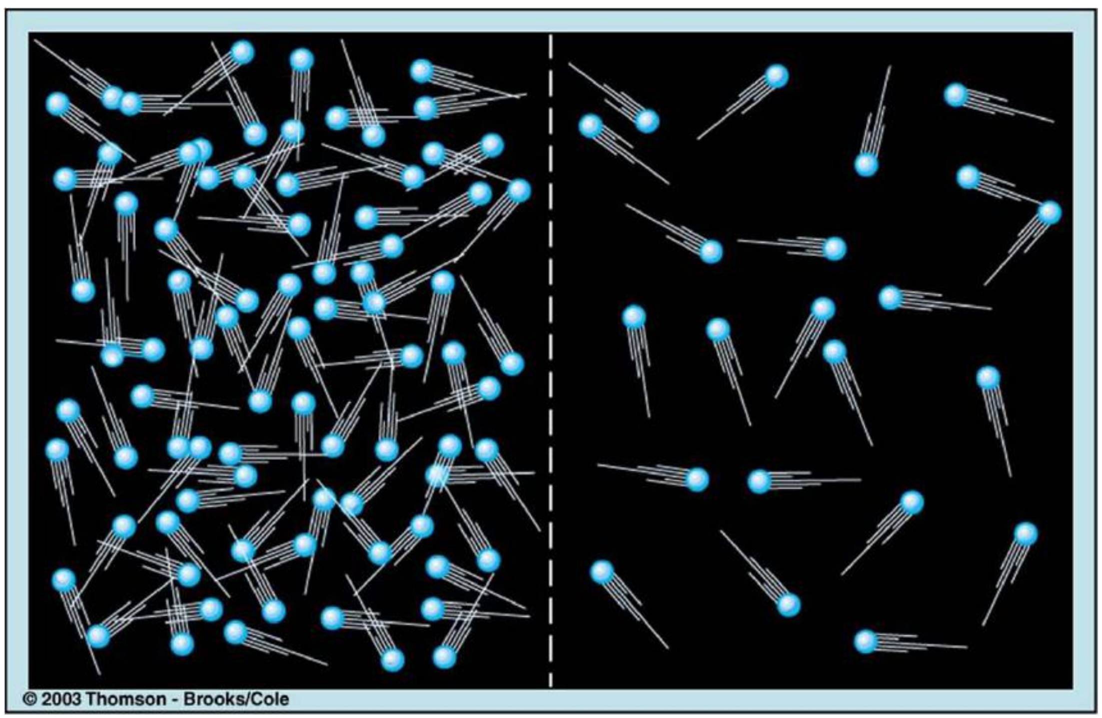
$$F_D = 6\pi\eta rv$$

- Where η is the viscosity of the fluid that the sphere is falling through, r is the radius of the sphere, and v is the velocity of the sphere.

Diffusion

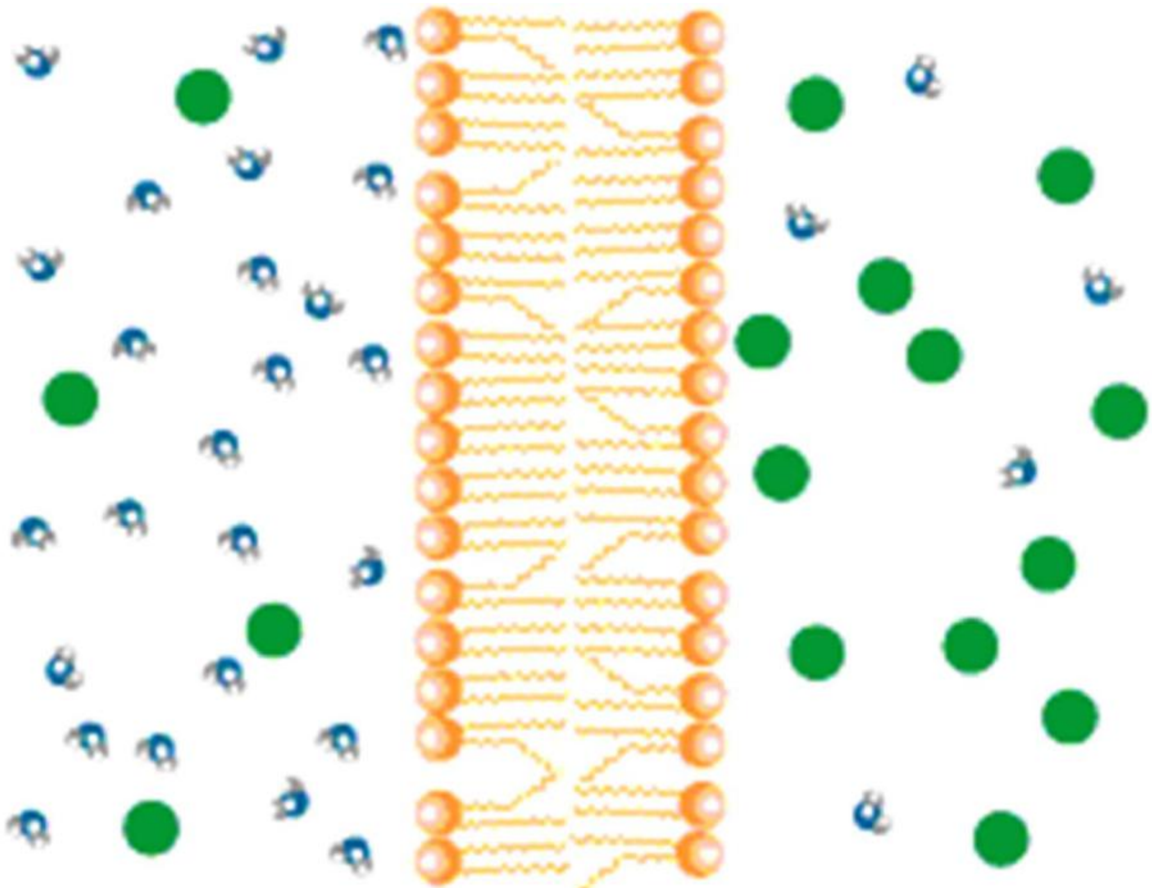
Molecules move from region of high concentration to region of low concentration. Fick's Law: (D = diffusion coefficient)

$$Diffusion\ rate = \frac{Mass}{time} = DA\left(\frac{C_2 - C_1}{L}\right)$$



Osmosis

Movement of water through a boundary while denying passage to specific molecules, e.g. salts



Basic Physics 1
Lecture Module

Rahadian N, S.Si. M.Si.

Temperature, Heat, Thermal

Temperature, Heat, Thermal

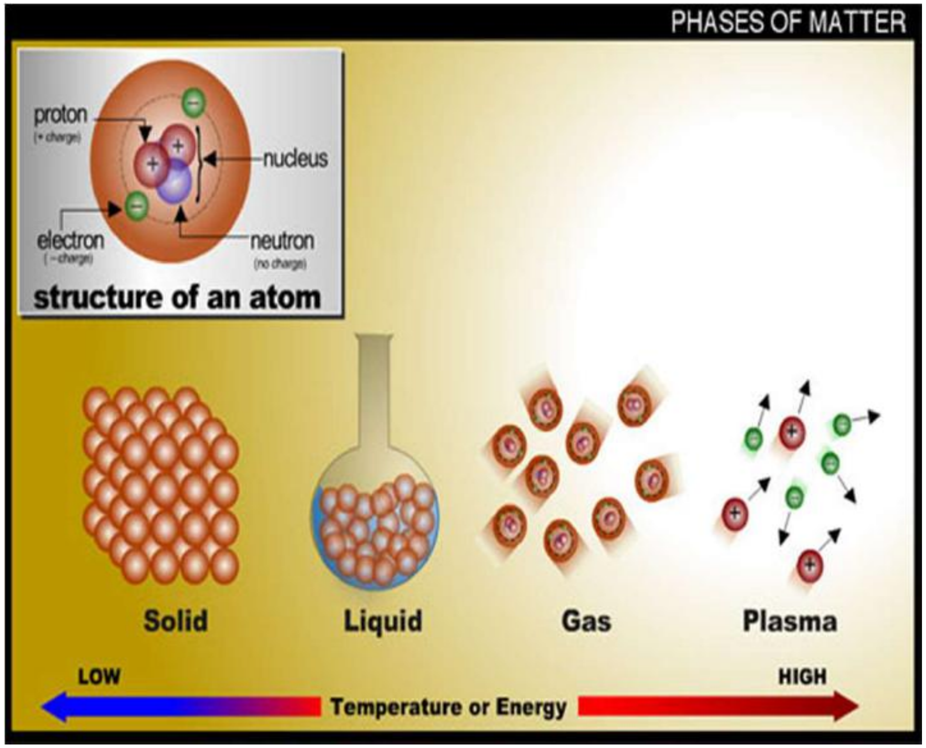
- Temperature
- Thermometers
- Heat
- Thermal energy
  - Conduction
  - Convection
  - Radiation

Temperature

- Temperature depends on Particle Movement
- All matter is made up of atoms that are moving, even solid objects have atoms that are vibrating.
- The motion from the atoms gives the object energy.

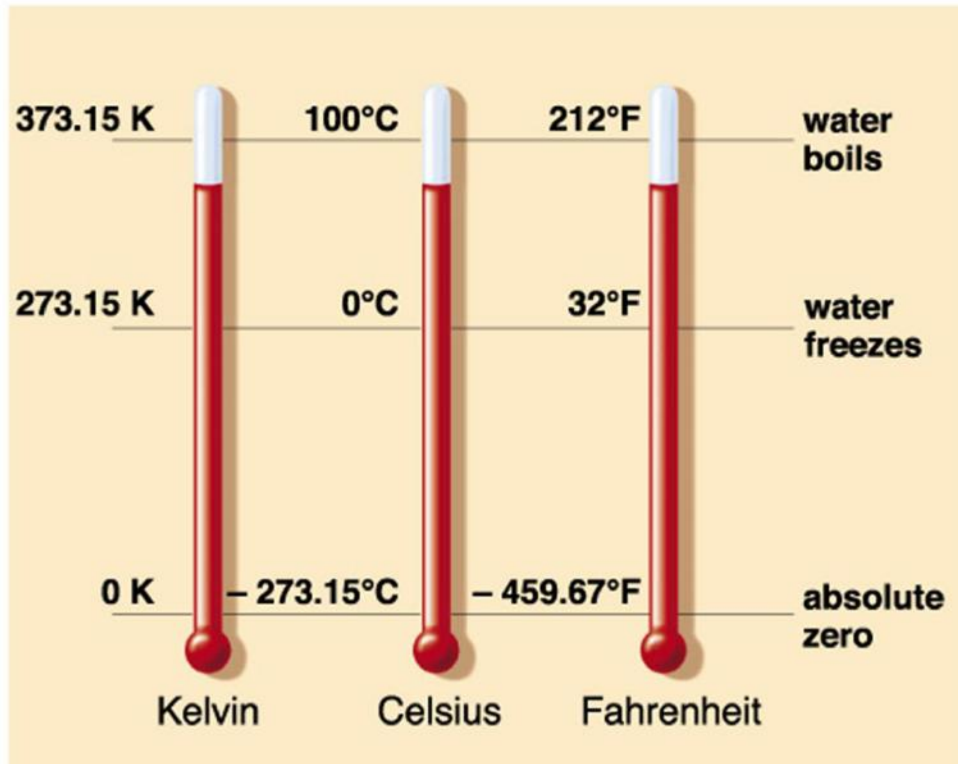
Particle Movement

- All of the particles that make up matter are constantly in motion
- Solid= vibrating atoms
- Liquid= flowing atoms
- Gas= move freely
- Plasma= move incredibly fast and freely



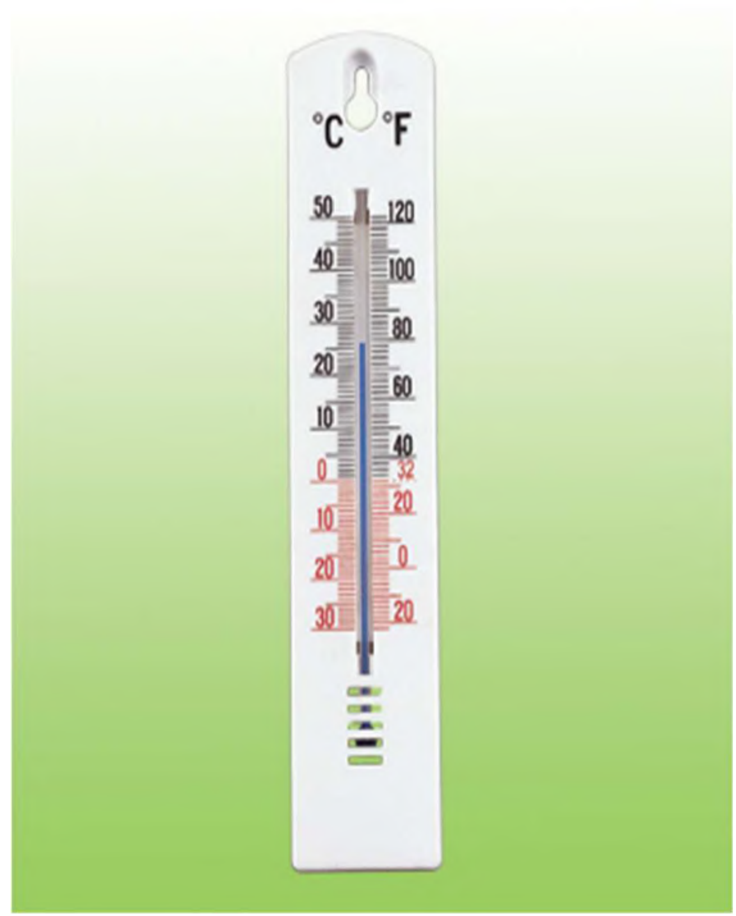
Movement Energy

- The Measure of the average kinetic energy of all the particles in the object
- The atoms mass and speed determine the temperature of the object



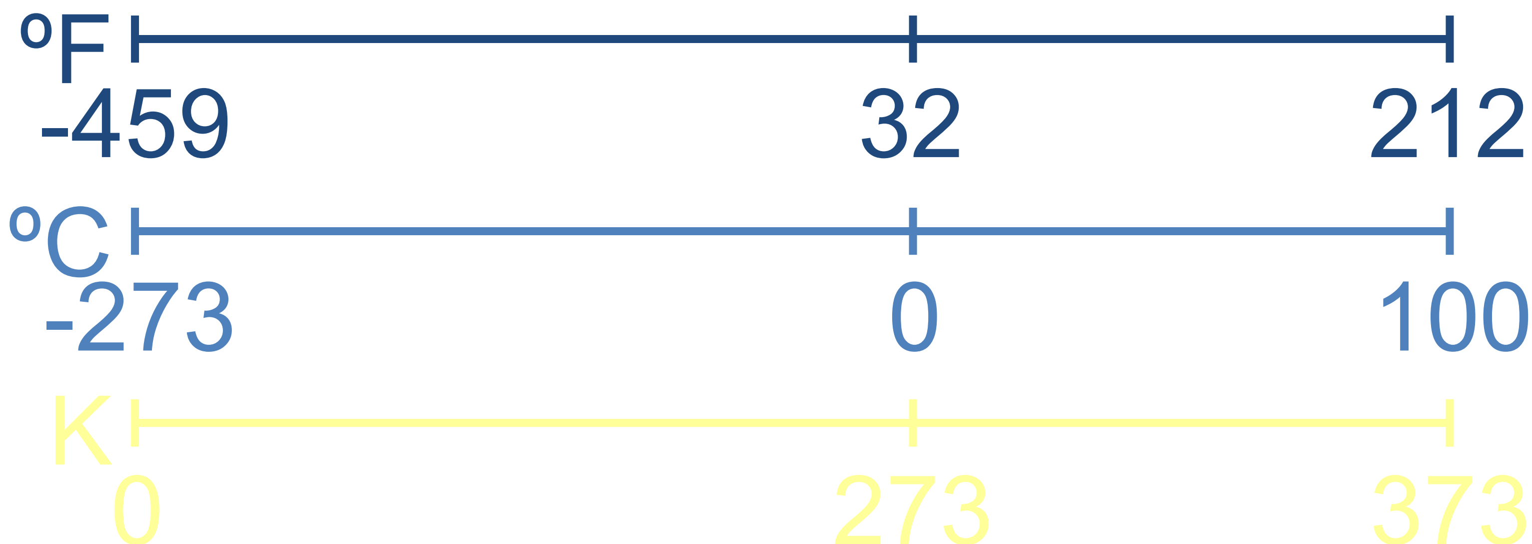
Measuring Temperature

- Temperature is measured in units called degrees (^oC,R,F,K)
- Celsius: Water freezes at 0^oC and boils at 100^oC
- Reaumur: Water freezes at 0^oC and boils at 80^oC
- Fahrenheit: Water freezes 32^oF and boils at 212^oF
- Temperature Scale C : R : F = 5 : 4 : 9



Absolute Temperature

- Always use absolute temperature (Kelvin) when working with gases.



$$°C = \frac{5}{9}(°F - 32) \qquad K = °C + 273$$

Thermometers

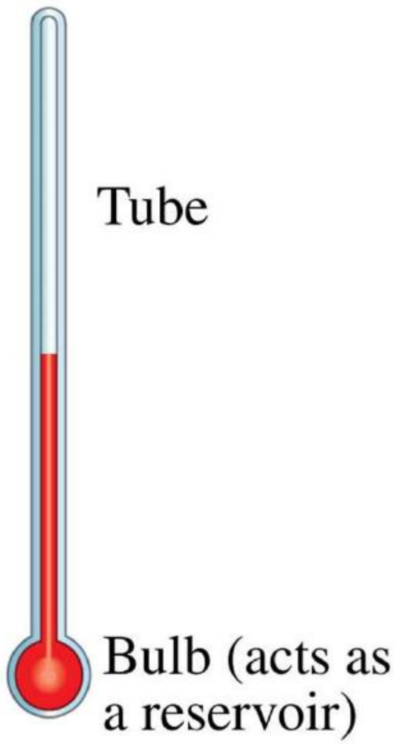
Thermometers are instruments designed to measure temperature. In order to do this, they take advantage of some property of matter that changes with temperature.

How does a Thermometer Work?

- The thermometer can measure temperature because the substance of the liquid inside always expands (increases) or contracts (decreases) by a certain amount due to a change in temperature.

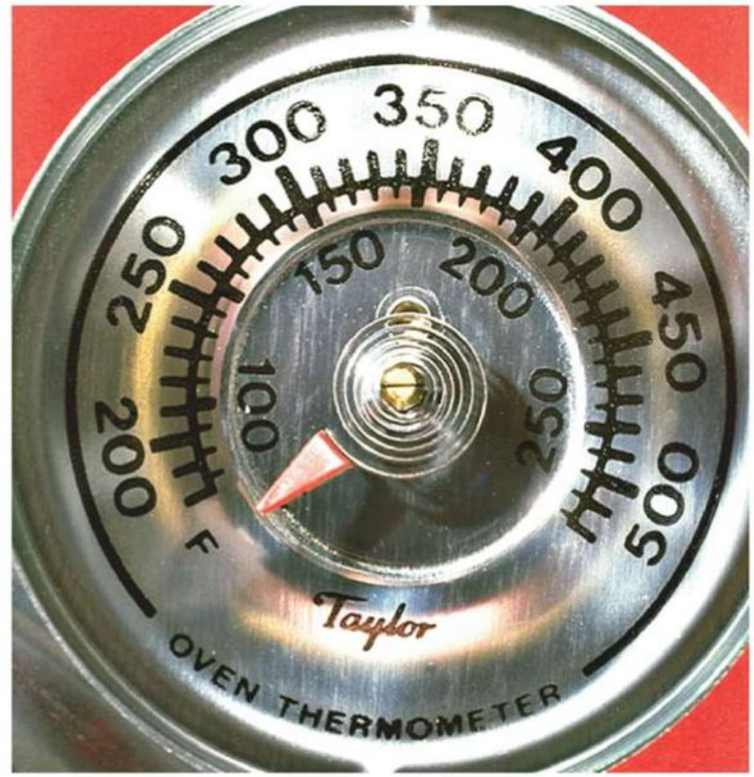
- Common thermometers used today include the liquid-in-glass type and the bimetallic strip.

The Example of Thermometers



(a)

(b)



Heat

- Heat is a flow of energy due to temperature differences. Energy from an object at a higher temperature to an object at a lower temperature.

- All gases, liquids, and most solids expand when their temperature increases. This is why bridges are built with short segments with small breaks to allow for expansion

Measuring Heat

- Heat is measured by the units of calorie and joule (J).

- calorie: The amount of energy needed to raise the temperature of 1 gram of water by 1^{o}C

- 1 calorie= 4.18 J

Specific Heat Capacity

- The specific heat of a substance is defined as the energy required to change the temperature of 1 kg of a substance by $1^{o}C$.

- Every substance has a unique specific heat capacity. This value tells you how much the temperature of a given mass of that substance will increase or decrease, based on how much heat energy is added or removed.

Calculation of Specific Heat

- **C** = specific heat capacity **Units [J/(kg.K)]**
- **m** = mass
- Δ **T** = change in temperature $T_{final} - T_{initial}$
- **Q =** heat energy transferred

$$\boxed{C = \frac{Q}{(m)(\Delta T)}}$$

$$\boxed{Q = (m)(\Delta T)(C)}$$

Thermal Energy

- The total energy of all the particles.
- If 2 samples of matter are at the same temperature they do not necessarily have the same total energy.
- Heat is thermal energy moving from a warmer object to a cooler object.
- There are 3 ways that heat can move.
  - Conduction
  - Convection
  - Radiation

Conduction

- The process that moves energy from one object to another when they are touching physically.
- It is also described as the transfer of thermal energy through matter, from a region of higher temperature to a region of lower temperature, and acts to equalize temperature differences
- Conductors: materials that transfer energy easily.
- Insulators: materials that do not transfer energy easily.
- Hot cup of cocoa transfers heat energy to cold hands

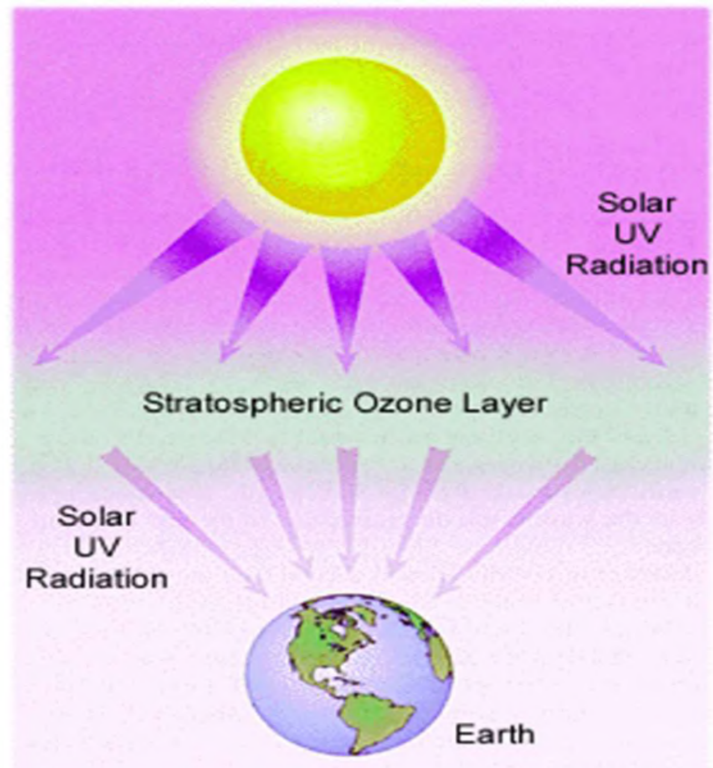
Convection

- The process that transfers energy by the movement of large numbers of particles in the same direction within a liquid or gas. the warm fluid rises and cooler fluids flow in to replace it. This creates a circular flow. Cycle in Nature

Soup is heated in the pan by convection. The hot soup rises. Cool soup falls to take the hot soup's place.

Radiation

- The energy that travels by electromagnetic waves (visible light, microwaves, and infrared light)
- Radiation from the sun strikes the atoms in your body and transfers energy



Comparison

Conduction	Convection	Radiation
• Energy transferred by direct contact • Energy flows directly from warmer to cooler objects • Continues until object temperatures are equal	• Occurs in gases and liquids • Movement of large number of particles in same direction • Cycle occurs while temperature differences exist	• Energy transferred by electromagnetic waves (visible light, microwaves, infrared) • All objects radiate energy • Can transfer energy through empty space

Basic Physics 1
Lecture Module

Rahadian N, S.Si. M.Si.

Gas Mechanic Theory

Gas Mechanic Theory

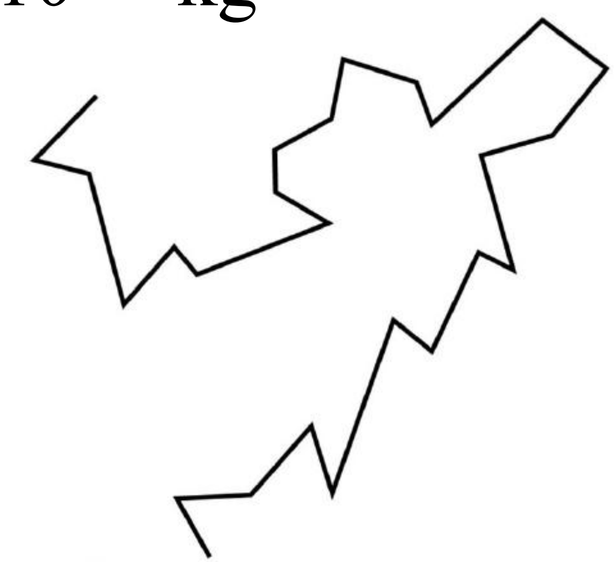
- Atomic Theory of Matter
- Thermal Expansion
- Kinetic Theory
- Distribution of Molecular Speeds
- Real Gases and Changes of Phase
- Vapor Pressure and Humidity
- Diffusion

Atomic Theory of Matter

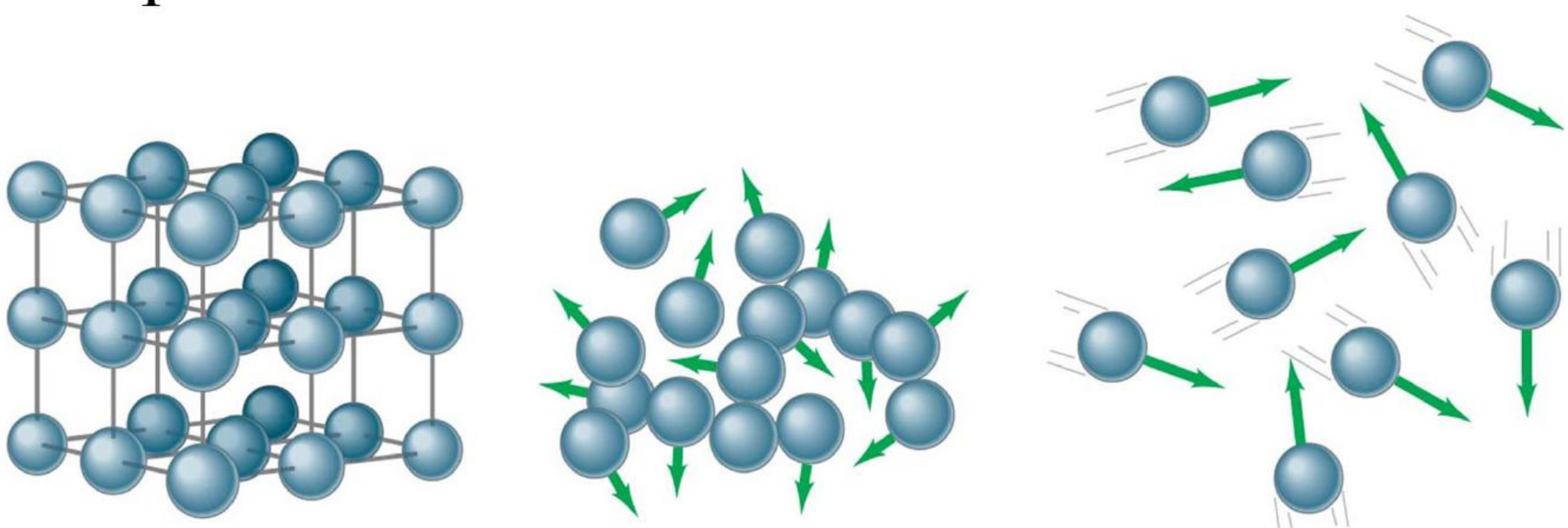
Atomic and molecular masses are measured in unified atomic mass units (u). This unit is defined so that the carbon-12 atom has a mass of exactly 12.0000 u. Expressed in kilograms:

$$1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg}$$

Brownian motion is the jittery motion of tiny flecks in water; these are the result of collisions with individual water molecules.

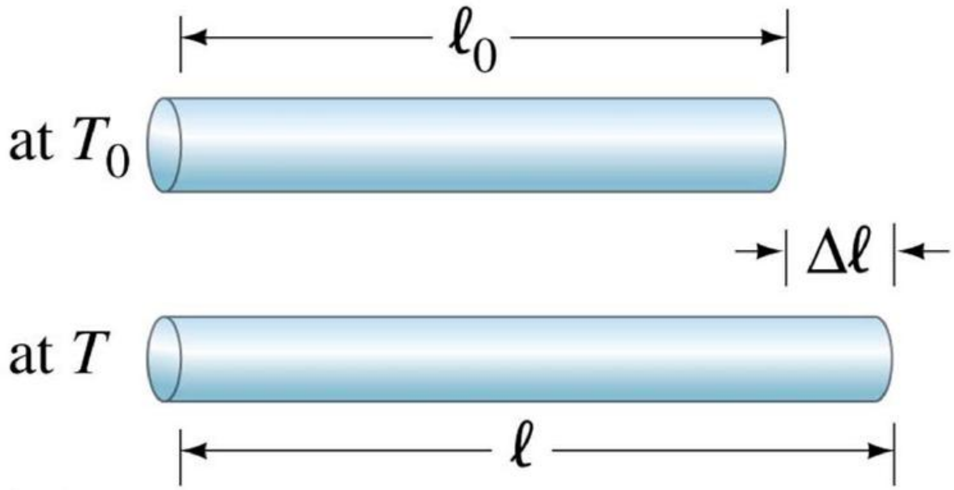


On a microscopic scale, the arrangements of molecules in solids (a), liquids (b), and gases (c) are quite different.



(a) (b) (c)

Thermal Expansion



Linear expansion occurs when an object is heated.

$$\ell = \ell_0(1 + \alpha\,\Delta T), \quad \text{(13-1b)}$$

Here, α is the coefficient of linear expansion.

Thermal Expansion for Uniform Solid

Volume expansion is similar, except that it is relevant for liquids and gases as well as solids: For uniform solids, $\beta \approx 3^{\alpha}$.

$$\Delta V = \beta V_0 \, \Delta T,$$

β is the coefficient of volume expansion.

Material	Coefficient of Linear Expansion, α $(\mathrm{C}^\circ)^{-1}$	Coefficient of Volume Expansion, β $(\mathrm{C}^\circ)^{-1}$
Solids		
Aluminum	25×10^{-6}	75×10^{-6}
Brass	19×10^{-6}	56×10^{-6}
Copper	17×10^{-6}	50×10^{-6}
Gold	14×10^{-6}	42×10^{-6}
Iron or steel	12×10^{-6}	35×10^{-6}
Lead	29×10^{-6}	87×10^{-6}
Glass (Pyrex®)	3×10^{-6}	9×10^{-6}
Glass (ordinary)	9×10^{-6}	27×10^{-6}
Quartz	0.4×10^{-6}	1×10^{-6}
Concrete and brick	$\approx 12 \times 10^{-6}$	$\approx 36 \times 10^{-6}$
Marble	$1.4\text{–}3.5 \times 10^{-6}$	$4\text{–}10 \times 10^{-6}$
Liquids		
Gasoline		950×10^{-6}
Mercury		180×10^{-6}
Ethyl alcohol		1100×10^{-6}
Glycerin		500×10^{-6}
Water		210×10^{-6}
Gases		
Air (and most other gases at atmospheric pressure)		3400×10^{-6}

Thermal Expansion for Material

A material may be fixed at its ends and therefore be unable to expand when the temperature changes. It will then experience large compressive or tensile stress—thermal stress—when its temperature changes.

The force required to keep the material from expanding is given by:

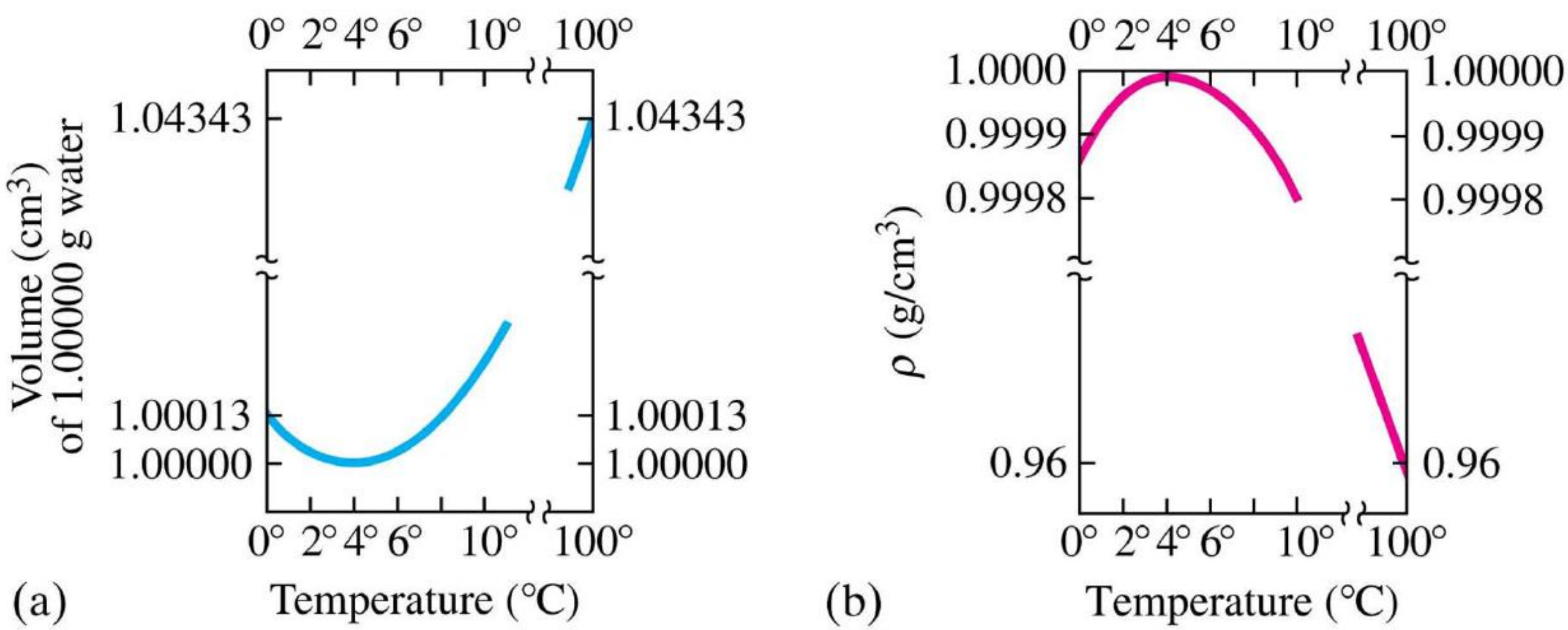
$$\Delta \ell = \frac{1}{E} \frac{F}{A} \ell_0 ,$$

where E is the Young's modulus of the material. Therefore, the stress is:

$$\frac{F}{A} = \alpha E \, \Delta T.$$

Thermal Expansion for Water

Water behaves differently from most other solids—its minimum volume occurs when its temperature is 4°C. As it cools further, it expands, as anyone who has left a bottle in the freezer to cool and then forgets about it can testify.

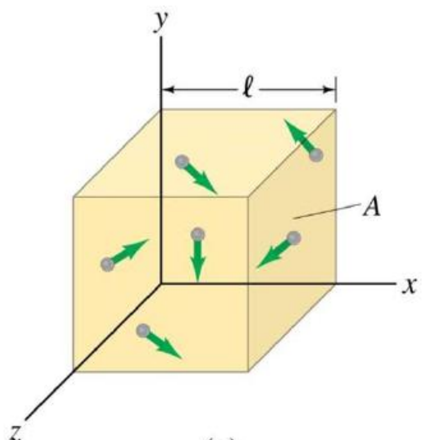


Kinetic Theory

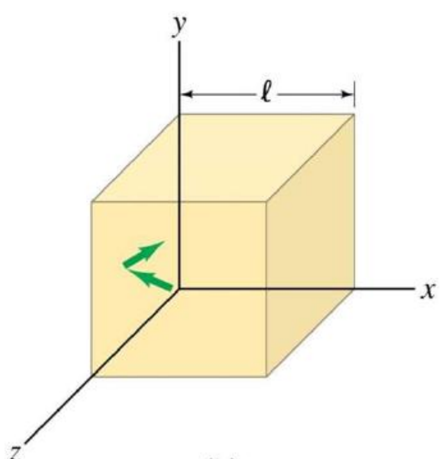
Assumptions of kinetic theory:

- large number of molecules, moving in random directions with a variety of speeds
- molecules are far apart, on average
- molecules obey laws of classical mechanics and interact only when colliding
- collisions are perfectly elastic

Kinetic Theory: Force



(a)



(b)

The force exerted on the wall by the collision of one molecule is

$$F = \frac{\Delta(mv)}{\Delta t} = \frac{2mv_x}{2\ell/v_x} = \frac{mv_x^2}{\ell}. \qquad \text{[due to one molecule]}$$

Then the force due to all molecules colliding with that wall is

$$F = \frac{m}{\ell} N\overline{v_x^2}$$

Kinetic Theory: Pressure

The averages of the squares of the speeds in all three directions are equal:

$$F = \frac{m}{\ell} N \frac{\overline{v^2}}{3}.$$

So the pressure is:

$$P = \frac{F}{A} = \frac{1}{3} \frac{Nm\overline{v^2}}{A\ell}$$

or

$$P = \frac{1}{3} \frac{Nm\overline{v^2}}{V}, \qquad \left[\begin{array}{l}\text{pressure in an}\\ \text{ideal gas}\end{array}\right]$$

Kinetic Theory: Energy

Rewriting, $$PV = \tfrac{2}{3}N\left(\tfrac{1}{2}m\overline{v^2}\right).$$

$$\tfrac{2}{3}\left(\tfrac{1}{2}m\overline{v^2}\right) = kT,$$

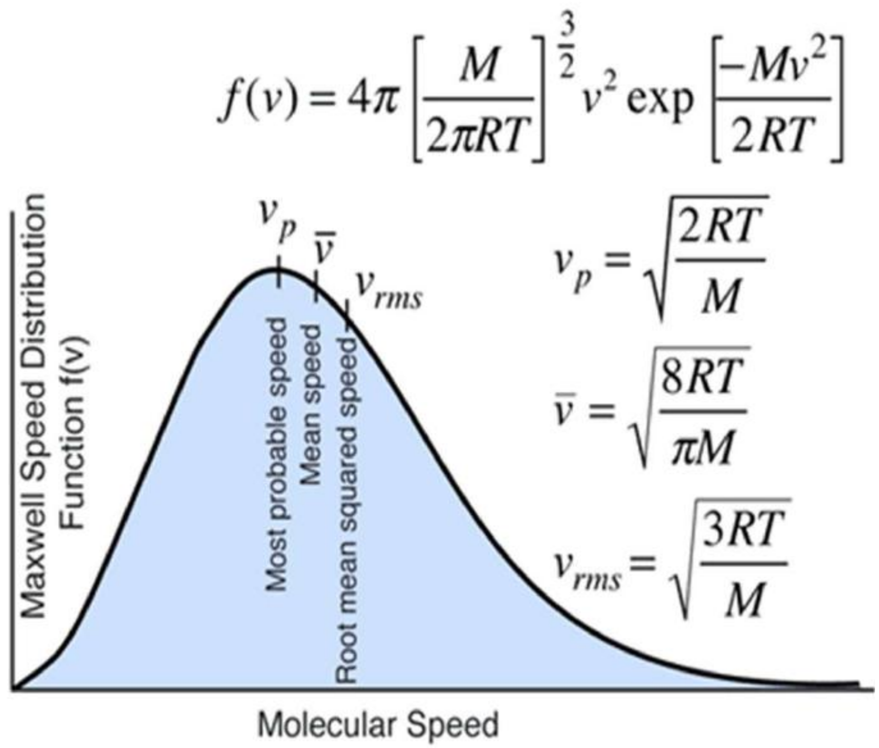
, or

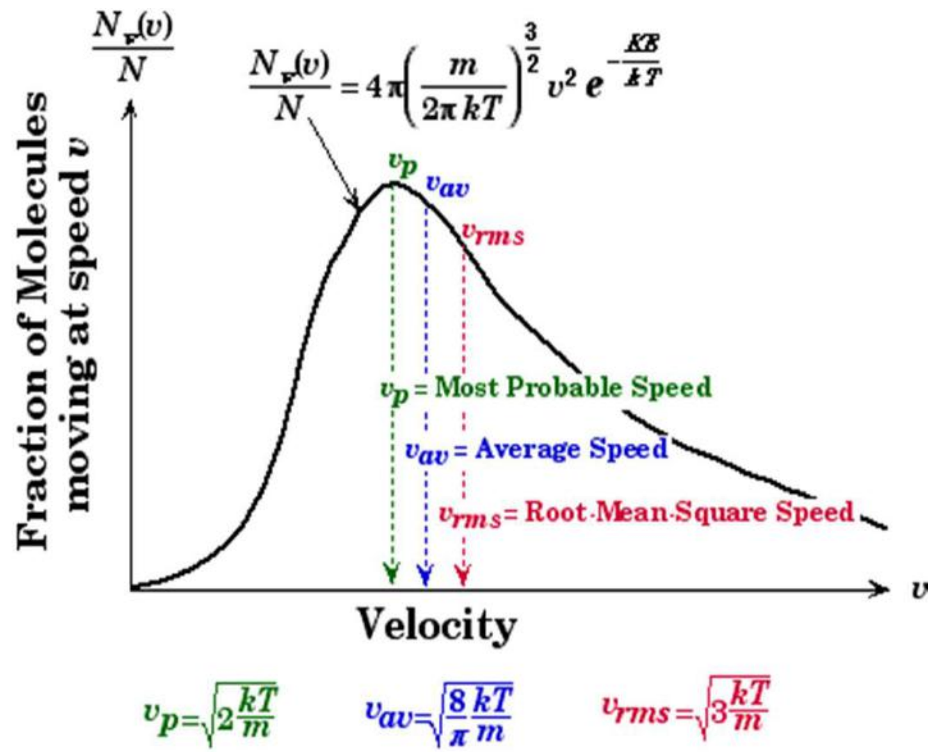
$$\overline{\mathrm{KE}} = \tfrac{1}{2}m\overline{v^2} = \tfrac{3}{2}kT. \qquad \text{[ideal gas]}$$

molecules in an ideal gas is directly proportional to the temperature of the gas.

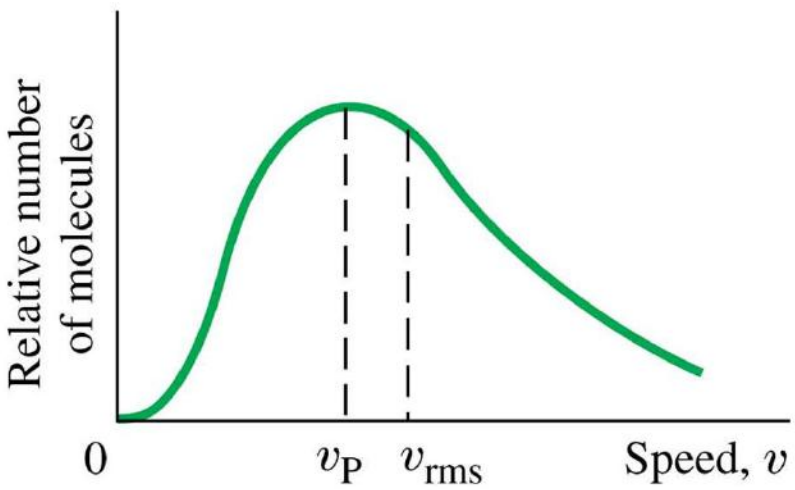
Kinetic Theory: Velocity

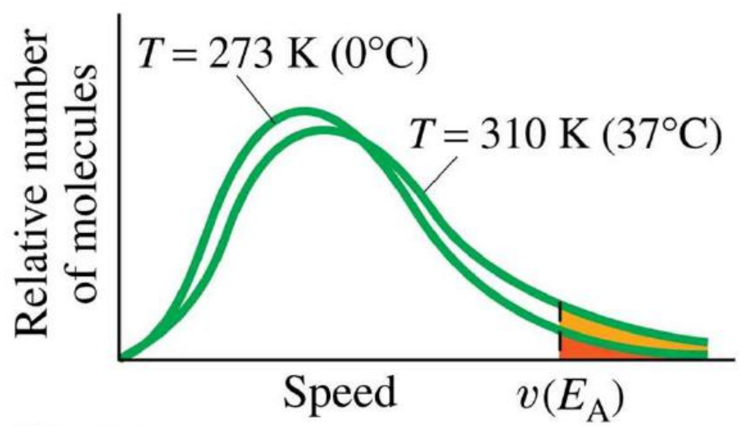
We can invert this to find the average speed of molecules in a gas as a function of temperature:





Distribution of Molecular Speeds





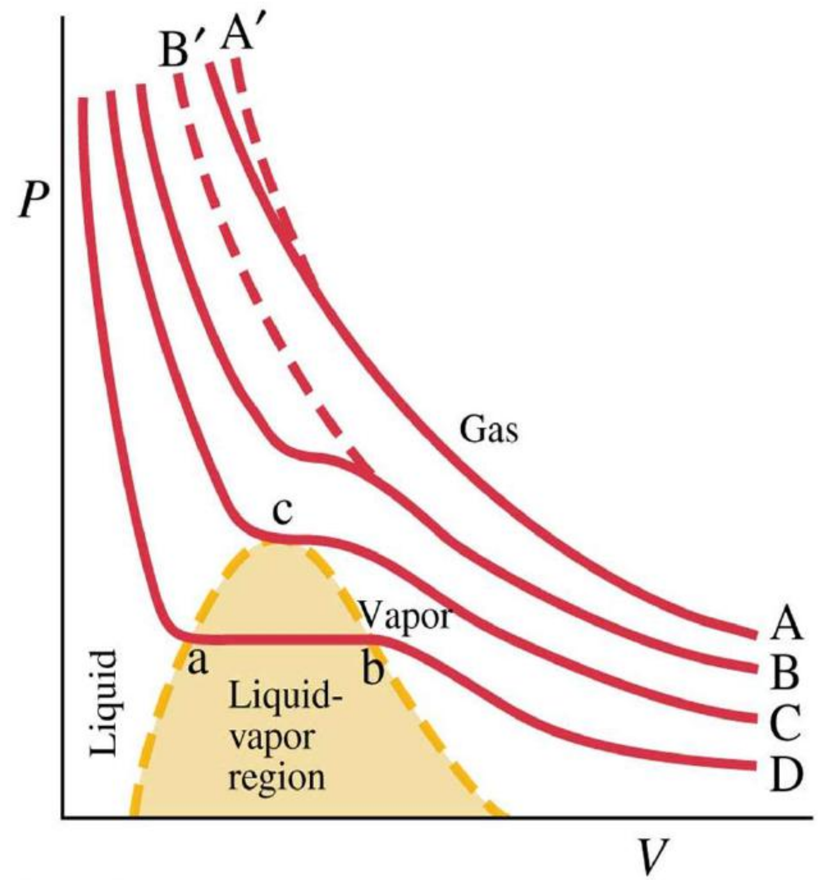
These two graphs show the distribution of speeds of molecules in a gas, as derived by Maxwell. The most probable speed, v_P, is not quite the same as the rms speed.

As expected, the curves shift to the right with temperature.

Real Gases and Changes of Phase

The curves here represent the behavior of the gas at different temperatures. The cooler it gets, the farther the gas is from ideal.

In curve, the gas becomes liquid; it begins condensing at (b) and is entirely liquid at (a). The point (c) is called the critical point.



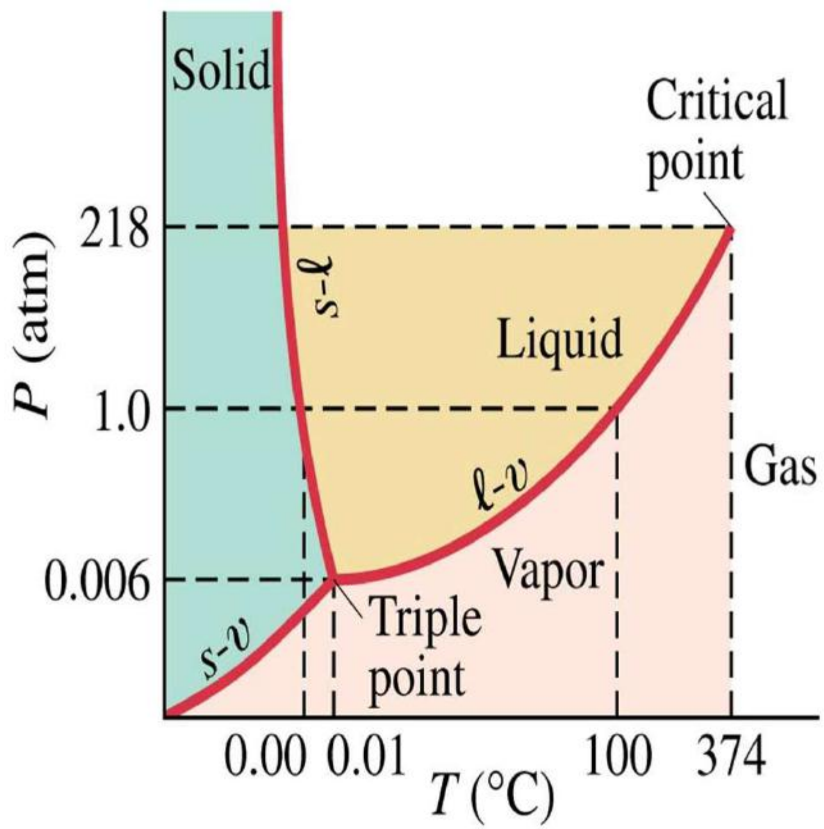
Critical Temperature & Pressure

Below the critical temperature, the gas can liquefy if the pressure is sufficient; above it, no amount of pressure will suffice.

Substance	Critical Temperature °C	Critical Temperature K	Critical Pressure (atm)
Water	374	647	218
CO_2	31	304	72.8
Oxygen	−118	155	50
Nitrogen	−147	126	33.5
Hydrogen	−239.9	33.3	12.8
Helium	−267.9	5.3	2.3

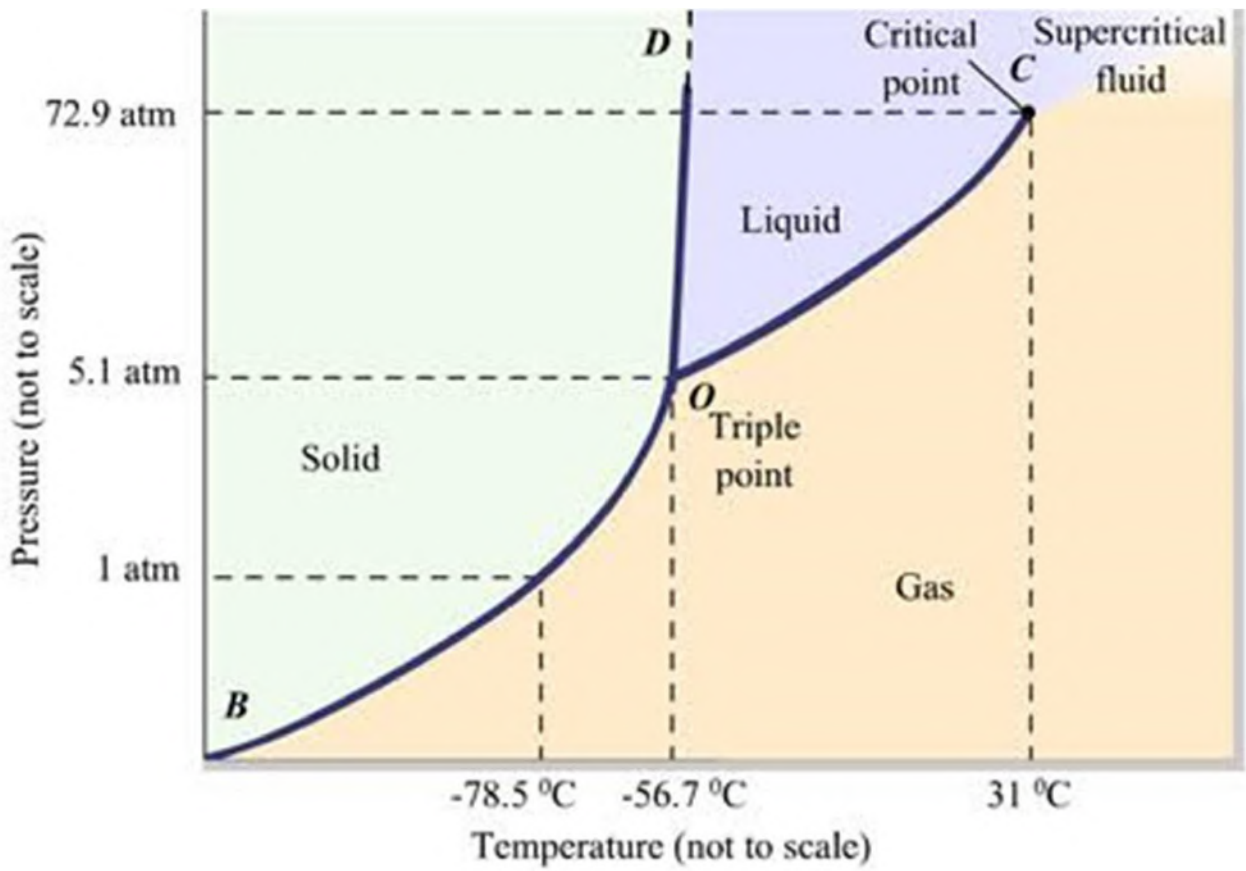
Phase Diagram of Water

PT diagram is called a phase diagram; it shows all three phases of matter. The solid-liquid transition is melting or freezing; the liquid-vapor one is boiling or condensing; and the solid-vapor one is sublimation.

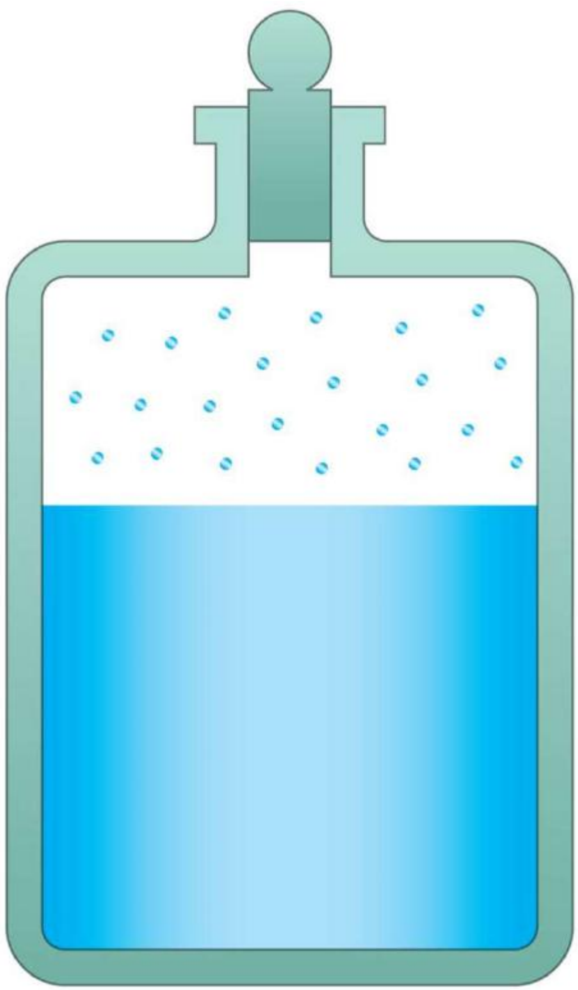


Phase Diagram of Carbon Oxide

The triple point is the only point where all three phases can coexist in equilibrium.



Vapor Pressure and Humidity



An open container of water can evaporate, rather than boil, away. The fastest molecules are escaping from the water's surface, so evaporation is a cooling process as well.

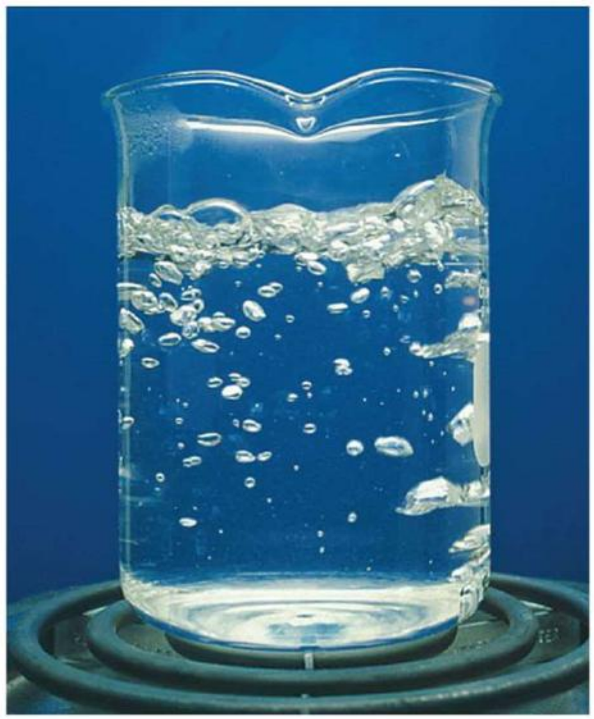
The inverse process is called condensation.

When the evaporation and condensation processes are in equilibrium, the vapor just above the liquid is said to be saturated, and its pressure is the saturated vapor pressure.

Saturated Vapor Pressure for Water

The saturated vapor pressure increases with temperature.

A liquid boils when its saturated vapor pressure equals the external pressure.



Temperature (°C)	Saturated Vapor Pressure torr (= mm-Hg)	Saturated Vapor Pressure Pa (= N/m²)
−50	0.030	4.0
−10	1.95	2.60×10^2
0	4.58	6.11×10^2
5	6.54	8.72×10^2
10	9.21	1.23×10^3
15	12.8	1.71×10^3
20	17.5	2.33×10^3
25	23.8	3.17×10^3
30	31.8	4.24×10^3
40	55.3	7.37×10^3
50	92.5	1.23×10^4
60	149	1.99×10^4
70†	234	3.12×10^4
80	355	4.73×10^4
90	526	7.01×10^4
100‡	760	1.01×10^5
120	1489	1.99×10^5
150	3570	4.76×10^5

† Boiling point on summit of Mt. Everest.
‡ Boiling point at sea level.

Partial Pressure & Relative Humidity

Partial pressure is the pressure each component of a mixture of gases would exert if it were the only gas present. The partial pressure of water in the air can be as low as zero, and as high as the saturated vapor pressure at that temperature.

Relative humidity is a measure of the saturation of the air.

$$\text{Relative humidity} = \frac{\text{partial pressure of } H_2O}{\text{saturated vapor pressure of } H_2O} \times 100\%.$$

Humidity


(a)


(b)


(c)

When the humidity is high, it feels muggy; it is hard for any more water to evaporate.

The dew point is the temperature at which the air would be saturated with water.

If the temperature goes below the dew point, dew, fog, or even rain may occur.

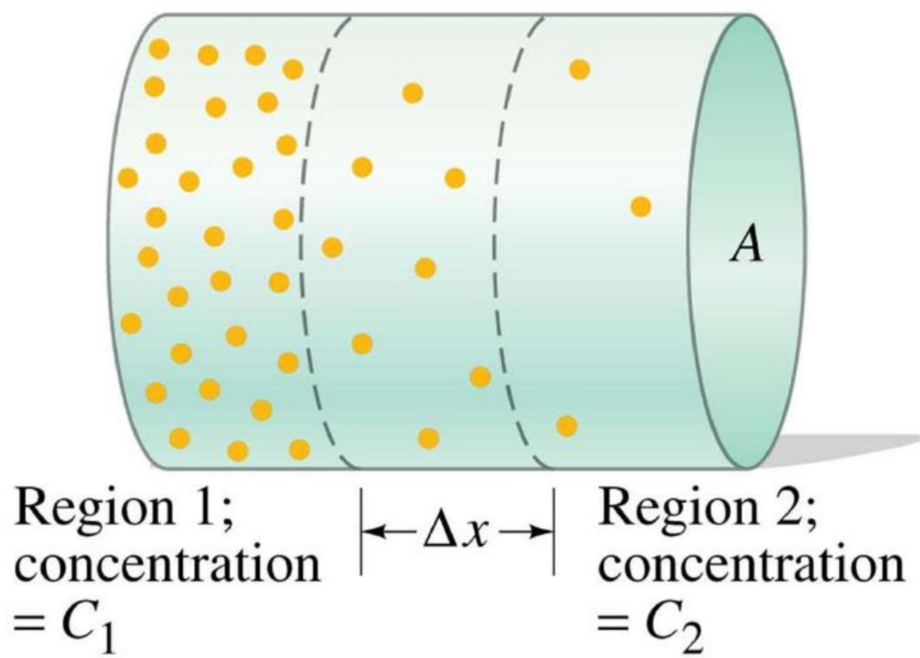
Diffusion

Even without stirring, a few drops of dye in water will gradually spread throughout. This process is called diffusion.

(a) (b) (c)

Diffusion Process

Diffusion occurs from a region of high concentration towards a region of lower concentration.



Rate of Diffusion

The rate of diffusion is given by:

$$J = DA\frac{C_1 - C_2}{\Delta x}.$$

In this equation, *D* is the diffusion constant.

Diffusing Molecules	**Medium**	***D*** **(m^2/s)**
H_2	Air	6.3×10^{-5}
O_2	Air	1.8×10^{-5}
O_2	Water	100×10^{-11}
Glycine (an amino acid)	Water	95×10^{-11}
Blood hemoglobin	Water	6.9×10^{-11}
DNA (mass 6×10^6 u)	Water	0.13×10^{-11}

Basic Physics 1
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Gas Laws

Gas Laws

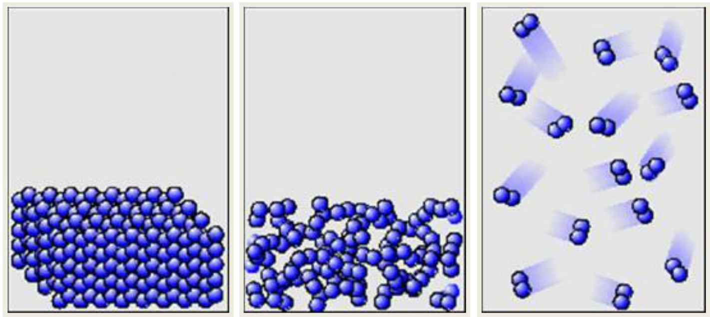
- Physical Properties
- Temperature and Pressure
- Boyle, Charles, Gay-Lussac
- Ideal Gas
- Avogadro’s Number

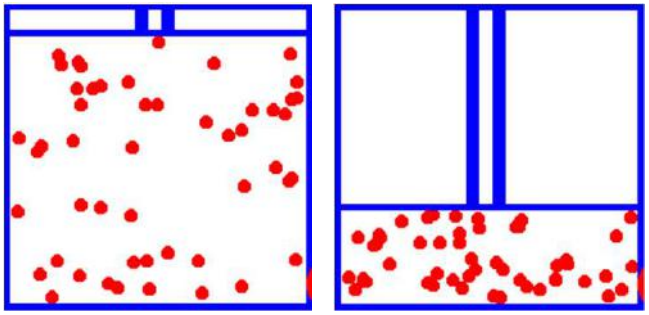
Molecular Theory of Gas

- Particles in an ideal gas…
  - have no volume.
  - have elastic collisions.
  - are in constant, random, straight-line motion.
  - don't attract or repel each other.
  - have an average of kinetic energy directly related to Kelvin temperature.

Characteristics of Gases

- Gases expand to fill any container.
  - random motion, no attraction
- Gases are fluids (like liquids).
  - no attraction
- Gases have very low densities.
  - no volume = lots of empty space
- Gases can be compressed.
  - no volume = lots of empty space
- Gases undergo diffusion & effusion.
  - random motion





Temperature

- Temperature depends on Particle Movement

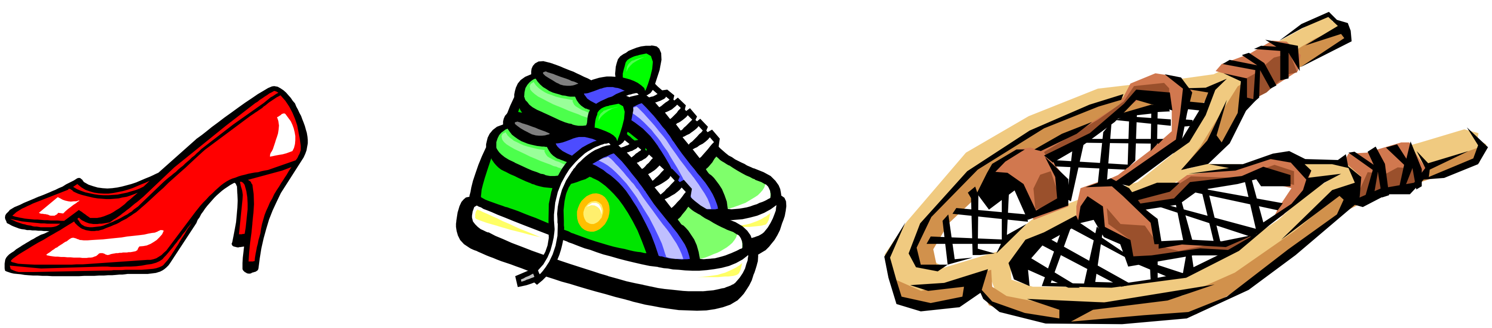
- All matter is made up of atoms that are moving, even solid objects have atoms that are vibrating.

- The motion from the atoms gives the object energy.

- More about Temperature, please refer to our previous lecture

Pressure

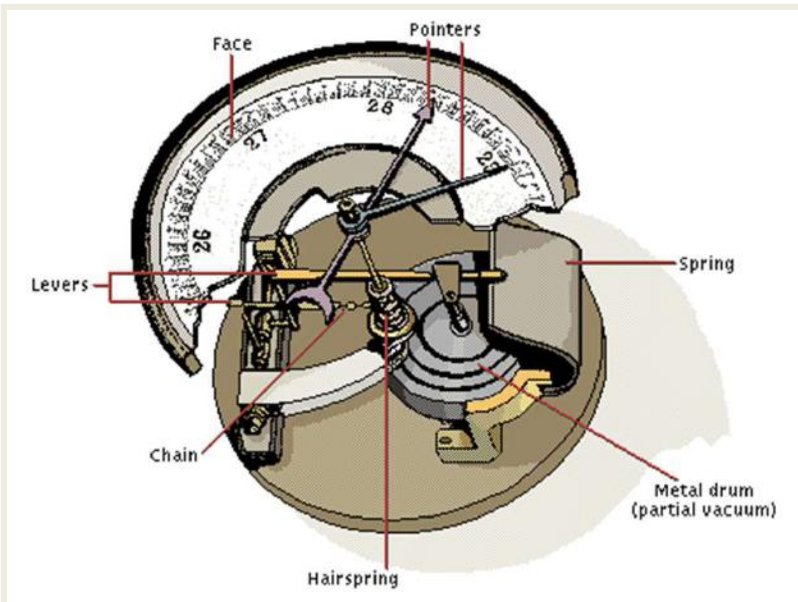
$$\textit{pressure} = \frac{\textit{force}}{\textit{area}}$$



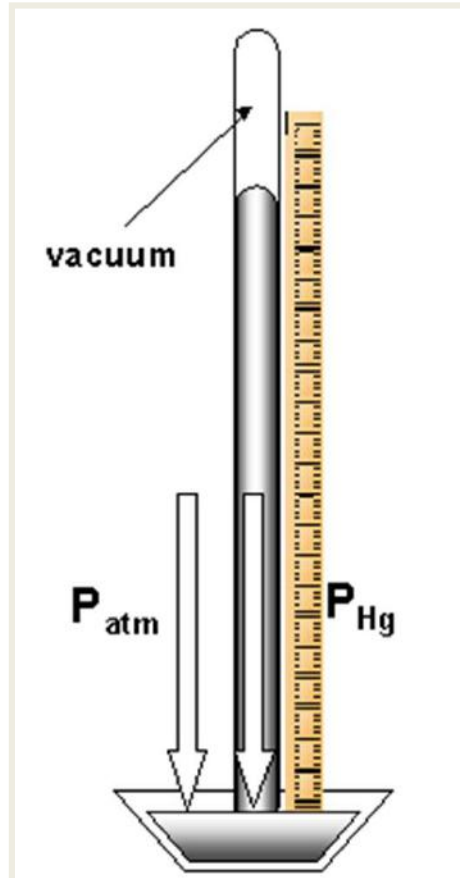
Which shoes create the most pressure?

Measuring Pressure

- **Barometer**
  - measures atmospheric pressure

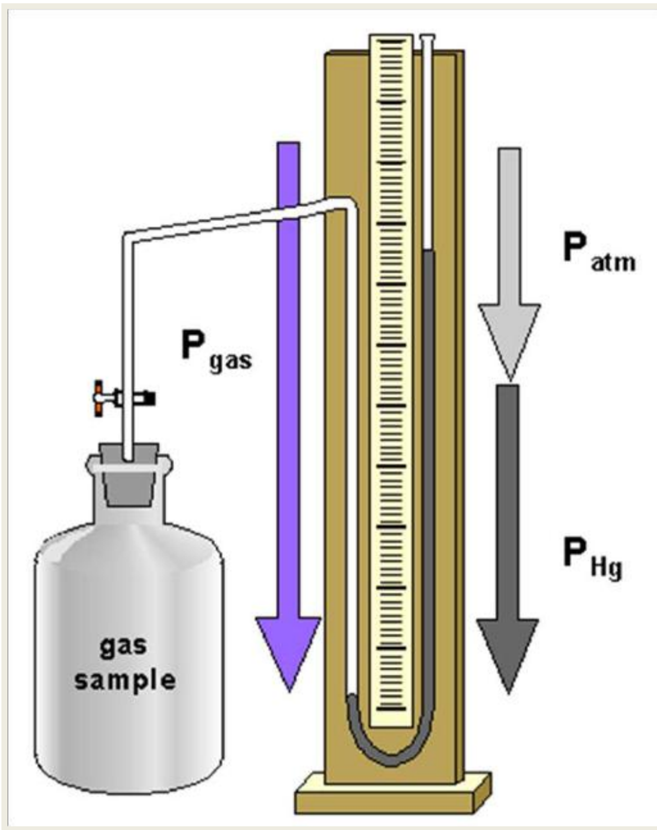


Aneroid Barometer

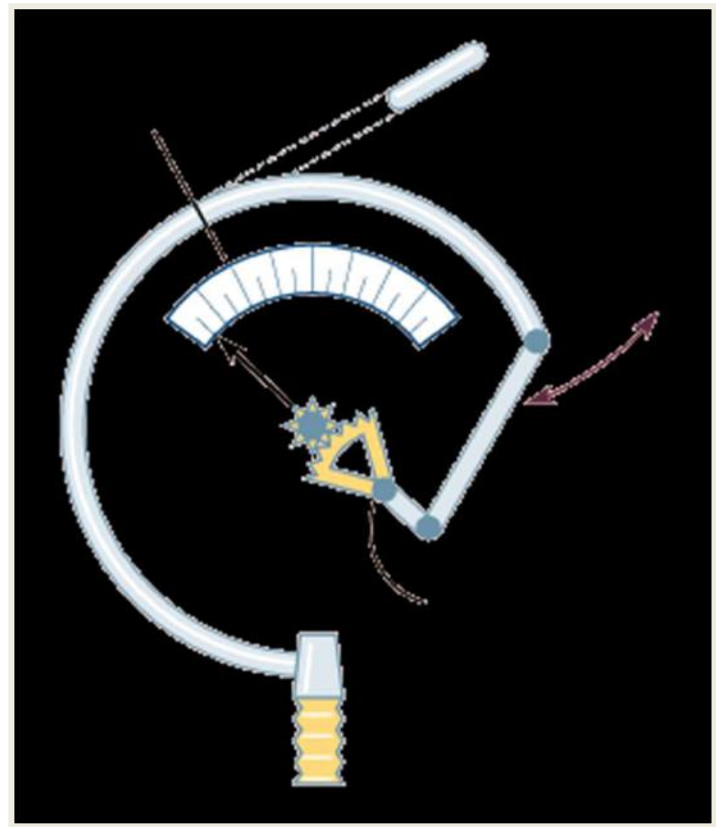


Mercury Barometer

- **Manometer**
  - measures contained gas pressure



U-tube Manometer



Bourdon-tube gauge

Pressure Units

- **KEY UNITS**

  101.325 kPa (kilo Pascal)

  1 atm

  760 mm Hg

  760 torr

  14.7 psi

$$kPa = \frac{N}{m^2}$$

The Gas Laws

- Standard Temperature & Pressure (STP)

0°C	-OR-	273 K
1 atm		101.325 kPa

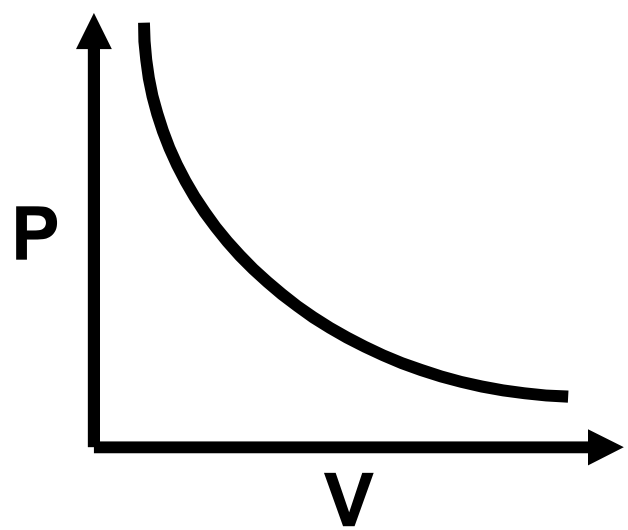
- Boyle, Charles, Gay Lusac. Based on Temperature, Pressure, Volume.

Boyle's Experiment

Volume (mL)	**Pressure (torr)**	**P·V (mL·torr)**
10.0	760.0	7.60×10^3
20.0	379.6	7.59×10^3
30.0	253.2	7.60×10^3
40.0	191.0	7.64×10^3

Boyle's Law

- The pressure and volume of a gas are inversely related
  - at constant mass & temp



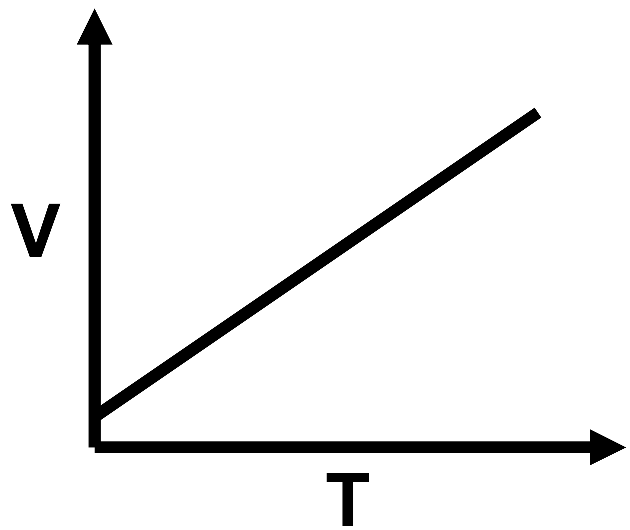
$$PV = k$$

Charles' Experiment

Volume (mL)	Temperature (K)	V/T (mL/K)
40.0	273.2	0.146
44.0	298.2	0.148
47.7	323.2	0.148
51.3	348.2	0.147

Charles' Law

- The volume and absolute temperature (K) of a gas are directly related
  - at constant mass & pressure



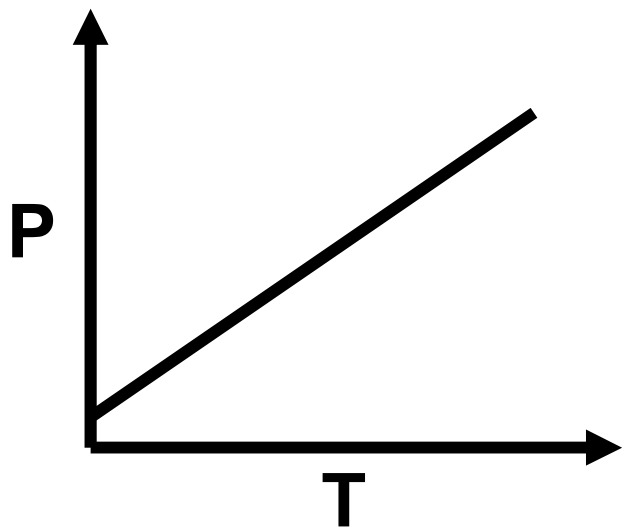
$$\frac{V}{T} = k$$

Gay-Lussac's Experiment

Temperature (K)	Pressure (torr)	P/T (torr/K)
248	691.6	2.79
273	760.0	2.78
298	828.4	2.78
373	1,041.2	2.79

Gay-Lussac's Law

- The pressure and absolute temperature (K) of a gas are directly related
  - at constant mass & volume



$$\frac{P}{T} = k$$

Combined Gas Law

$$\frac{PV}{T} = k$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$P_1V_1T_2 = P_2V_2T_1$$

Ideal Gas

We can combine the three relations just derived into a single relation:

$$PV \propto T$$

What about the amount of gas present? If the temperature and pressure are constant, the volume is proportional to the amount of gas:

$$PV \propto mT$$

Mole

A mole (mol) is defined as the number of grams of a substance that is numerically equal to the molecular mass of the substance:

1 mol H_2 has a mass of 2 g

1 mol Ne has a mass of 20 g

1 mol CO_2 has a mass of 44 g

The number of moles in a certain mass of material:

$$n\ (\text{mole}) = \frac{\text{mass (grams)}}{\text{molecular mass (g/mol)}}.$$

The Ideal Gas Law

We can now write the ideal gas law:

$$\boxed{PV = nRT,}$$

where n is the number of moles and R is the universal gas constant.

$$\begin{aligned} R &= 8.314\ \mathrm{J/(mol \cdot K)} && \text{[SI units]} \\ &= 0.0821\ \mathrm{(L \cdot atm)/(mol \cdot K)} \\ &= 1.99\ \mathrm{calories/(mol \cdot K)}. \end{aligned}$$

Gas Law Problems

- A gas occupies 473 cm^3 at 36°C. Find its volume at 94°C.
- A gas occupies 100. mL at 150. kPa. Find its volume at 200. kPa.
- A gas occupies 7.84 cm^3 at 71.8 kPa & 25°C. Find its volume at STP.

Problem Solving with the Ideal Gas Law

Useful facts and definitions:

- Standard temperature and pressure (STP)

$$T = 273\ \text{K}\ (0°\text{C})$$

$$P = 1.00\ \text{atm} = 1.013 \times 10^5\ \text{N/m}^2 = 101.3\ \text{kPa}$$

- Volume of 1 mol of an ideal gas is 22.4 L
- If the amount of gas does not change:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}. \qquad [\text{fixed } n]$$

- Always measure *T* in kelvins
- *P* must be the absolute pressure

Avogadro's Number

Since the gas constant is universal, the number of molecules in one mole is the same for all gases. That number is called Avogadro's number:

$$N_{\mathrm{A}} = 6.02 \times 10^{23}$$

The number of molecules in a gas is the number of moles times Avogadro's number:

$$N = nN_{\mathrm{A}}$$

Ideal Gas Law in Terms of Molecules: Avogadro's Number

Therefore we can write:

$$PV = nRT = \frac{N}{N_A} RT,$$

or

$$\boxed{PV = NkT,}$$

where *k* is called Boltzmann's constant.

$$k = \frac{R}{N_A} = \frac{8.314\ \mathrm{J/mol \cdot K}}{6.02 \times 10^{23}/\mathrm{mol}} = 1.38 \times 10^{-23}\ \mathrm{J/K}.$$

Basic Physics 1
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Thermodynamics, Entropy, Free Energy

Thermodynamics, Entropy, Free Energy

- Zeroth Law of Thermodynamics
- First Law of Thermodynamics
- Entropy
- Second Law of Thermodynamics
- Spontaneous & irreversible Process
- Hess Law
- Third Law of Thermodynamics
- Gibbs Free Energy

Zeroth Law of Thermodynamics

Two objects placed in thermal contact will eventually come to the same temperature. When they do, we say they are in thermal equilibrium.

The zeroth law of thermodynamics says that if two objects are each in equilibrium with a third object, they are also in thermal equilibrium with each other.

First Law of Thermodynamics

- Energy cannot be created nor destroyed.
- Therefore, the total energy of the universe is a constant.
- Energy can, however, be converted from one form to another or transferred from a system to the surroundings or vice versa.

Entropy

- *Entropy* (*S*) is a term coined by Rudolph Clausius in the 19th century.
- Clausius was convinced of the significance of the ratio of heat delivered and the temperature at which it is delivered.
- Entropy can be thought of as a measure of the randomness of a system. It is related to the various modes of motion in molecules.
- It is symbolized by <u>S</u>.

More about Entropy

- Entropy is a measure of randomness or disorder of a system. So, if there is a increase in disorder, ΔS is positive

- Some Examples of an increase of entropy
  - Diffusion – the process of dispersion
  - Reduction of pressure of a gas
  - Production of more gas in a chemical reaction
  - Total number of moles of products is greater than the number of moles of reactants [Example:
    $2NI_3\ (s) \rightarrow N_2\ (g) + 3I_2\ (g)$]

Calculation of Entropy

- For a process occurring at constant temperature (an isothermal process): q_{rev} = the heat that is transferred when the process is carried out **reversibly** at a constant temperature. T = temperature in Kelvin.

$$\Delta S = \frac{q_{rev}}{T}$$

- Like total energy, *E*, and enthalpy, *H*, entropy is a state function. Therefore,

$$\Delta S = S_{\text{final}} - S_{\text{initial}}$$

Entropy on the Molecular Scale

- Ludwig Boltzmann described the concept of entropy on the molecular level.

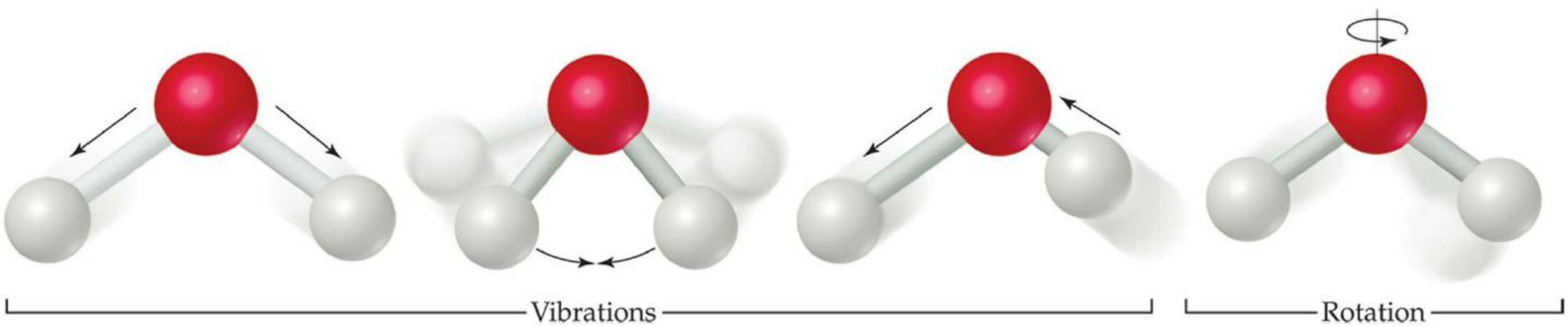
- Temperature is a measure of the average kinetic energy of the molecules in a sample.

- Boltzmann envisioned the motions of a sample of molecules at a particular instant in time.
  - This would be akin to taking a snapshot of all the molecules.

- He referred to this sampling as a microstate of the thermodynamic system.

Motion of Molecules

- Molecules exhibit several types of motion:
  - Translational: Movement of the entire molecule from one place to another.
  - Vibrational: Periodic motion of atoms within a molecule.
  - Rotational: Rotation of the molecule on about an axis or rotation about σ bonds.



Physical State

- Each thermodynamic state has a specific number of microstates, W, associated with it.

- Entropy is

$$S = k \ln W$$

where k is the Boltzmann constant, 1.38×10^{-23} J/K.

- Entropy increases with the freedom of motion of molecules. Therefore, $S(g) > S(l) > S(s)$

- The number of microstates and, therefore, the entropy tends to increase with increases in
  – Temperature.
  – Volume (gases).
  – The number of independently moving molecules.

Implication

- more particles

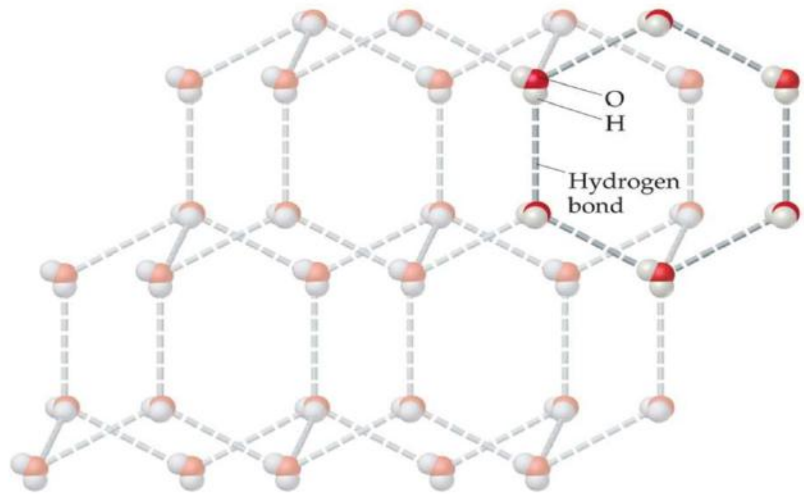
  -> more states -> more entropy

- higher T

  -> more energy states -> more entropy

- less structure (gas vs solid)

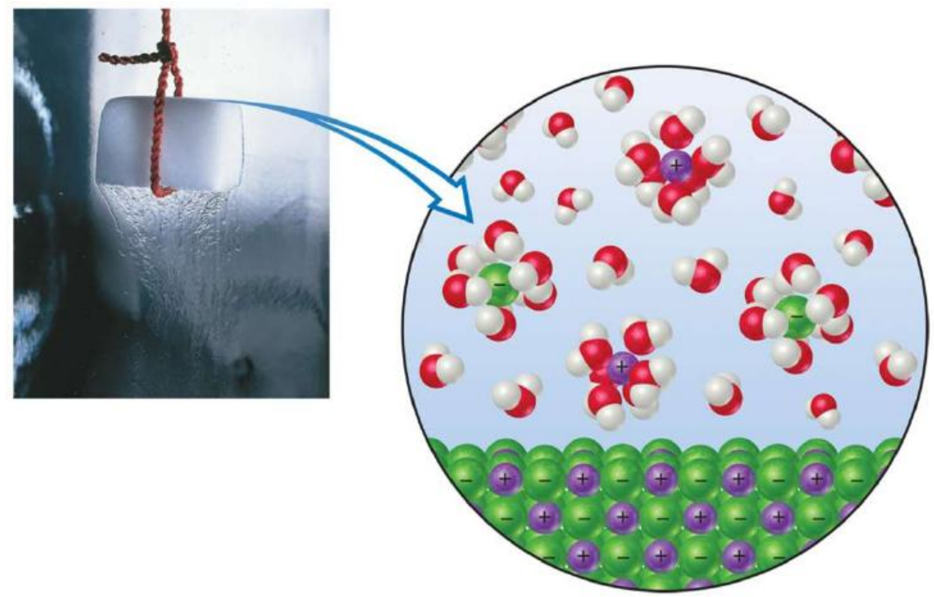
  -> more states -> more entropy



Dissolution

Dissolution of a solid:

– Ions have more entropy (more states) *But,* Some water molecules have less entropy (they are grouped around ions).



Usually, there is an overall increase in S.
(The exception is very highly charged ions that make a lot of water molecules align around them.)

Second Law of Thermodynamics

- The entropy of the universe does not change for reversible processes and increases for spontaneous processes.
- The entropy of the universe increases (real, spontaneous processes). But, entropy can decrease for individual systems.

Reversible (ideal):

$$\Delta S_{univ} = \Delta S_{system} + \Delta S_{surroundings} = 0$$

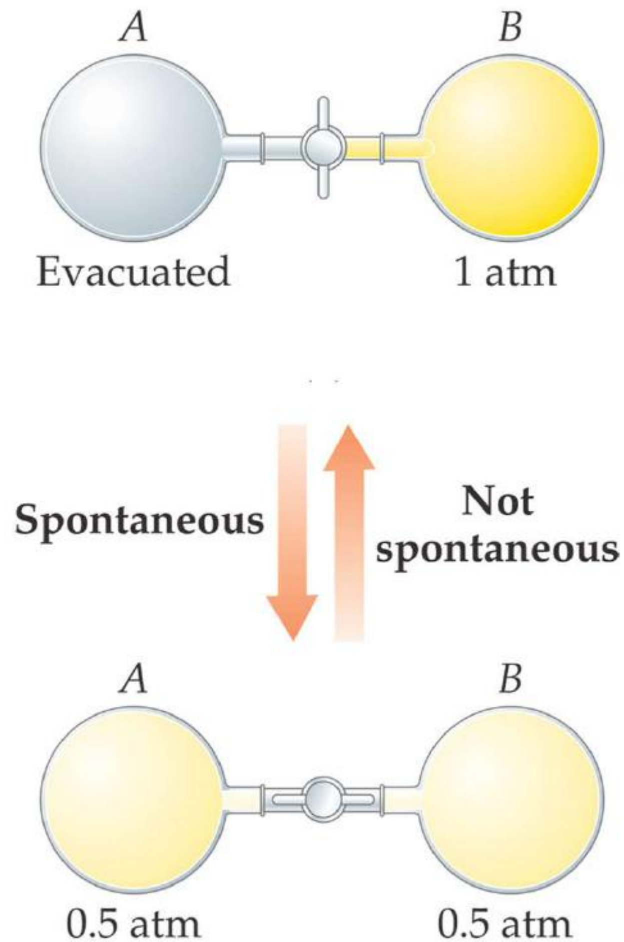
Irreversible (real, spontaneous):

$$\Delta S_{univ} = \Delta S_{system} + \Delta S_{surroundings} > 0$$

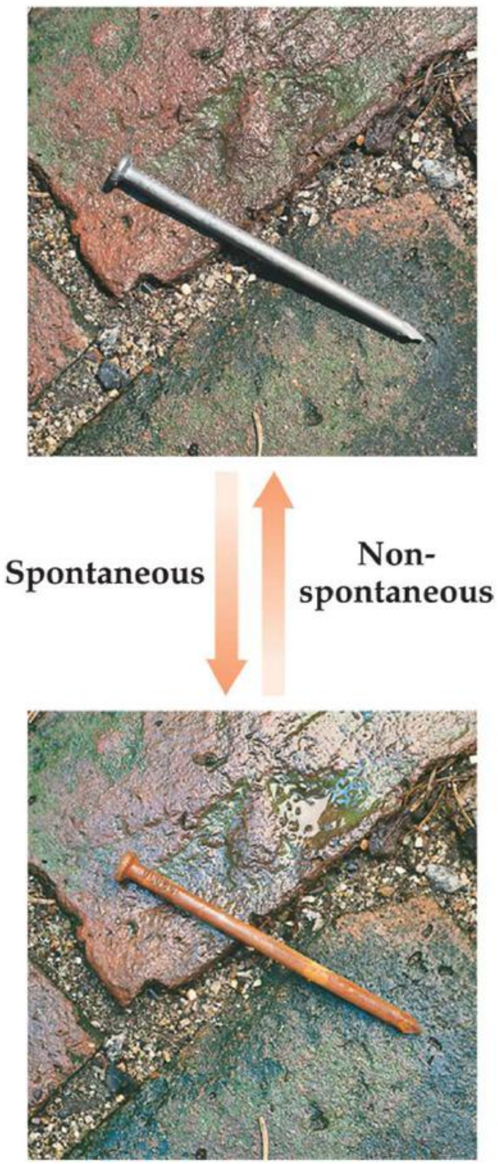
Spontaneous Processes

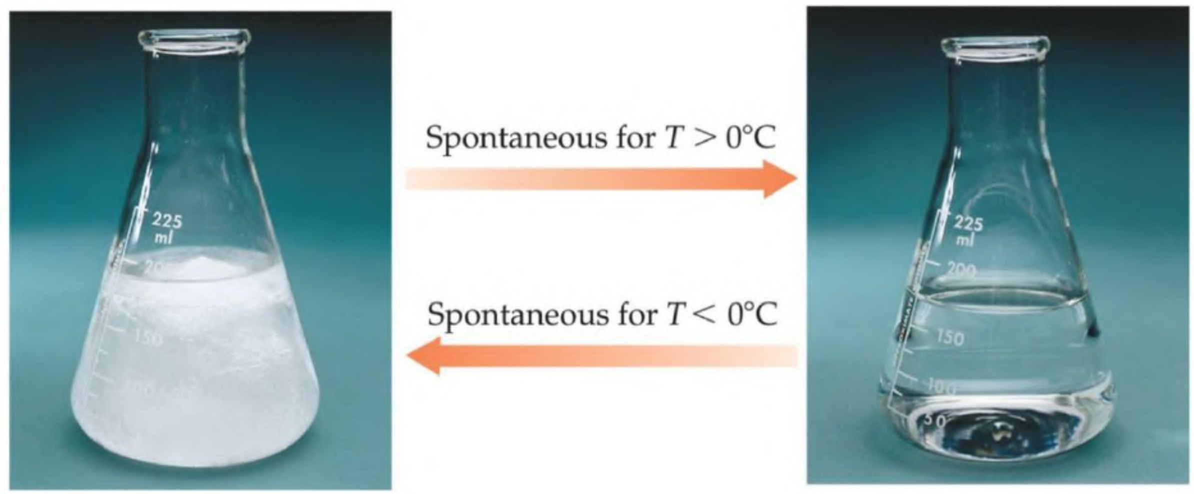
- Spontaneous processes are those that can proceed without any outside intervention.

- The gas in vessel *B* will spontaneously effuse into vessel *A*, but once the gas is in both vessels, it will *not* spontaneously



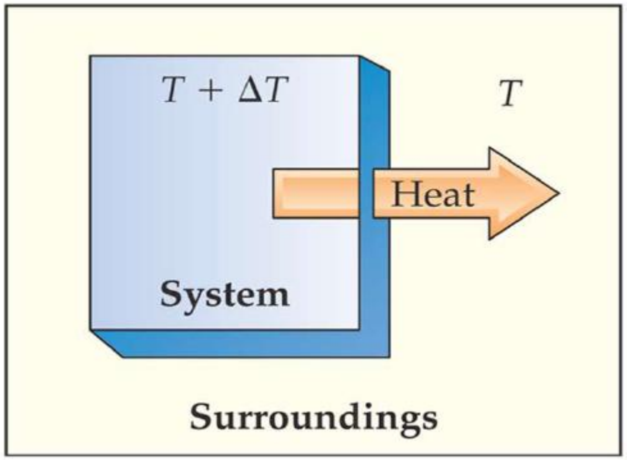
Illustration

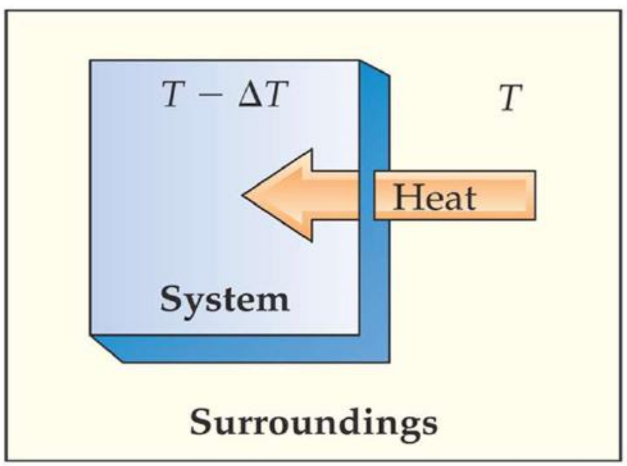




Processes that are spontaneous in one direction are nonspontaneous in the reverse direction.

Reversible Processes

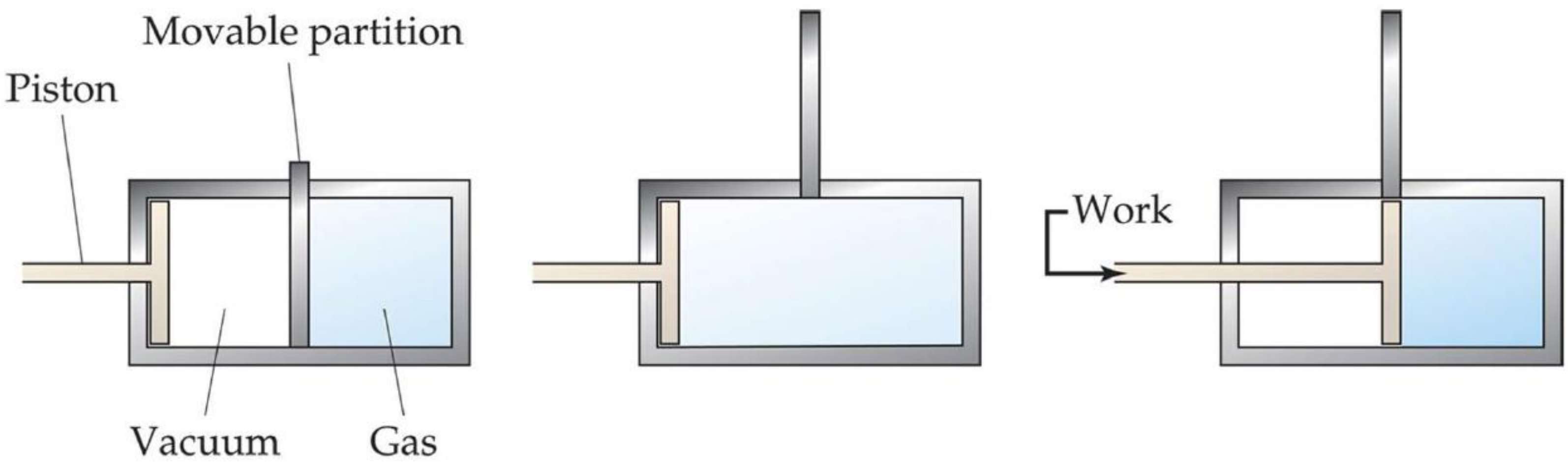




- In a reversible process the system changes in such a way that the system and surroundings can be put back in their original states by exactly reversing the process.

- Changes are infinitesimally **small** in a reversible process.

Illustration



- Irreversible processes cannot be undone by exactly reversing the change to the system.
- All **Spontaneous** processes are **irreversible**.
- All **Real** processes are **irreversible**.

Hess's Law and Entropy

- Hess's law can also be applied to entropy in the same way as it is applied to enthalpy.

- Therefore, $\Delta S_{reaction} = \Delta S_{products} - \Delta S_{reactants}$

- Example:

  Calculate the change in entropy for the following reaction using values from page 360 and that CH_3OH has an S^0 of 126.8 J/mol·K:

$$CO\ (g) + 2H_2\ (g) \rightarrow CH_3OH\ (l)$$

How can we tell if a reaction will be spontaneous?

- Spontaneous means that a reaction will occur without continuous outside assistance.
- Well, if there is a decrease in enthalpy, ΔH is negative. This helps it to be spontaneous.
- If there is an increase in entropy, ΔS is positive, that helps too.
- A higher temperature also helps.
- Josiah Willard Gibbs came up with an answer by putting these facts together. He came up with what is now known as Gibbs energy, or free energy, or Gibbs free energy

Gibbs Energy

- The equation is $\Delta G = \Delta H - T\Delta S$,

  where ΔG is the change in Gibbs energy, ΔH is the change in enthalpy, T is the temperature, and ΔS is the change in entropy
- If ΔG is a negative number, the reaction will be spontaneous.
- If ΔG is a positive number, the reaction will be nonspontaneous.
- If ΔG is a zero, the reaction is at equilibrium.

Relating Enthalpy and Entropy Changes to Spontaneity

ΔH	ΔS	ΔG	Spontaneous?
Negative	Positive	Negative	Yes, at all temperatures
Negative	Negative	Either	Only if $T < \Delta H / \Delta S$
Positive	Positive	Either	Only if $T > \Delta H / \Delta S$
Positive	Negative	Positive	Never

Example

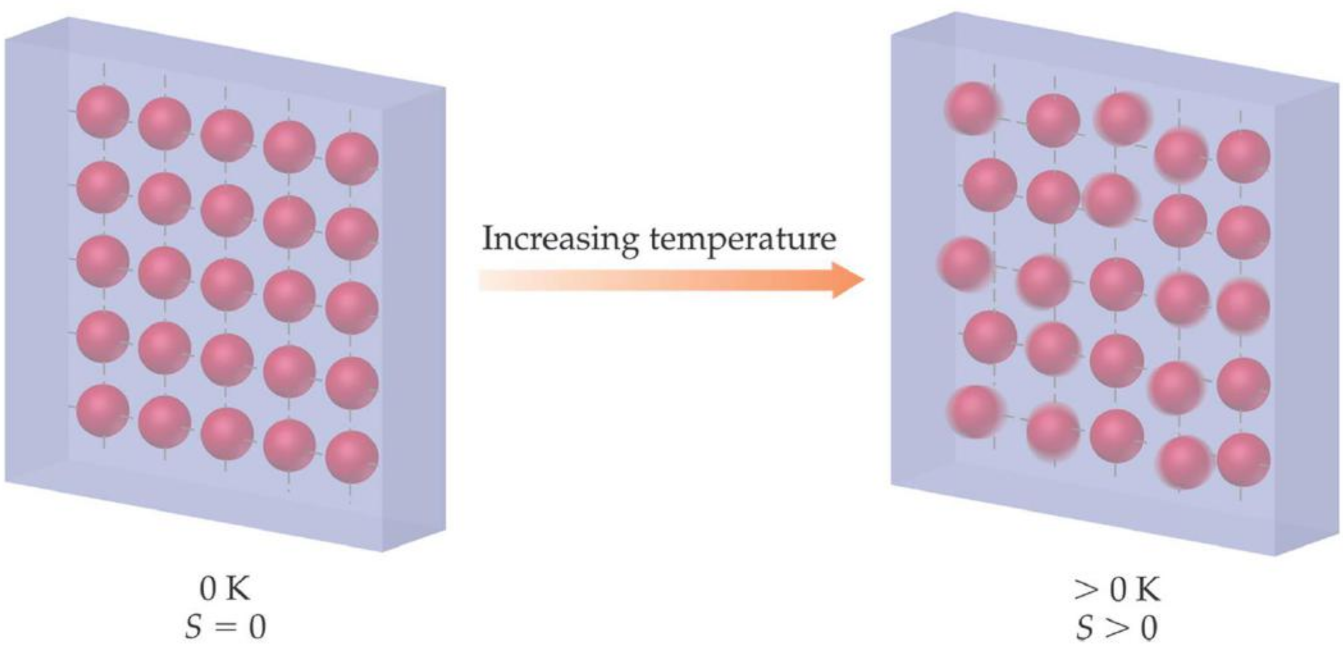
- Using the following values, compute the ΔG value for each reaction and predict whether they will occur spontaneously.

Reaction	ΔH (kJ)	Temperature	ΔS (J/K)
1	+95	298 K	+45
2	-96.1	157 K	+119
3	-266	400 °C	+54

Third Law of Thermodynamics

The entropy of a pure crystalline substance at absolute zero is 0.

$$S = k\ lnW = k\ ln1 = 0$$



Standard Entropies

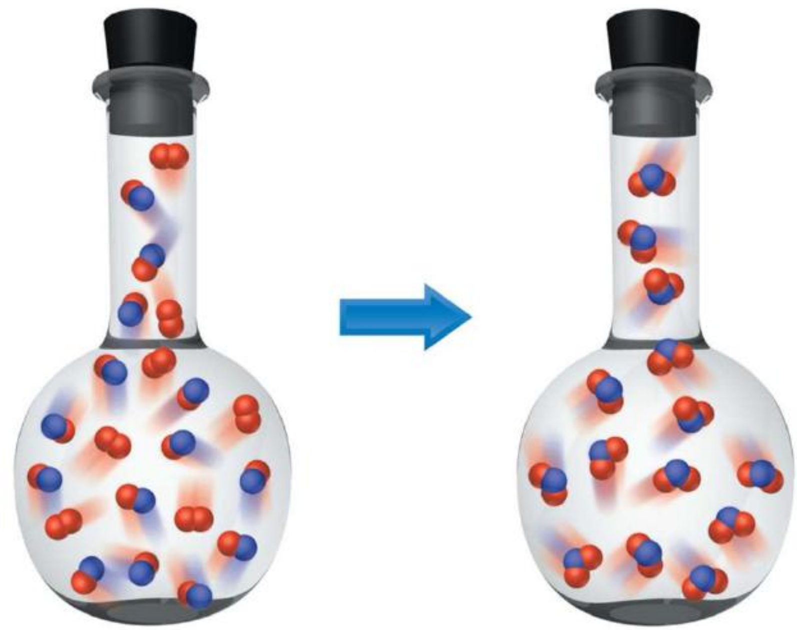
- These are molar entropy values of substances in their standard states.

- Standard entropies tend to increase with increasing molar mass.

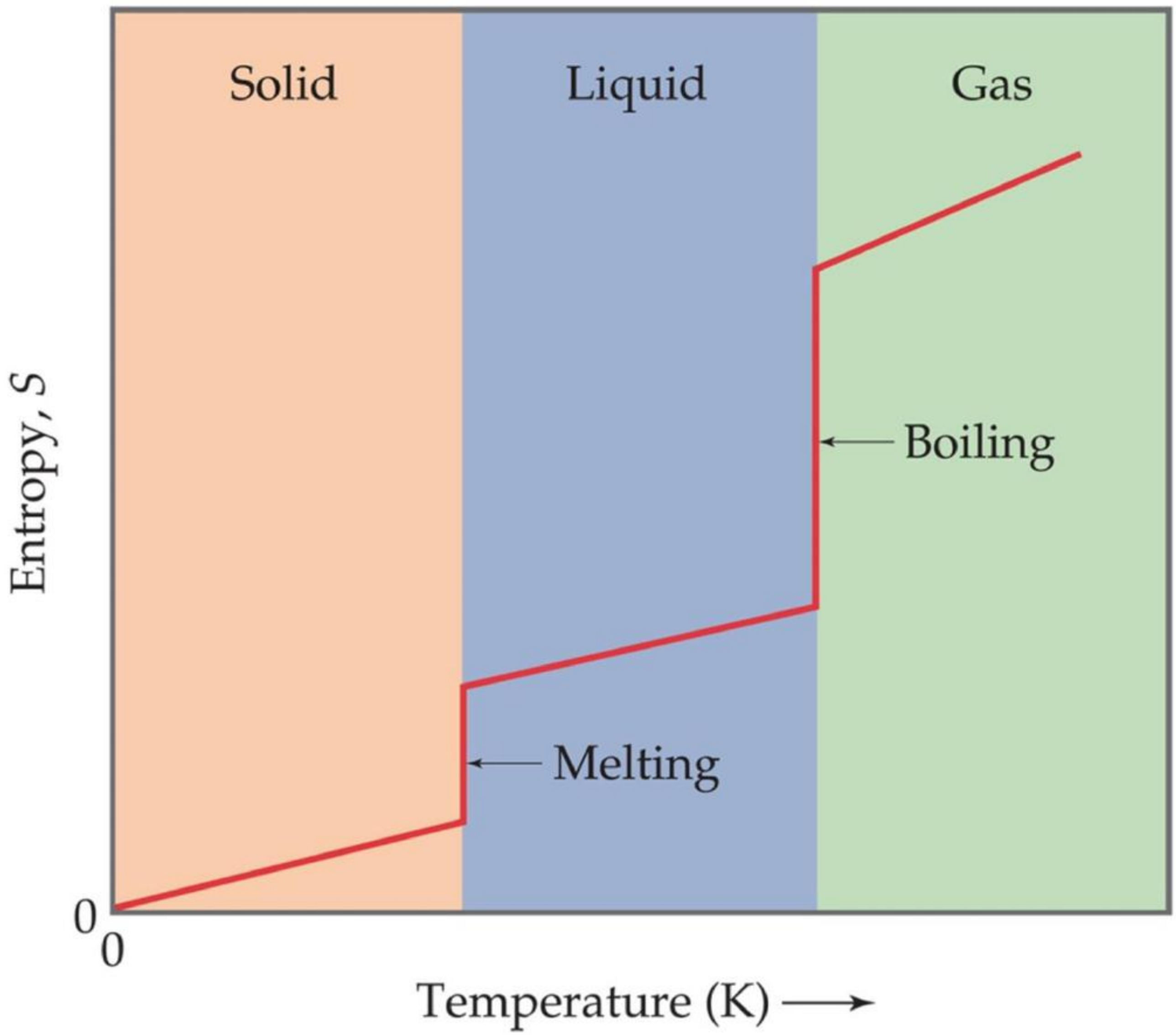
Substance	$S°$, J/mol-K
Gases	
$H_2(g)$	130.6
$N_2(g)$	191.5
$O_2(g)$	205.0
$H_2O(g)$	188.8
$NH_3(g)$	192.5
$CH_3OH(g)$	237.6
$C_6H_6(g)$	269.2
Liquids	
$H_2O(l)$	69.9
$CH_3OH(l)$	126.8
$C_6H_6(l)$	172.8
Solids	
$Li(s)$	29.1
$Na(s)$	51.4
$K(s)$	64.7
$Fe(s)$	27.23
$FeCl_3(s)$	142.3
$NaCl(s)$	72.3

Entropy Changes

- In general, entropy *increases* when
  - Gases are formed from liquids and solids.
  - Liquids or solutions are formed from solids.
  - The number of gas molecules increases.
  - The number of moles increases.



Entropy Phase Changes



Entropy Changes Calculation

Entropy changes for a reaction can be calculated the same way we used for ΔH*:*

$$\Delta S_{rxn} = \Sigma S^{\circ}_{reactants} - \Sigma S^{\circ}_{products}$$

S° for each component is found in a table.

Note for pure elements:

$$S^{\circ} \neq 0$$

$$\Delta H^{\circ} = 0$$

Changes in Surroundings

- Heat that flows into or out of the system also changes the entropy of the surroundings.
- For an isothermal process:

$$\Delta S_{surr} = \frac{-q_{sys}}{T}$$

- At constant pressure, q_{sys} is simply ΔH° for the system.

$$\Delta S_{surr} = \frac{-q_{sys}}{T} = \frac{-\Delta H^\circ}{T}$$

Link S and ΔH: Phase changes

$$\Delta S_{surr} = \frac{-q_{sys}}{T} = \frac{-\Delta H^{\circ}_{sys}}{T}$$

A phase change is isothermal
(no change in T).

For water:

ΔH_{fusion} = 6 kJ/mol
ΔH_{vap} = 41 kJ/mol

If we do this reversibly: $\Delta S_{surr} = -\Delta S_{sys}$

Entropy Change in the Universe

$$\Delta S_{universe} = \Delta S_{system} + \Delta S_{surround}$$

$$\Delta S_{surr} = \frac{-\Delta H^{\circ}_{sys}}{T}$$

$$\Delta S_{universe} = \Delta S_{system} + \frac{-\Delta H^{\circ}_{sys}}{T}$$

$$\underbrace{T\Delta S_{universe}}_{} = T\Delta S_{system} + -\Delta H^{\circ}_{sys}$$

= – Gibbs Free Energy

$$\underbrace{T\Delta S_{universe}}_{= -\,\text{Gibbs Free Energy}} = T\Delta S_{system} + -\Delta H^{\circ}_{sys}$$

= – Gibbs Free Energy

Make this equation nicer:

$$-T\Delta S_{universe} = \Delta H^{\circ}_{sys} - T\Delta S_{system}$$

$$\boxed{\Delta G = \Delta H^{\circ}_{sys} - T\Delta S_{system}}$$

- $T\Delta S_{\text{universe}}$ is defined as the Gibbs free energy, ΔG.

- For spontaneous processes: $\boldsymbol{\Delta S_{\text{universe}} > 0}$

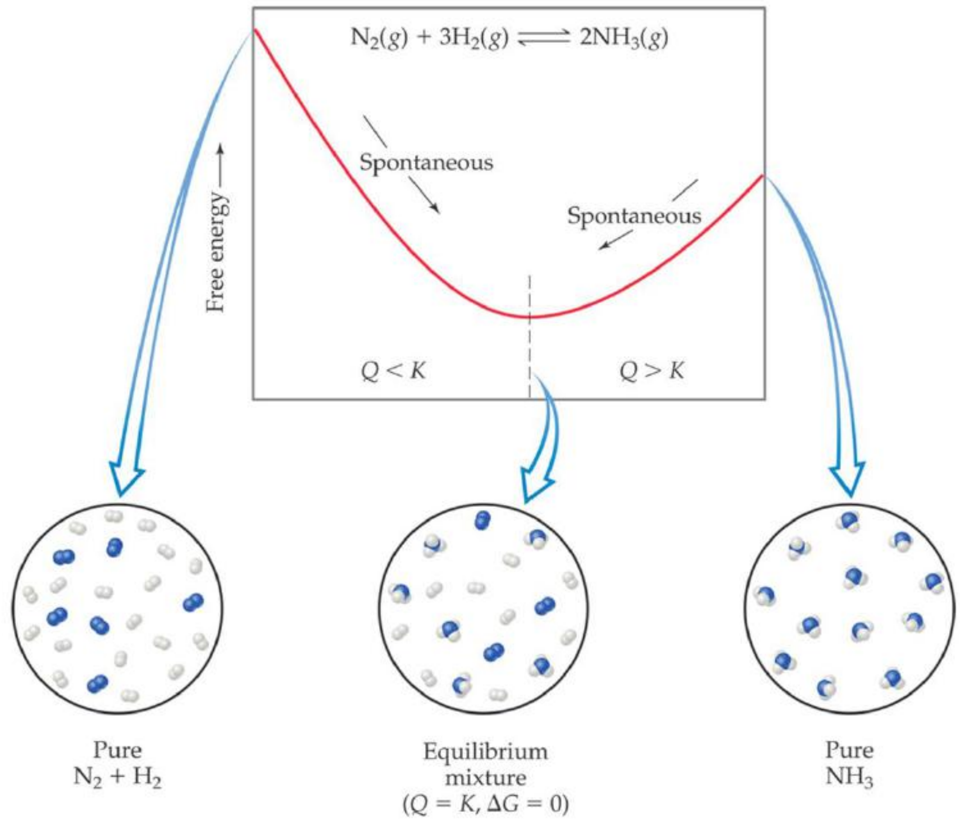
And therefore: $\boldsymbol{\Delta G < 0}$

- *ΔG is easier to determine than $\Delta S_{universe}$.*

- Use ΔG to decide if a process is spontaneous.

Gibbs Free Energy

1. If ΔG is negative, the forward reaction is spontaneous.
2. **If ΔG is 0, the system is at equilibrium.**
3. If ΔG is positive, the reaction is spontaneous in the reverse direction.



Standard Free Energy Changes

- Standard free energies of formation, ΔG_f° are analogous to standard enthalpies of formation, ΔH_f°.

$$\Delta G_f^\circ = \Sigma \Delta G^\circ_{reactants} - \Sigma \Delta G^\circ_{products}$$

- ΔG° can be looked up in tables, *or* calculated from S° and ΔH°.

Free Energy Changes

$$\boxed{\Delta G = \Delta H^{\circ}_{sys} - T\Delta S_{system}}$$

- This equation shows how ΔG° changes with temperature. (We assume S° & ΔH° are independent of T)

- There are two parts to the free energy equation:
  - ΔH°— the enthalpy term
  - $T\Delta S^{\circ}$ — the entropy term

- The temperature dependence of free energy comes from the entropy term.

Free Energy and Temperature

ΔH	ΔS	$-T\Delta S$	$\Delta G = \Delta H - T\Delta S$	Reaction Characteristics	Example
−	+	−	−	Spontaneous at all temperatures	$2\,O_3(g) \longrightarrow 3\,O_2(g)$
+	−	+	+	Nonspontaneous at all temperatures	$3\,O_2(g) \longrightarrow 2\,O_3(g)$
−	−	+	+ or −	Spontaneous at low *T*; nonspontaneous at high *T*	$H_2O(l) \longrightarrow H_2O(s)$
+	+	−	+ or −	Spontaneous at high *T*; nonspontaneous at low T	$H_2O(s) \longrightarrow H_2O(l)$

- There are two parts to the free energy equation:
  - ΔH°— the enthalpy term
  - $T\Delta S^\circ$ — the entropy term

- The temperature dependence of free energy comes from the entropy term.

- By knowing the sign (+ or -) of ΔS and ΔH, we can get the sign of ΔG and determine if a reaction is spontaneous.

Free Energy and Equilibrium

- If ΔG is 0, the system is at equilibrium. So ΔG must be related to the equilibrium constant K . The *standard* free energy, $\Delta G°$, is directly linked to K_{eq} by:

$$\Delta G^\circ = -RT \ln K$$

- Under non-standard conditions, we need to use $\Delta \boldsymbol{G}$ instead of $\Delta \boldsymbol{G°}$.

$$\Delta G^\circ = -RT \ln K$$

$$\Delta G = \Delta G^\circ + RT \ln Q$$

Basic Physics 1
Lecture Module

Rahadian N, S.Si. M.Si.

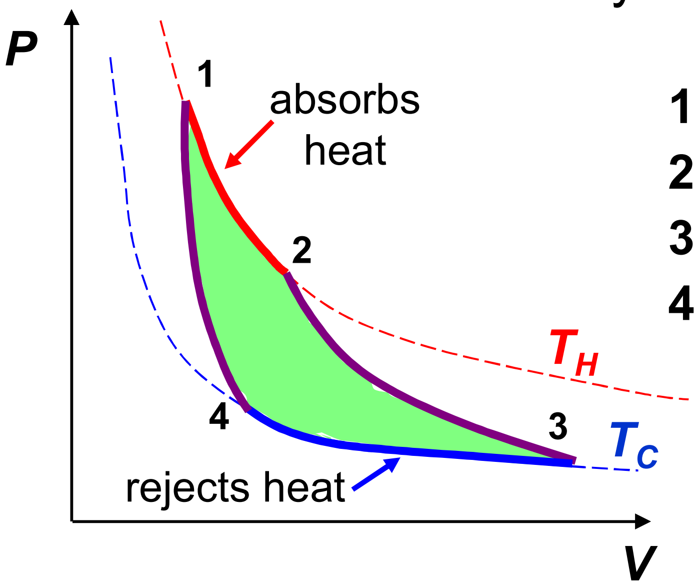
Carnot, Heat Engine, Refrigeration & Air Conditioning

Carnot, Heat Engine, Refrigeration & Air Conditioning

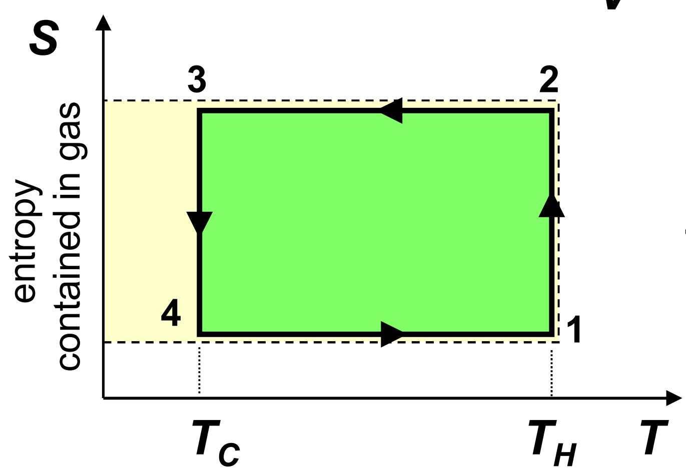
- Carnot
- Heat Engine
- Refrigeration
- Air Conditioning

Carnot

- is not very practical (too slow), but operates at the ***maximum*** efficiency allowed by the Second Law.



1 – 2 isothermal expansion (in contact with T_H)
2 – 3 isentropic expansion to T_C
3 – 4 isothermal compression (in contact with T_C)
4 – 1 isentropic compression to T_H
(isentropic ≡ adiabatic+quasistatic)



On the ***S*** **-*T*** diagram, the work done is the area of the loop:

$$\oint dU = 0 = \oint TdS - \oint PdV$$

The heat consumed at T_H (**1 – 2**) is the area surrounded by the broken line:

$$Q_H = T_H\left(S_H - S_C\right)$$

Carnot Engine

- The efficiency of a typical automobile engine is less than 30%.
  - This seems to be wasting a lot of energy.
  - What is the best efficiency we could achieve?
  - What factors determine efficiency?

- The cycle devised by Carnot that an ideal engine would have to follow is called a ***Carnot cycle***.

- An (ideal, not real) engine following this cycle is called a ***Carnot engine***.

Carnot Efficiency

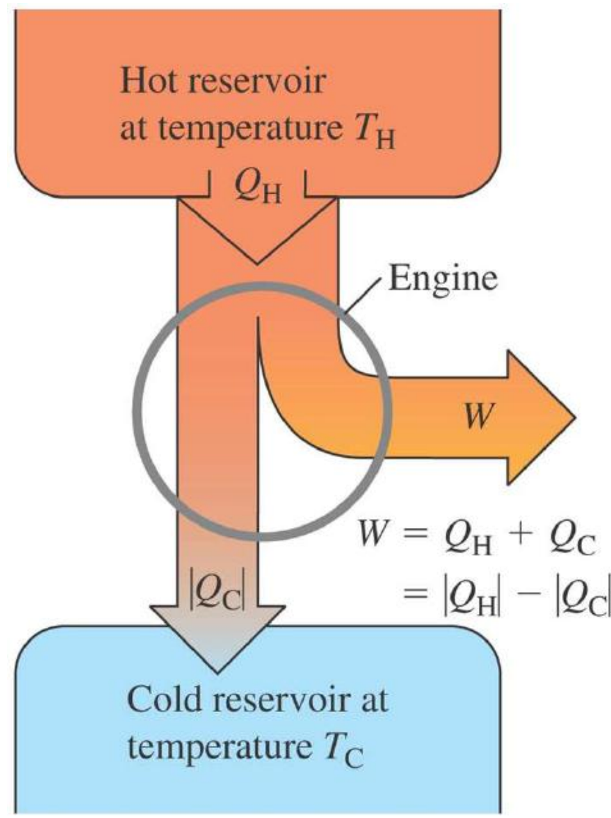
- The efficiency of Carnot's ideal engine is called the ***Carnot efficiency*** and is given by:

$$e_C = \frac{T_H - T_C}{T_H}$$

- This is the ***maximum efficiency possible*** for ***any*** engine taking in heat from a reservoir at absolute temperature T_H and releasing heat to a reservoir at temperature T_C.
  - The temperature must be measured in absolute degrees.
- Even Carnot's ideal engine is less than 100% efficient.

Heat engines

- A *heat engine* is any device that partly transforms heat into work or mechanical energy.
- Simple heat engines operate on a *cyclic process* during which they absorb heat Q_H from a hot reservoir and discard some heat Q_C to a cold reservoir.
- Picture at the right shows a schematic energy-flow diagram for a heat engine.



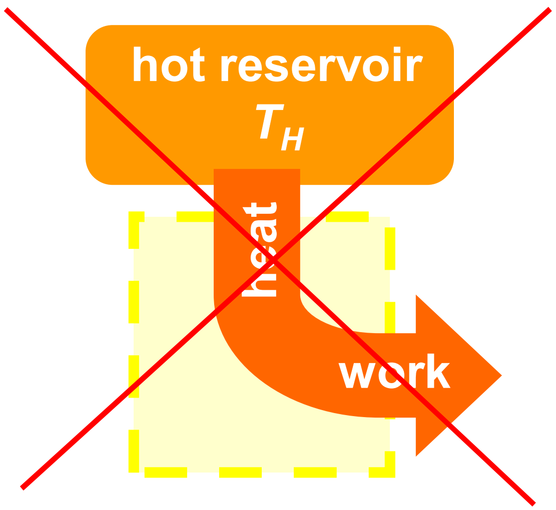
Perpetual Motion Machines

- Perpetual Motion Machines of the first type – these designs seek to create the energy required for their operation out of nothing.

violation of the First Law (energy conservation)

- Perpetual Motion Machines of the second type - these designs extract the energy required for their operation in a manner that decreases the entropy of an isolated system.

violation of the Second Law



impossible cyclic heat engine

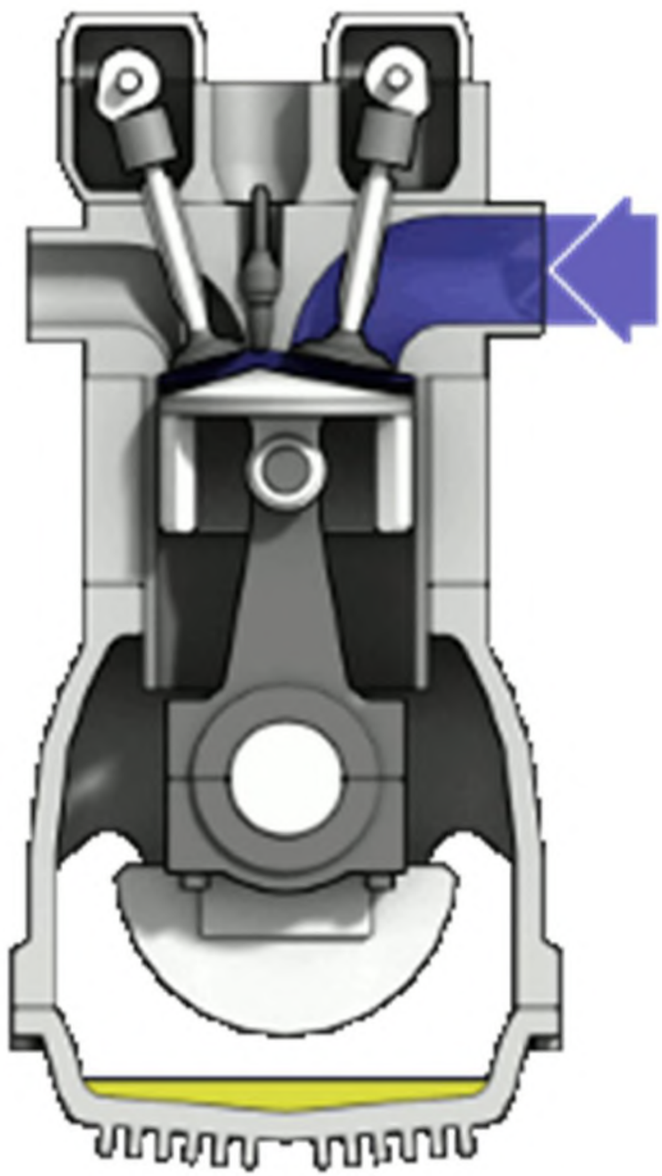
Word of caution: for *non-cyclic* processes, 100% of heat *can* be transformed into work without violating the Second Law.

Example: an ideal gas expands isothermally being in thermal contact with a hot reservoir. Since U = const at T = const, all heat has been transformed into work.

Mechanical Heat Engines

- **INTAKE stroke**:
  the piston descends from the top to the bottom of the cylinder, reducing the pressure inside. A mixture of fuel and air, is forced by atmospheric pressure into the cylinder through the intake port. The intake valve then close.
- **COMPRESSION stroke:**
  with both intake and exhaust valves closed, the piston returns to the top of the cylinder compressing the fuel-air mixture.
- **POWER stroke**:
  the compressed air–fuel mixture in a gasoline engine is ignited by a spark plug. The compressed fuel-air mixture expand and move the piston back
- **EXHAUST stroke:**
  during the *exhaust* stroke, the piston once again returns to top while the exhaust valve is open and expel the spent fuel-air mixture out through the exhaust valve(s).

1



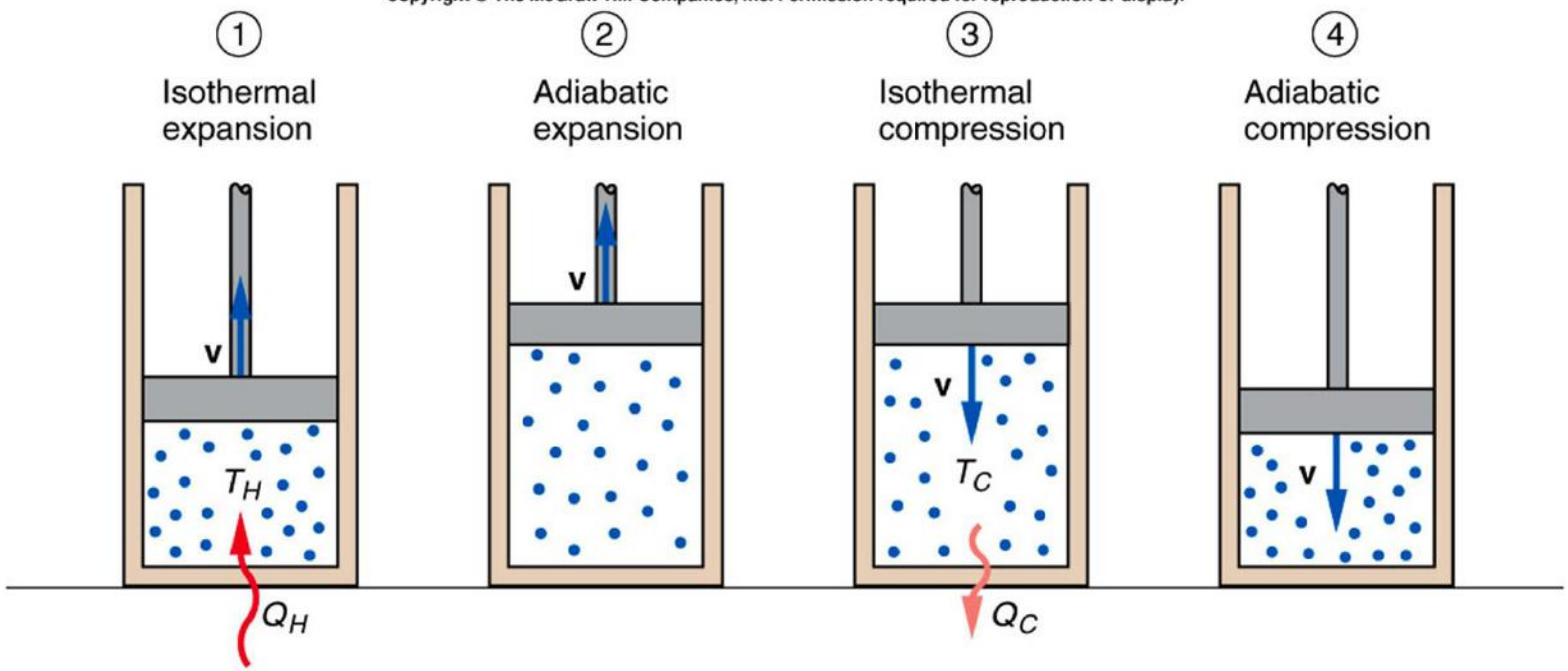
Thermal Process

- If the process is ***adiabatic***, no heat flows into or out of the gas

- In an ***isothermal*** process, the temperature does not change.
  - The internal energy must be constant.
  - The change in internal energy, ΔU, is zero.
  - If an amount of heat Q is added to the gas, an equal amount of work W will be done by the gas on its surroundings, from $\Delta U = Q - W$.

- In an ***isobaric*** process, the pressure of the gas remains constant.
  - The internal energy increases as the gas is heated, and so does the temperature.
  - The gas also expands, removing some of the internal energy.

1. Heat flows into cylinder at temperature T_H. The fluid ***expands isothermally*** and does work on the piston.
2. The fluid continues to ***expand, adiabatically***.
3. Work is done by the piston on the fluid, which undergoes an ***isothermal compression***.
4. The fluid returns to its initial condition by an ***adiabatic compression***.



Entropy Transfer of Heating & Work

Transferring purely mechanical energy to or from a system does not (necessarily) change its entropy: ΔS = 0 for reversible processes. For this reason, all forms of work are ***thermodynamically equivalent*** to each other - they are freely convertible into each other and, in particular, into mechanical work.

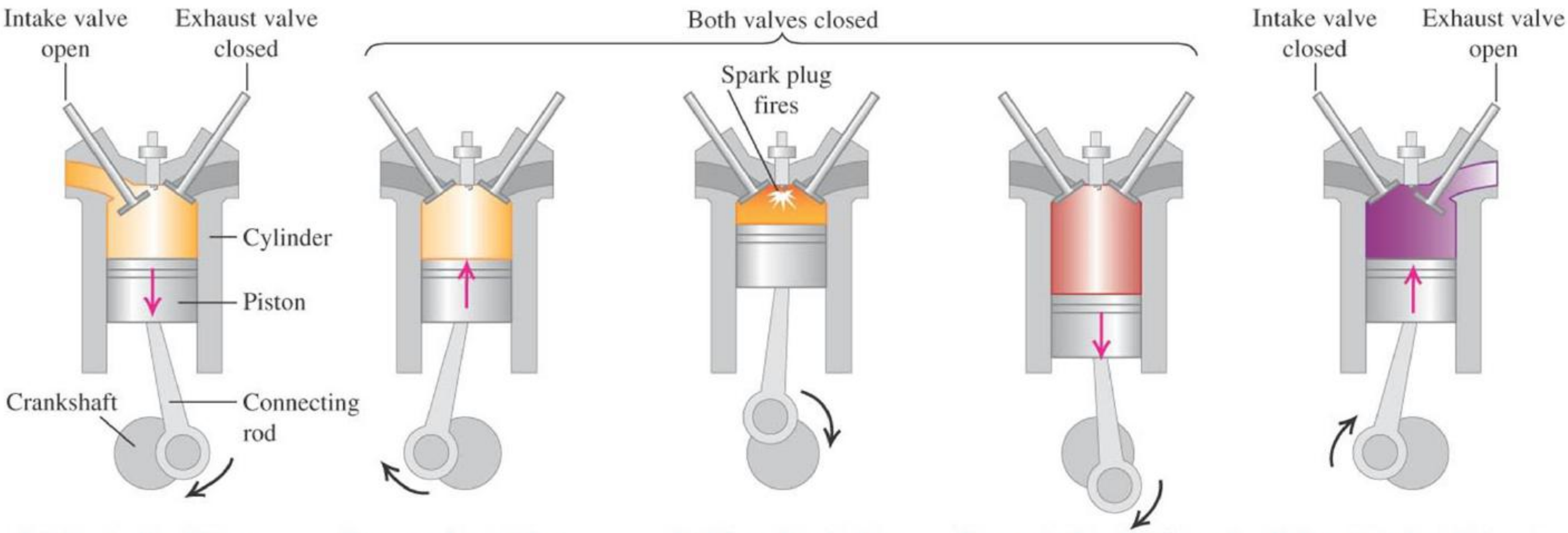
$$dS = \frac{\delta Q}{T}$$

Work can be completely converted into heat, but the inverse is not true. The transfer of energy by heating is accompanied with the entropy transfer

Thus, entropy enters the system with heating, but does not leave the system with the work. On the other hand, for a continuous operation of a heat engine, **the net entropy change during a cycle must be zero!** How is it possible???

Internal-combustion Engines

- Below illustrates a four-stroke internal-combustion engine. The *compression ratio r* is the ratio of the maximum volume to the minimum volume during the cycle.



Intake stroke: Piston moves down, causing a partial vacuum in cylinder; gasoline–air mixture enters through intake valve.

Compression stroke: Intake valve closes; mixture is compressed as piston moves up.

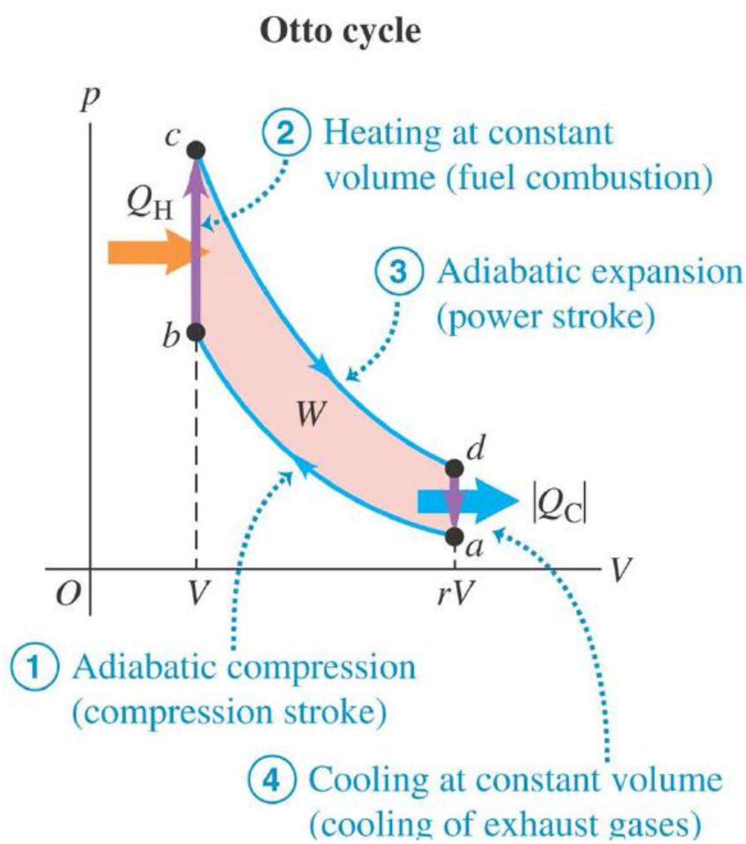
Ignition: Spark plug ignites mixture.

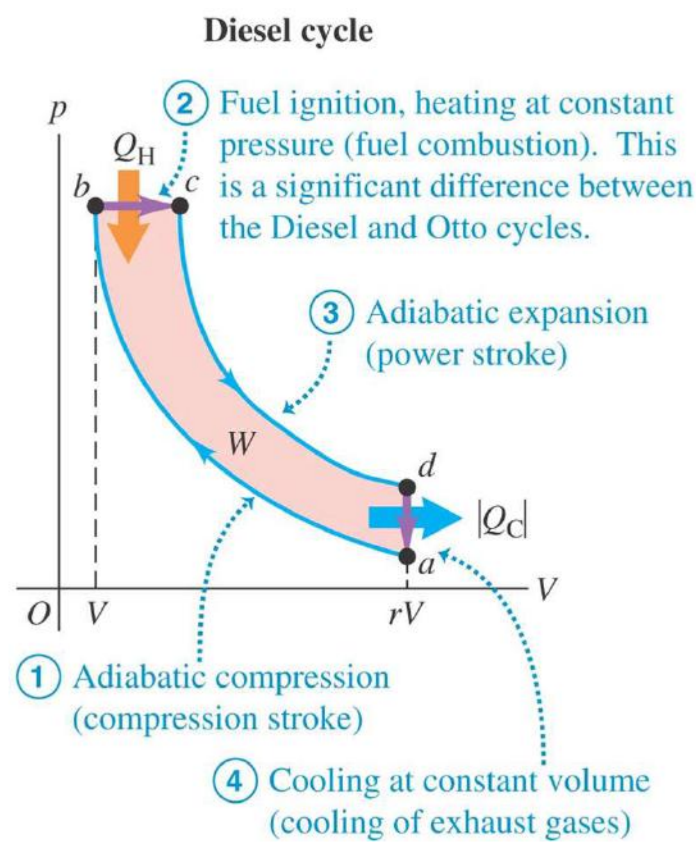
Power stroke: Hot burned mixture expands, pushing piston down.

Exhaust stroke: Exhaust valve opens; piston moves up, expelling exhaust and leaving cylinder ready for next intake stroke.

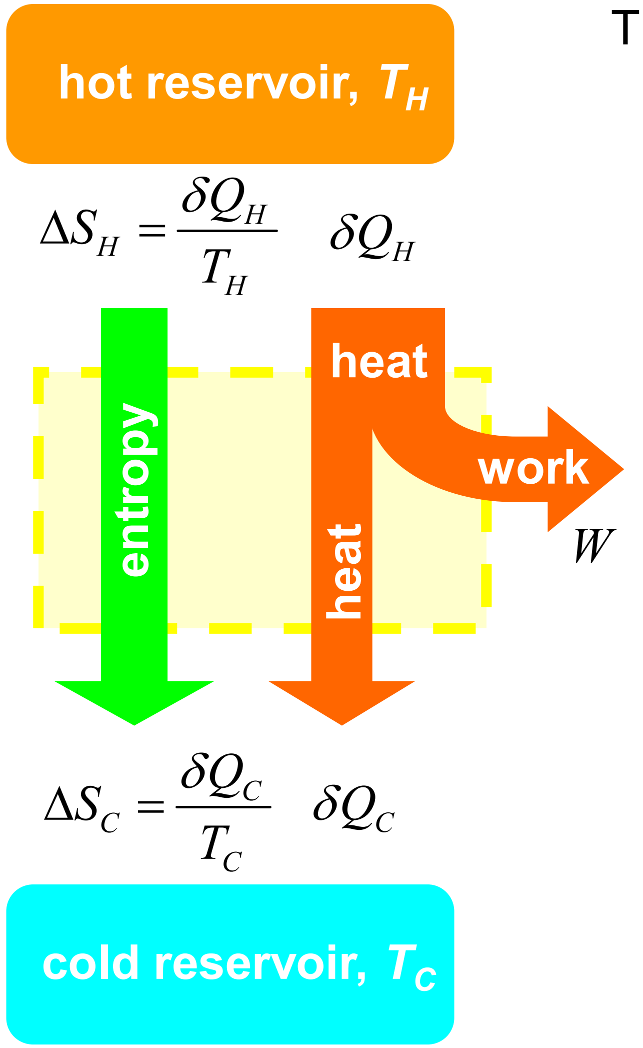
The Otto cycle and the Diesel cycle

- Below show *pV*-diagrams for idealized *Otto cycle* and *Diesel cycle* engines. In both cases, the efficiency depends on the compression ratio *r*.





Perfect Engines (no extra *S* generated)



The condition of continuous operation:

$$\Delta S_H = \Delta S_C \qquad \frac{\delta Q_H}{T_H} = \frac{\delta Q_C}{T_C}$$

$$\boxed{\delta Q_C = \frac{T_C}{T_H}\delta Q_H}$$

The work generated during one cycle of a reversible process:

$$\boxed{\delta W = \delta Q_H - \delta Q_C = \frac{T_H - T_C}{T_H}\delta Q_H}$$

Carnot efficiency:

the highest possible value of the energy conversion efficiency

$$\boxed{e_{\max} \equiv \frac{\delta W}{\delta Q_H} = 1 - \frac{T_C}{T_H} < 1}$$

Real Engines

Real heat engines have lower efficiencies because the processes within the devices are not perfectly reversible – the entropy will be generated by ***irreversible*** processes:

$$e = \frac{\delta W}{\delta Q_H} \leq 1 - \frac{T_C}{T_H} = e_{\max}$$

e = $\boldsymbol{e_{max}}$ only in the limit of ***reversible operation***. ***Some sources of irreversibility***:

- heat may flow directly between reservoirs;
- not all temperature difference $T_H - T_C$ may be available (temperature drop across thermal resistances in the path of the heat flow);
- part of the work generated may be converted to heat by friction;
- gas may expand irreversibly without doing work (as gas flow into vacuum).

The Price Should be Paid...

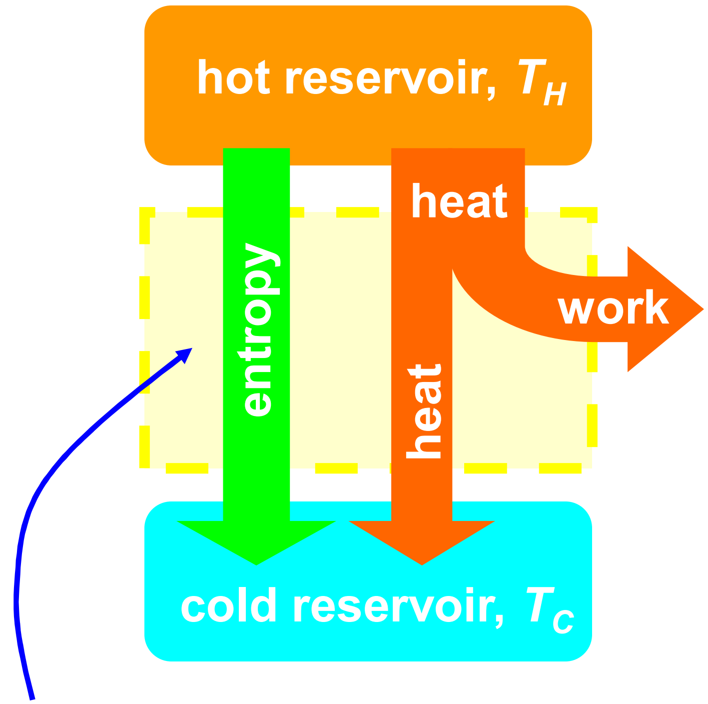
An engine can get rid of ***all*** the entropy received from the hot reservoir by transferring only ***part*** of the input thermal energy to the cold reservoir.

$$d\,S = \frac{\delta Q}{T}$$

Thus, the only way to get rid of the accumulating entropy is to absorb more internal energy in the heating process than the amount converted to work, and to "flush" the entropy with the flow of the waste heat out of the system.

An essential ingredient: a temperature difference between hot and cold reservoirs.

Essential parts of a heat engine (any continuously operating reversible device generating work from "heat")



"Working substance" – the system that absorbs heat, expels waste energy, and does work (e.g., water vapor in the steam engine)

Consequence

- Any difference $T_H - T_C \neq 0$ can be exploited to generate mechanical energy.
- The greater the $T_H - T_C$ difference, the more efficient the engine.
- Energy waste is ***inevitable***.

Example: In a typical nuclear power plant, $T_H = 300^0C$ (~570K), $T_C = 40^0C$ (~310K), and the maximum efficiency e_{max}=0.45. If the plant generates 1000 MW of "work", its waste heat production is at a rate

$$\delta Q_C = \delta Q_H - \delta W = \delta W \left(\frac{1}{e} - 1\right) \approx 1220 \text{ MW}$$

- more fuel is needed to get rid of the entropy then to generate useful power.

Efficiency

General definition: $\text{efficiency} = \dfrac{\text{benefit}}{\text{cost}}$

	benefit	cost	efficiency
heat engine	W	Q_h	W/Q_h
refrigerator	Q_c	W	Q_c/W
heat pump	Q_h	W	Q_h/W

Natural Refrigeration

- The natural method includes the utilization of ice or snow obtained naturally in cold climate. Ice melts at zero degree centigrade. So, when it is placed in a system or space warmer than that temperature, heat is absorbed by the ice and the space is cooled.

- The different methods of natural refrigeration include:
  - Use of ice transported from colder regions
  - Use of ice harvested in winter and stored in ice houses
  - Use of ice produced by nocturnal cooling
  - Use of evaporative cooling
  - Cooling by salt solution

Mechanical Refrigeration

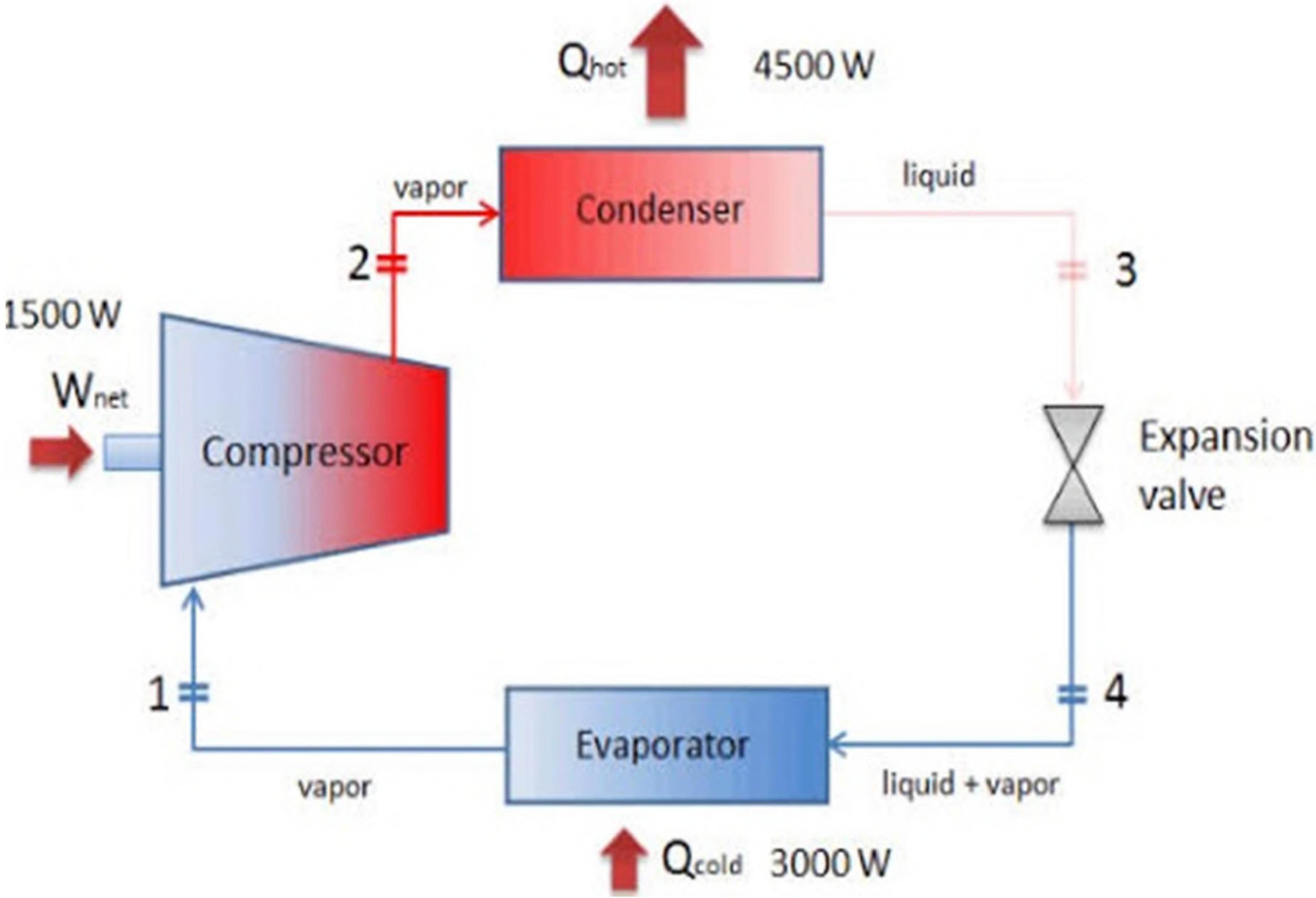
- This consists of a refrigeration cycle , where heat is removed from a low temperature space or source and rejected to a high temperature sink with the help of external work.

- Heat naturally flows from hot to cold. Work is applied to cool a living space or storage volume by pumping heat from a lower temperature heat source into a higher temperature heat sink. Different types of artificial refrigeration include: Vapor compression refrigeration, Vapor absorption refrigeration, Gas cycle refrigeration, Thermoelectric refrigeration, Magnetic refrigeration

Vapor compression refrigeration:

The vapor compression cycle is used in most household refrigerators as well as in many large commercial and industrial refrigeration systems.

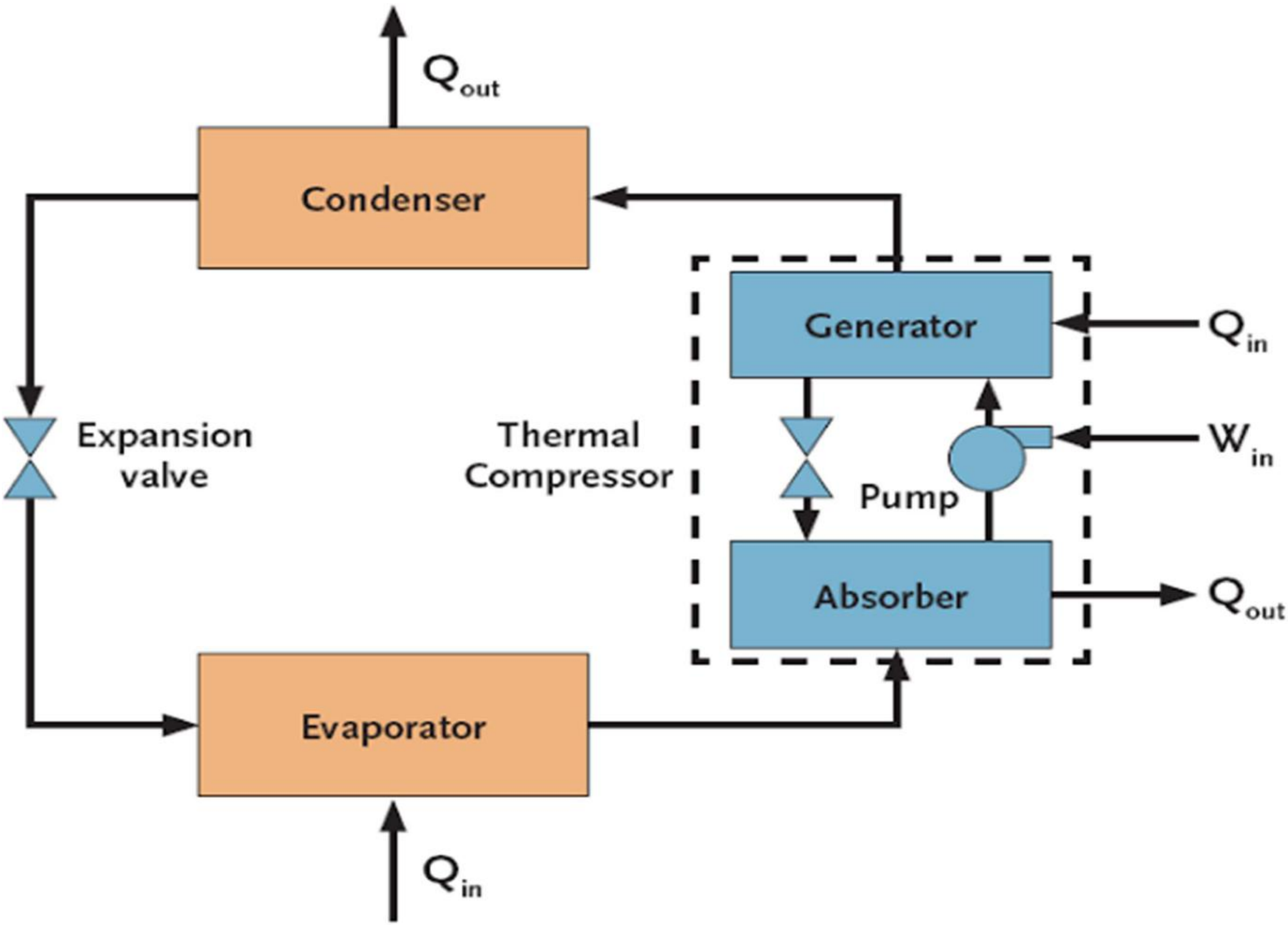
- In this method,a **circulating refrigerant** such as **Freon** enters the **compressor** as a vapor during which it is compressed at constant entropy and exits the compressor as a vapor at a higher temperature.
- This heated vapor travels through the condenser which cools the vapor and condenses it into a liquid by removing additional heat at constant temperature and pressure.
- This liquid refrigerant now goes through the expansion valve or throttle valve, where its pressure abruptly decreases,which results in a mixture of liquid and vapor at a lower temperature and pressure.
- This cold liquid vapor mixture then enters the evaporator coil and is completely vaporized by cooling the warm air being blown by a fan across the evaporator coil.
- The evaporator is the main component of the system that produces the cooling effect by extracting heat from the working space.
- The resulting refrigerant vapor returns to the compressor inlet and the cycle repeats.



Qhot
4500 W
vapor
Condenser
liquid
2
3
1500 W
Wnet
Compressor
Expansion
valve
1
4
Evaporator
vapor
liquid + vapor
Qcold
3000 W

Vapor Absorption Refrigeration

- This absorption cycle is almost similar to the compression cycle, except for the method of raising the pressure of refrigerant vapor.
- In this,the compressor is replaced by an absorber which dissolves the refrigerant in a suitable liquid.
- A liquid pump raises the pressure and a generator,which on heat addition,drives off the refrigerant vapor from the high pressure liquid.
- A suitable combination of refrigerant and absorbent is used in this method. The most common combinations are ammonia as a refrigerant with water as an absorbent and water as refrigerant with lithium bromide as an absorbent.



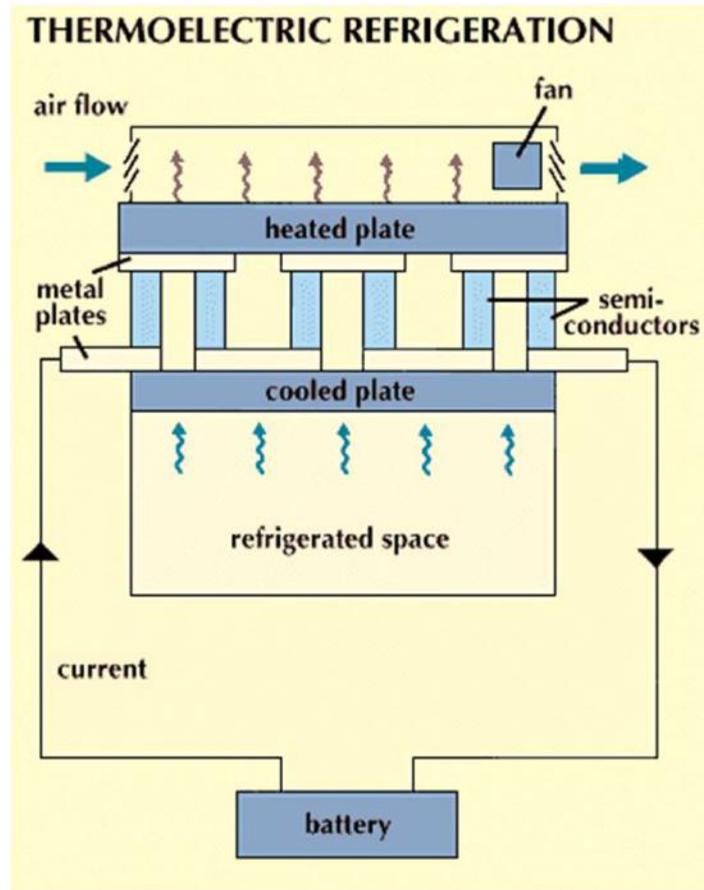
Q_{out}
Condenser
Generator
Q_{in}
Expansion valve
Thermal Compressor
W_{in}
Pump
Absorber
Q_{out}
Evaporator
Q_{in}

Gas Cycle Refrigeration

- In this, the working fluid is a gas that is compressed and expanded but doesn’t change phase.
- Air is most often the working fluid.
- As there is no condensation and evaporation, the components corresponding to the condenser and evaporator are the hot and cold gas-to-gas heat exchangers.

Thermoelectric Refrigeration

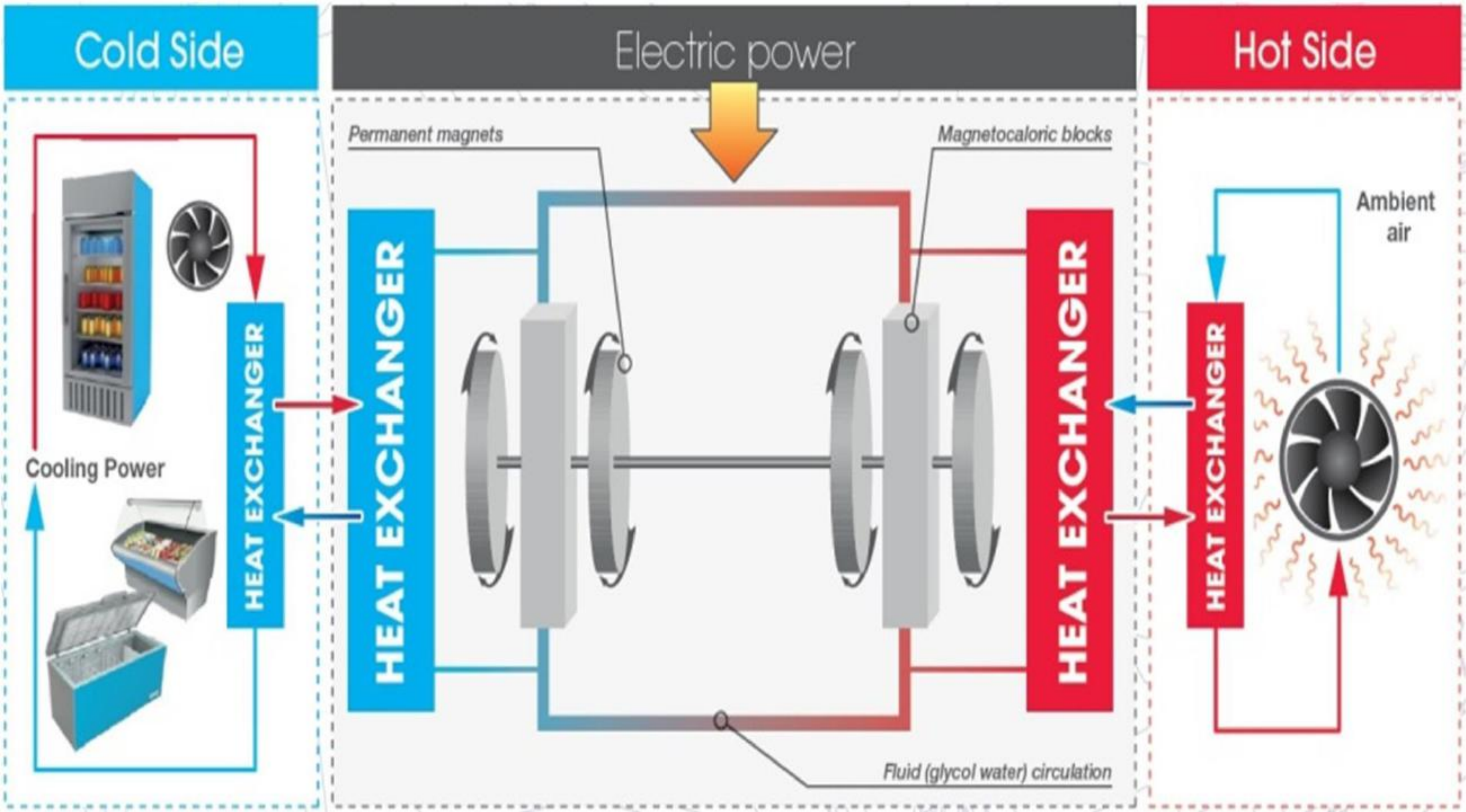
- When an electrical current is applied across the junction of two dissimilar metals, heat is removed from one of the metals and transferred to the other.
- Cooling is achieved electronically using the "Peltier" effect - heat is pumped with electrical energy.



Magnetic Refrigeration

- Magnetic refrigeration is also known as adiabatic demagnetization.
- It is based on the principle of magnetocaloric effect
- The refrigerant often used is a paramagnetic salt, such as cerium magnesium nitrate.
- A strong magnetic field is applied to the refrigerant, forcing its various magnetic dipoles to align and putting the degrees of freedom of it into a state of lowered entropy.
- A heat sink then absorbs the heat released due to its loss of entropy.
- The application of this is limited to cryogenics and research because only a few materials exhibit the desired properties at room temperature.

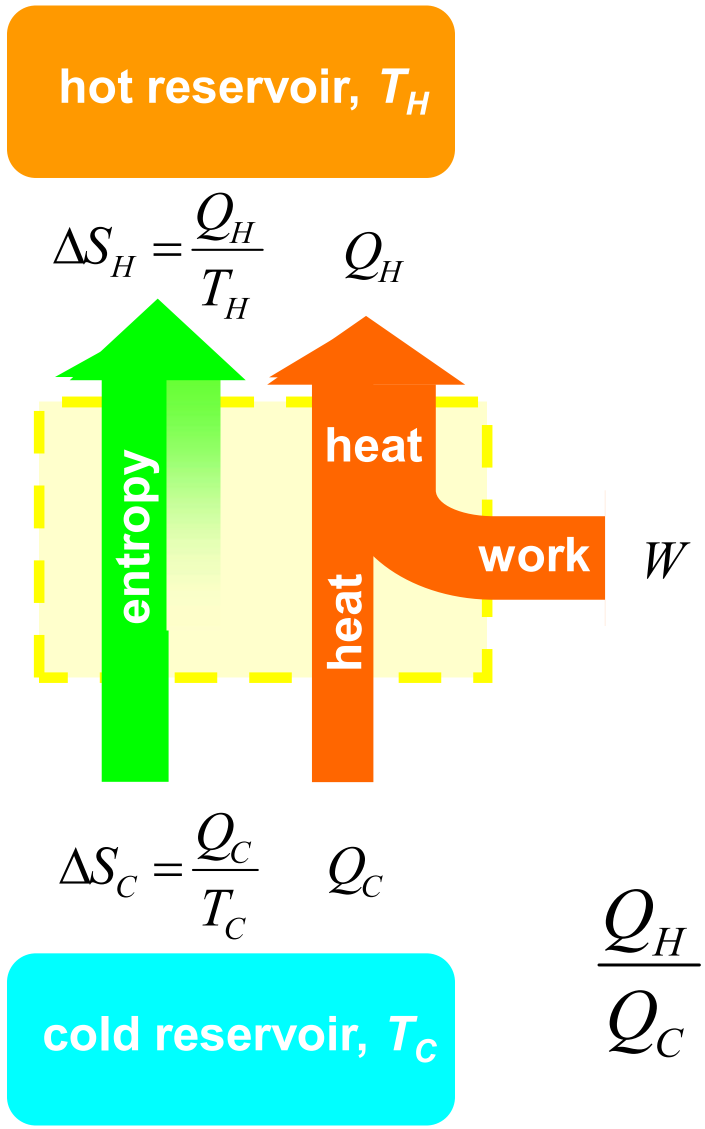
Thermoelectric refrigeration systems: It uses the Peltier effect to absorb heat at the junction between two wires made of different metals. These devices are lightweight, but not very efficient.



Refrigerators

- A ***refrigerator*** is also a form of a heat pump.
- It also moves heat from a cooler reservoir to a warmer reservoir by means of work supplied from some external source.
- It keeps food cold by pumping heat out of the cooler interior of the refrigerator into the warmer room.
- An electric motor or gas-powered engine does the necessary work.
- We can create a refrigerator by running a Carnot engine backwards: the gas extracts heat from the cold reservoir and deposit it in the cold reservoir.

More on Refrigerators



The purpose of a refrigerator is to make thermal energy flow from cold to hot. A *refrigerator* takes heat from a cold place (inside the refrigerator) and gives it off to a warmer place (the room). An *input* of mechanical work is required to do this. A refrigerator is essentially a heat engine operating in reverse.

$$COP \equiv \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C} = \frac{1}{Q_H / Q_C - 1}$$

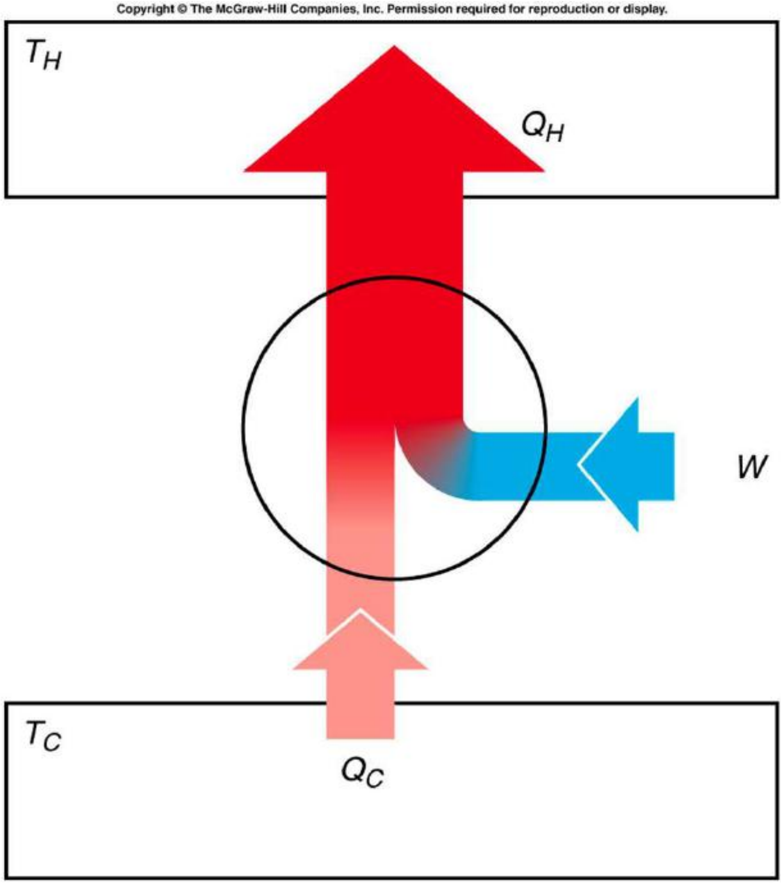
$$\frac{Q_H}{Q_C} \geq \frac{T_H}{T_C}$$

$$\boxed{COP \leq COP_{\max} = \frac{T_C}{T_H - T_C}}$$

Heat Pumps, and Entropy

- If a heat engine is run in reverse, then work W is done ***on*** the engine as heat Q_C is removed from the lower-temperature reservoir and a ***greater*** quantity of heat Q_H is released to the higher-temperature reservoir.
- A device that moves heat from a cooler reservoir to a warmer reservoir by means of work supplied from some external source is called a ***heat pump***.

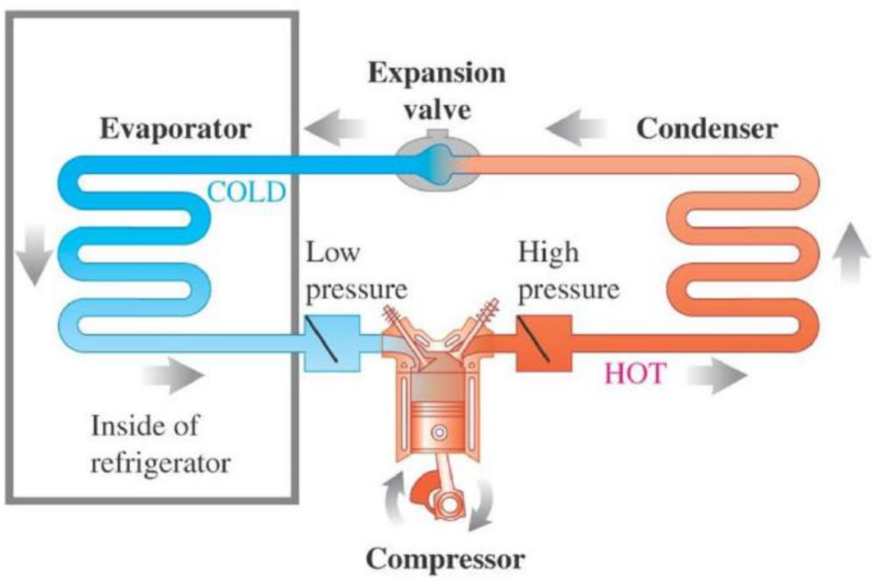
$$W + Q_C = Q_H$$



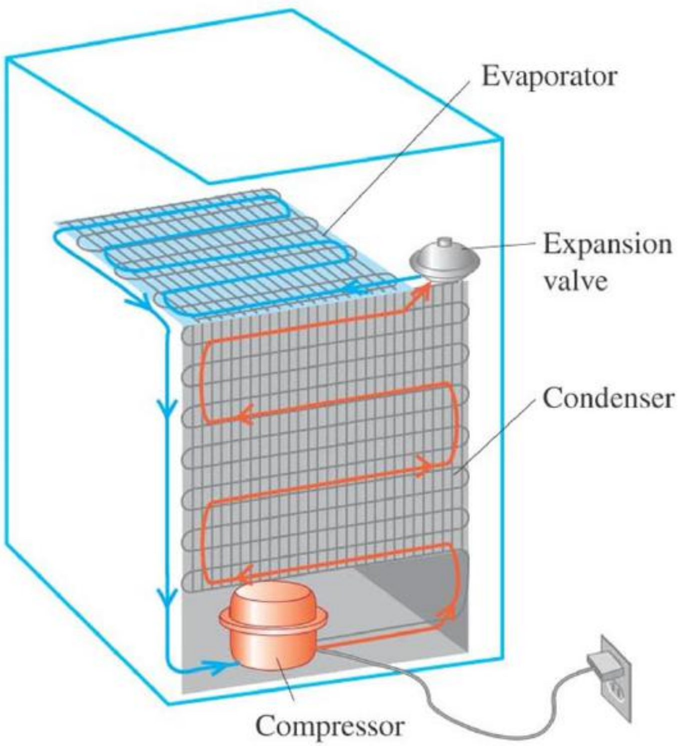
Mechanical Refrigerator

- Picture below shows the principle of the mechanical refrigeration cycle and how the key elements are arranged in a practical refrigerator.

(a)

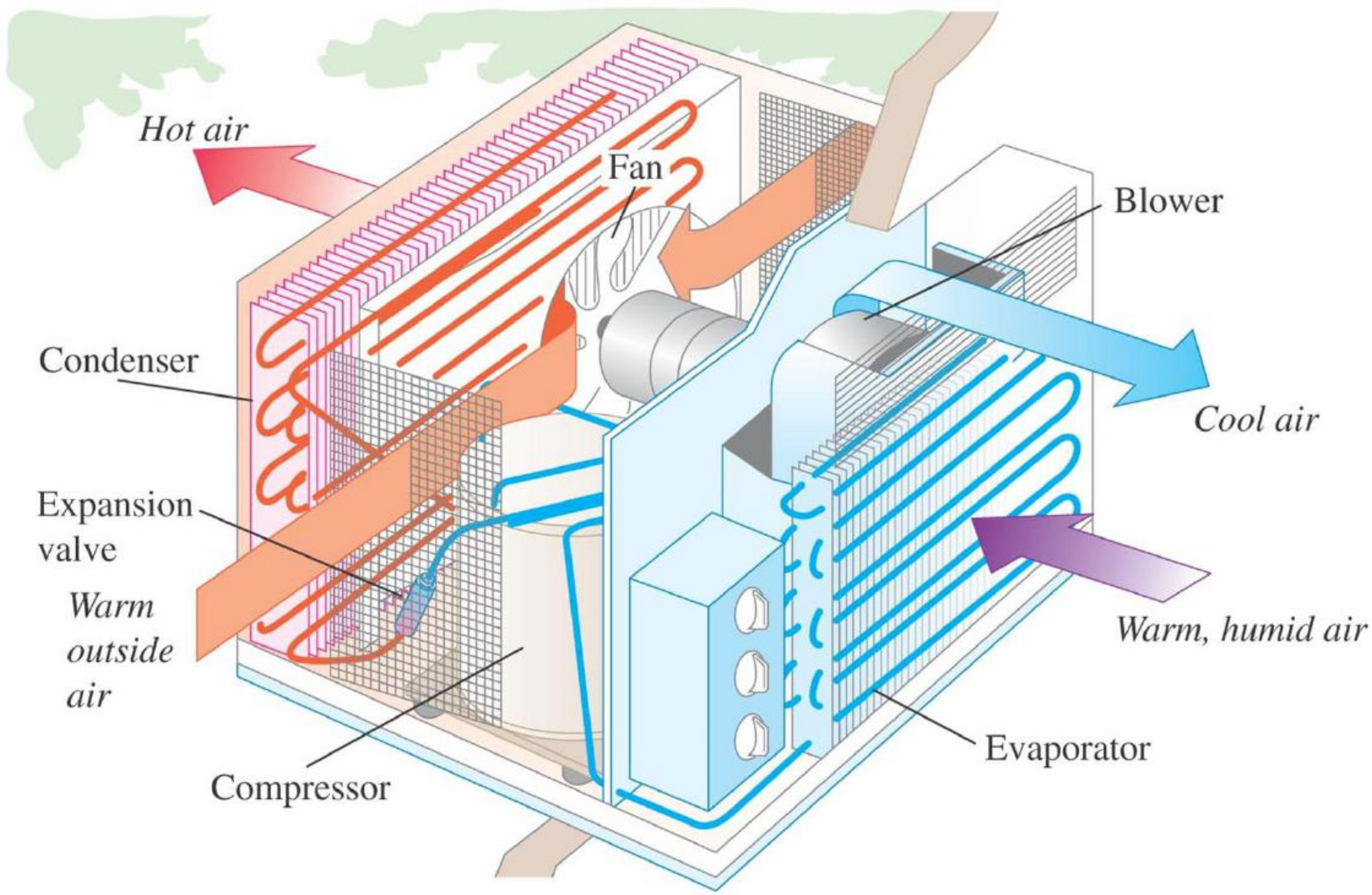


(b)



Mechanical air conditioner

- An air conditioner works on the same principle as a refrigerator. A *heat pump* operates in a similar way.



Applications of Refrigeration

- Central Air Conditioning
- Food Storage
- Making of ice
- Ice-Cream plants
- Industrial applications
- Hospital operation Theatre
- Research Laboratories
- Computer Rooms
- Production Of Rocket fuels(Cryogenic Fuel)
- Cryonics Project

Uses in Farm

- In order to reduce humidity levels and spoiling due to bacterial growth, refrigeration is used
- for meat production and dairy processing in farming today. Refrigeration systems are used
- heaviest in the warmer months for farming produce, which must be cooled as soon as possible
- in order to meet quality standards and increase the shelf life. Meanwhile, dairy farms refrigerate
- milk year round to avoid spoiling.

Differences between Refrigeration & Air Conditioning

- **Sources**
  - **Refrigeration**, in general, refers to any process where thermal energy is taken away from a place and transferred to a place with a higher temperature.
  - **Air conditioning** is a *type* of refrigeration where thermal energy is taken away from the air (typically in a room or a vehicle) in order to keep the air cooler.

Process

- **Refrigeration** is a process where thermal energy is transferred from a place with lower temperature to a place with higher temperature using energy, against the natural flow of heat.
- **Air conditioning** is a type of refrigeration which is used to cool large volumes inhabited by people.

Functions

- **Refrigeration** is concerned only with regulating the temperature of a volume of air.
- **Air conditioning** is concerned with not only maintaining the temperature of a volume of air, but also maintaining the humidity and purity.